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COCKER's

Decimal Arithmetick.

WHEREIN

ſhewed the Nature and Uſe of Decimal Fractions in the uſual Rules of Arithmetick, and the Menſuration of Plains and Solids.

ogether with Tables of Intereſt and Rebate for the Valuation of Leaſes and Annuities, Preſent or in Reverſion, and Rules for calculating thoſe Tables.

Whereunto is added,

is Artificial Arithmetick, ſhewing the Geneſis or Fabrick of the Logarithms, and their Uſe in the Extraction of Roots, the ſolving of Queſtions in Anatociſm, and in other Arithmetical Rules, in a Method not uſually practiſed.

ALSO

's Algebraical Arithmetick, containing the Doctrine of compoſing and reſolving an Equation; with all other Rules requiſite for the underſtanding that myſterious Art, according to the Method uſed by Mr. *John Kerſey* in his incomparable Treatiſe of ALGEBRA.

---

mpoſed by *EDWARD COCKER*, late Practitioner in the Arts of *Writing*, *Arithmetick*, and *Engraving*.

Peruſed, Corrected, and Publiſhed
JOHN HAWKINS, Writing-Maſter at St. *George's* Church in *Southwark*.

Cum tua non edas, cur hæc mea, Zoile, carpis?
Carpere vel noli noſtra, vel ede tua.

Μωμεῖσθαι μὲν ῥᾴδιον ἐστι, μιμεῖσθαι δὲ χαλεπο

The SIXTH EDITION.

LONDON:
nted for J. DARBY, A. BETTESWORTH, and F. CLAY, in *Truſt for* RICHARD, JAMES, and *BETHEL WELLINGTON*. M.DCC.XXIX.

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To the Right Worſhipful

Sir *Peter Daniel* Kt.

2436c

AND

***Peter Rich* Eſq;**

Aldermen of the

City of *LONDON*;

***Thomas Lee* Eſq;**

AND

***James Reading* Eſq;**

Juſtices of the Peace for the

County of *SURRY*;

JOHN HAWKINS

Humbly *Dedicateth this* TREATISE
*of ARITHME*TICK.

TO THE READER.

Courteous Reader,

I*N the Year of our Lord 1677. I publiſhed Mr.* Cocker'*s* Vulgar Arithmetick; *and therein gave an account of the ſpeedy Publication of his* Decimal, Logarithmical *and* Algebraical Arithmetick; *but other extraordinary Occurrences intervening, occaſioned its not ſeeing the Light before this time.*

By the Vulgar Part *the ingenious Learner may be qualified with ſo much of that moſt neceſſary Art of* Arithmetick, *as is ſufficient for the Management of Buſineſs in the greateſt Concerns of Trade and Commerce: And for thoſe ingenious Souls, whoſe active Fancies lead them to a further Scrutiny into the Study of the* Arts Mathematical, *was this Treatiſe compoſed; which will fairly lead them by the Hand, without any other Guide or Company, into the Contemplation of thoſe moſt ſublime Speculations; an Inheritauce entailed only upon the ingenious and induſtrious Sons of Art.*

The Method throughout the whole is plain, perſpicuous, and clear, and I hope will prove ſatisfactory to thoſe who ſhall ſeriouſly apply themſelves to the Rules, *Precepts, and Examples therein contained.*

The Uſe of Decimals *(in the Solution of Queſtions Arithmetical, and ſuch Geometrical as are neceſſary in the Menſuration of the moſt uſual Plains and Solids) is as plainly laid down as the Author or my ſelf could poſſibly contrive it; and particularly in all the Varieties of Intereſt, both ſimple and compound, with Tables and Rules for the Calculation thereof, according to the Method of ſeveral famous Authors, (who have beſtowed much Pains in the Management thereof) and eſpecially of that moſt famous, and no leſs laborious Mathematician of our Age and Nation, Mr.* John Kerſey, *whoſe Memory deſerves highly to be honoured by all the Profeſſors of this Science.*

The Geneſis, *or* Fabrick *of the* Logarithms, *and their Uſe in Arithmetick, is laid down after a different but more intelligible manner than hath hitherto been uſed by other Authors; and I hope the ſtudious Reader will receive that Satisfaction therein, which our Author earneſtly aimed at, or himſelf can expect.*

And as for the Algebraical Part, *I think there is nothing therein expreſſed that is ſuperfluous. nor any thing omitted that could be thought neceſſary to render it plain, perſpicuous, and clear; ſo that what other Authors, treating upon this Subject, have left intricate, and difficult to be underſtood, is here made obvious (by clear Demonſtration) to the meaneſt Capacity. Therefore, courteous Reader, if thou intendeſt to be a Proficient in the* Mathematicks, *begin chearfully, proceed gradually, and with Reſolution, the End will crown thy Endeavours with Succeſs: and be not ſo ſlothfully ſtudious, as at every Difficulty thou meeteſt withal to cry out,* Ne plus ultra; *for Pains and Diligence will overcome the greateſt Difficulty. To conclude, that thou may'ſt ſo read as to underſtand, and ſo underſtand* as to become a

Proficient, is the hearty Deſire of him who wiſheth thy Welfare, and the Progreſs of Arts.

From my School at
St. *George*'s Church in
Southwark, *October* 27.
1684.

J. HAWKINS.

* * *

Anixo guo Anamſiggino Jorammi Lehkeg *Lofoxrofeholrii Torgiemgig Im Xonifafu Diſohemiemgi Puwinaſig feho.*

G I H,

IT you Lepeag ſo zegfod gone ot youh glahe rougheg im lehugims fre toppodims feheafige you dipp frem ze fre xeffeh azpe ſo juws rod I rave glemf nime, amw it ny laimeg freheim nay ze Lhotifazpe fro fre luzpixk I rave ny digr, zuf it moſ, if ig moſ a ſoow frims mod imweew I wo gay go.

Oxfoz. 30. 1684.
Thop Pomwom.

G I H,

I an

Youb ruuz pe gehuamf,

JORM. RADKING.

The Advice of a Friend of the Author's to ſuch as are deſirous to attain to the Perfection of this moſt uſeful Art, &c.

YOU who peruſe this curious Work, obſerve
That he not meanly does of Men deſerve,
Whoſe ſtudious Labour brought it to an end,
And as his Maſter-piece did it commend
To thoſe who are deſirous to employ
Their Time the beſt of curious Arts t'enjoy:
An Art by which Man's Fortune's often rais'd,
An Art by all who trade or traffick prais'd;
An Art, or an Acquirement, whoſo wants,
His Buſineſs (if important) quickly faints:
'Tis what's ſo uſeful, that not to be known,
Wou'd ruin each Man's Occupation.
Therefore let thoſe who fain would riſe, embrace
This, and Preferment they have in their Chace.
Long ſince it was invented for our good,
Yet till late days not rightly underſtood;
And not till now to its Perfection brought,
Tho many ways with tedious Trouble ſought.
In theſe choice Pages all is to be found
That does concern the Subject: Theſe do bound
The largeſt Field of true Arithmetick,
No Numbers wanting that Mankind would ſeek.
The curious Artiſt with a ſearching Eye,
Altho turned Critick, here no Faults can ſpy;
Or if there any be, they are ſo ſmall,
That nearly they reſemble none at all:
For all who have perus'd it, have confeſt,
That of this kind this much exceeds the reſt.

J. A. Teacher of the Mathematicks

In Commendation of his Friend Mr. John Hawkins, *upon the publication of this Treatiſe.*

THE learned *Chymiſt* can't more truly ſay,
He can the unſeen Powers of Herbs diſplay;
Or by diſſolving their external Face,
Bring ſubtile Spirits, Sulphurs, Salts in place;
Exalt their intern Energy, ſublime
From putrefactive *Nunc*, eternal Time;
Than you by *Algebra* and Numbers prove
Th' *Equations* true of all the Orbs above.
You, by Subtracting, Add, and do Divide,
The ſelf-ſame way by which you multiply'd:
From Numbers ſmall you mighty Powers make,
And from the ſame the Quinteſſence you take.
By Infinites, you finite Numbers bind,
By things unknown, you unknown things do find.
Proportions you find out, and as exact
As *Chymiſts*, you *Equations* do extract.
Thus you the Powers of Numbers do unfold,
And, like them, change baſe Metals into Gold.
The Spring's unſeen; for no Man fully knows
From whence the ſacred Source of Number flows.
But my poor Mite you need not, nor my Praiſe,
To you my Lines can't laſting Trophies raiſe.
Nor need your Numbers my unlearn'd Defence,
Numerick Truth in its abſtracted Senſe
Derives its Spring from an eternal Font,
Without Beginning, endleſs in Account:
The univerſal World it does comprize,
It no *Beginning had,* nor ever dies.
All things i'th' Sphere of ſacred Numbers ſtand,
The moſt immenſe, and the minuteſt Sand;

Heaven, Earth, the Seas, their Furniture ſubmit
And their numerous Offspring flows with it:
It meaſures Place and Time; in Shades of Night
It ſees no Darkneſs but illuſtrious Light.
Both Life and Death to it the ſame appear,
All Subjects are within its mighty Sphere.
Thus my Affections, Friend, make me intrude,
Tho with unpoliſhed Lines, and Numbers rude.
On ſuch a Theme! who could forbear to ſing?
To ſacred Fire, who ſhould not Incenſe bring?
Inſpired by thy Art, my ſublime Muſe
Th' eternal Truth of Numbers ſhall diffuſe:
Whilſt I applaud the Object of thy Pen,
The unknown Depths of *Algebra* and *Men*.
Here fix thy Pillars; in this Art aſpire
To light our Tapers with Celeſtial Fire.
In the ſame Zeal proceed; thy Numbers fit
With ſpeaking Symbols to the meaneſt Wit.

October 13. Yours and Truth's Servant,
1684.

W. SALMON, *M. P.*

* * *

To the ingenious Author of theſe Decimals *and* Algebra, *the famous Arithmetician, and his ſingular Friend by Choice,* Edw. Cocker.

WITH Admiration ſtruck, I here ſhould pauſe,
Not daring truſt my Muſe in your Applauſe;
Whoſe Fame already has ſo loud been ſung
By the Divineſt of the ſacred Throng:
Did not your rich and matchleſs Art inſpire
My drowſy Soul with a Poetick Fire.

For who in Silence can remain, that views
A Subject worthy, ſuch as can infuſe
A moving Rapture of the firſt degree
Into a Breaſt, before from *Phœbus* free?
So great a Maſter-piece as this, Mankind
In all their tedious Search could never find.
Arithmetick's here to Perfection brought;
Here's to be found what never yet was taught;
The curious Work ſo to the Life is drawn,
That all beſides are like the Morning's Dawn,
Compar'd to Day's clear Face when *Sol* ſits high
In his Meridian Throne; in vain ſome try
To reach your Art's Perfections, but the more
Their Genius flags when to your Heights they'd ſoar;
And at the beſt their Labours do appear
Foils to make your Diamonds ſhine more clear.
This Book of yours bears Record of your Fame,
And to all Ages will transfer your Name.
For why, your boundleſs Wit, and curious Pen
Do ſtill write you the Miracle of Men.

R. N. Philomath.

* * *

In Memory of the deceaſed Author, Mr. Edward Cocker; *and in Praiſe of this* (Poſthumal) *and his former Works.*

WHO e'er of old to the common Good ap-ply'd
Their Minds or Means, but they were deify'd?
And *chiefly thoſe who* new Inventions found;
Bacchus for Wine, Ceres who till'd the Ground,

Whoſe Fames and Memories will ever laſt,
Till the late Evening of the World be paſt.
Now this our Author by his fluent Pen,
In all Fair-Writing did exceed moſt Men;
And tho in Knotting, *Gething* did do well,
Cocker in That did *Gething* far excel:
And not with Pen alone, on Paper he
Could write and knot, but with the Graver too
On Copper-Plates he did all Men out-do.
What curious Copy-Books and Sculptures are
Extant in Print of his, which may compare
With any in the World? and no one Hand
Had Pen and Graver both at ſuch Command.
But leaving now his Writing, take a View
Of his Arithmetick, whoſe Books are two;
The one of Plain (or Vulgar Numbers) made
Fit for young Scholars, and for Men of Trade.
This other's in three Parts, more General:
I. Of artificial Numbers *Decimal*,
II. The ſecond's Numbers *Logarithmical*,
III. The third by Symbols *Algebraical*;
All fraught with Queſtions *Enigmatical*,
Of all Arithmeticks the *General*.
Conſider now what pains the Author took,
And praiſe him as thou benefits by his Book.
But ſince the Author's dead, I'll not defer
To praiſe and thank the ingenious Editor.

W. LEYBOURN.

* * *

Ad Amicum ſuum dilectiſſimum Dominum Joannem Hawkins, *de opere hoc, mira cum eruditione cum induſtria, correcto & reviſo.*

Ἐγκωμιαστικόν.

SI meruit Laurum, qui Lauro ſcribere digna
Novit, & ad ſophiam pandere callet iter;
Quod meruit qui non tantùm novit, ſed & ipſe
Præſtitit ingenio, vix facienda, ſuo?
Laura conveniunt non tantum ſerta capillis,
Aurea ſed potius, docte, corona tuis.
Aurum vos illi divites concedite, Laurum
Dent alii, nemo ſe meruiſſe neget.
Quod ſi nec Lauri noſtro tribuetis honorem
Autori, planè quem meruiſſe liquet,
Auri nec ſummam dabitis quam quiſque fatetur
Ingenii meritum non minus eſſe ſui:
Non Mæcenates eritis, non eſſe patronos
Poſſe putat, quorum tam ſit avara manus;
Sed potius (veniam petimus, dabimuſque viciſſim)
Nominat ingratos vos (ſcio cur) aſinos.

JOANNES ROBINSON.

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COCKER's

Decimal Arithmetick.

CHAP. I.

Notation of Decimals.

I. WHAT Arithmetick, and the Subject thereof (*viz.* Number) is, I have largely defined in the firſt Chapter of my Vulgar Arithmetick, in which Treatiſe I have applied the Species of Numeration to the various Rules of Vulgar Arithmetick, both in Integers and Fractions, for the Solution of various Practical Queſtions ſolvable thereby, by ſuch plain and eaſy Rules as many Years Experience in the Practice thereof had made me capable of; and which I *hope might render it* intelligible and ſerviceable *to the meaneſt* Capacity.

And in this I ſhall ſhew you the Uſe of Decima Fractions in all the Rules of Arithmetick, but prin cipally in the ſolving Queſtions of Intereſt an Rebate, according to ſeveral Rates of Intereſ both ſimple and compound, with the true Valua tion of Leaſes and Annuities, either Preſent c in Reverſion, and likewiſe their Uſe in the calcu lating of Tables for that purpoſe, *&c.*

II. In Decimal Fractions we ſuppoſe the Unit c Integer to be divided into ten equal parts, an each of theſe ten parts are again divided int ten other equal parts, ſo that then the Unit c Integer will be divided into a hundred equa parts; and then again each of thoſe hundre parts is ſuppoſed to be divided into ten other e qual parts, ſo that then the Unit or Integer wi be divided into a thouſand equal parts, *&c.* an ſo by decimating the firſt and ſubdecimating th ſecond, we proceed *ad infinitum.*

III. And hence it is evident that a Decimal Frac tion is always either ſo many tenths, or it is ſ many tenths of $\frac{1}{10}$, or 'tis ſo many tenths of $\frac{1}{}$ of $\frac{1}{10}$, or ſo many tenths of $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$, & which compound Decimal Fraction being reduc'd as is taught in the ſixth Rule of the nineteent Chapter of my Vulgar Arithmetick, will give i equivalent ſimple Decimal Fraction; as for exam ple, $\frac{8}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ is .008, that is $\frac{8}{1000}$; an hence it follows, that always a Decimal Fractic hath for its Denominator an Unit with a Cyphe or elſe Cyphers, annexed to it on the right han *viz.* either 10, or 100, or 1000, or 10000, c 100000, *&c. ad infinitum.*

IV. In Decimal Fractions the Denominator *never expreſs'd*, but may at firſt ſight be under *ſtood by the* number of Places contained in th

Numerator; the Denominator being always an Unit with as many Cyphers annexed to it, as there are real Places in the Numerator; as 8 being a Decimal $\frac{8}{10}$ *viz.* its Denominator is an Unit with one Cypher annexed to it, and $\frac{85}{100}$ is thus written, .85 and $\frac{140}{1000}$ thus written, .140; but if the Numerator of a Decimal Fraction conſiſteth not of ſo many Places as there are Cyphers in the Denominator, then ſuch Defect is ſupplied by prefixing ſo many Cyphers before the Numerator, (*viz.* on the left hand) as there are Places deficient: As for Example, $\frac{8}{100}$ if it were only ſet down thus, (.8) then it would be but $\frac{8}{10}$; but by prefixing a Cypher before it thus, (.08) it is $\frac{8}{100}$. And $\frac{8}{1000}$ is thus expreſſed, (.008) and $\frac{25}{1000}$ is thus written, (.025) and $\frac{25}{10000}$ thus, (.0025) *&c.*

V. A Decimal Fraction being written without its Denominator, is known from a whole Number, *by having* a Point or Prick prefixed before it, thus, .25 is $\frac{25}{100}$; but if it had been expreſs'd without a Point thus, (25) it would have ſignified ſo many Units. The ſame is to be obſerved in mixed Numbers, for 29 $\frac{16}{100}$ being written decimally will ſtand thus (29.16) and 48 $\frac{25}{1000}$ thus, (48.025) and 48 $\frac{28}{10000}$ thus (48.0028) and the like of any others.

But ſome Authors diſtinguiſh Decimals from whole Numbers, by prefixing a Virgula, or perpendicular Line before the Decimal (whether it be alone, or joined with a whole Number) thus |8 is $\frac{8}{10}$, and |025 is $\frac{25}{1000}$; and 29|16 is 29 $\frac{16}{100}$, *&c.* Others expreſs the ſame Decimal Fraction and mixed Numbers *thus, viz.* |8 $\underline{|025}$ 29 $\underline{\backslash 16}$, *&c. Others with a Point* over the place of Units *in the whole Number, and* then the former Fractions and mix'd Number will be thus writte *viz.* 8,025 29,16, and the like of others. An ſome Authors again put Points over all the Place or Figures in a Decimal Fraction thus: $8,\dot{0}\dot{2}$ $29,\dot{1}\dot{6}$ $48,\dot{0}\dot{2}\dot{5}$, *&c.* but being written accordin to the firſt Direction, I conceive they may be mo fit for Calculation.

VI. As whole Numbers do increaſe their Value in a decuple Proportion, by annexing a Cypher or Figure to the Place of Units; ſo by prefixing a Cypher or Figure on the left hand of Decimal, ſo as actually to take place in the Decimal, its Value is decreaſed in a ſubdecuple Proportion: ſo the Number 4, by annexing a Figur or Cypher to it, is increaſed from 4 to 40, *&* But if 4 had been a Decimal, *viz.* 4. and if ther had been 0 prefixed before it on the left hand its Value had been decreaſed from $\frac{4}{10}$ to $\frac{4}{100}$ o 4; and by prefixing 5 it is 54; and ſtil by prefixing more Figures or Cyphers, its Valu will decreaſe in the ſame Ratio *ad infinitum.*

VII. And as Cyphers being prefixed before whole Number, *viz.* on the left hand thereof, d neither increaſe nor decreaſe its Value; (for 4, 04 and 004 being Integers, do ſtill retain one and th ſame Value;) ſo a Decimal, by having a Cypher or Cyphers annexed to the right hand thereof have not their Value either increas'd or decreas'd Whence it is evident, that all Decimal Fraction may be reduc'd to an equal Denomination at firſt ſight; for ſuppoſe .15 and .008 and .73465 wer *Decimals given* to be reduced to one Denomination. *In this* caſe I conſider that the Denominato

to

given Decimal conſiſting of the moſt Places
›oo, and .15 and .008, whoſe Denomina-
: 100 and 1000, may be reduc'd to Decimals
ſame Value, having likewiſe 100000 for
enominator, by annexing ſo many Cyphers
ight hand of the Numerators, as (according
ourth Definition foregoing) may make each
ı to have 100000 for a Denominator; ſo
ı be .15000, and .068 will be .06800.

› As the order of places in whole Numbers
the right hand to the left, ſo the order of
ı a Decimal Fraction is from the left hand
right; the firſt Place being accounted
ırts of an Unity, and by ſome it is called
the ſecond Place is ſo many hundredth
f Unity, or it is called Seconds; the third
ſo many thouſandth Parts of Unity, or it
l Thirds, *&c.* which will more fully ap-
the following Table.

ble of the Notation of Integers and Decimals.

)f Units.						Of Unity.								
C of Thouſ.	X of Thouſ.	Thouſands	Hundreds	Tens	Units	Tenth parts	Hundr. parts	Thouſ. parts	X Thou. parts	C Thou. parts	Mill. parts.	X Mill. parts	C Mill. parts	Th. Mill. parts
3	7	5	8	6	4	8	2	3	0	5	6	3	5	4
Sixth place	Fifth place	Fourth place	Third place	Second place	Firſt place	Primes	Seconds	Thirds	Fourths	Fifths	Sixths	Sevenths	Eighths	Ninths
Integer						Decimals.								

In the foregoing Table is given a mix'd Number of Integers and Decimals ; the Integers bein ſeparated from the Decimals by a Point or a Prick according to the fifth Definition before-going ; ſ that 384375864 ſignify ſo many Integers, or Units and 823056354 ſignify ſo many Parts of Unity the Figure 8 in the firſt place being ſo man tenth Parts of Unity ; and the next Figure, *viz* the Figure 2, is ſo many hundredth parts of Unity, *&c.*

So in the Decimal Fraction .4378 the Figur 4 poſſeſſeth the firſt place, and is 4 primes, o four tenths of an Unit ; and 3 the ſecond Figur is called 3 ſeconds, or three hundredth parts o an Unit ; and 7, the third Figure, is called ſeve thirds, or ſeven thouſandth parts of an Unit and 8, the fourth Figure, is called eight fourth or eight ten thouſandth parts of an Unit, *&c.*

Whence it appeareth, that every Place in a Decimal Fraction being conſidered apart by itſel without any reſpect to the reſt, will of itſe make a particular Decimal Fraction ; ſo in th laſt mentioned Decimal Fraction, *viz.* .4378, eac Place being conſider'd by itſelf, will make thei following Decimal Fractions, *viz.* .4 .03 .00 and .0008 or $\frac{4}{10}$ $\frac{3}{100}$ $\frac{7}{1000}$ and $\frac{8}{10000}$; whic Fractions being added together, according t the Rules of Addition of Decimals hereafter delivered in the third Chapter, their Sum will b .4378, which is the given Decimal of which the are compos'd.

IX. A Decimal Fraction is expreſſed by ſom Authors by Primes, Seconds, Thirds, Fourth *&c.* As if this Decimal .748 were to be expreſ*ſed, they* ſay it is ſeven Primes, four Second *and eight Thirds.* Others there are who expre it thus, *viz.* ſeven hundred forty eight thirds; but the moſt approved way to expreſs or read a Decimal Fraction, is according to the Method of reading a Vulgar Fraction, and to give it the Denomination of the Figure in the laſt Place of their Decimal, and then the Decimal .748 will be thus read, *viz.* ſeven hundred forty eight thouſandths; and .036 is thus read, thirty ſix thouſandths, and ſo of any other. This Chapter being well underſtood, all the Parts of Numeration, *viz.* Addition, Subſtraction, Multiplication and Diviſion of Decimals will prove very eaſy.

CHAP. II.

Reduction of Decimals.

To reduce a given Vulgar Fraction to a Decimal, that ſhall be equivalent thereto.

I. WHEN in any Arithmetical Operation your Work is ſo mingled with Vulgar Fractions, as to render it tedious, or difficult, the beſt Remedy you can have, is to reduce your Vulgar Fraction or Fractions into a Decimal or Decimals; which having done, the Work will be as eaſy in every reſpect, as if you had to do with nothing but whole Numbers: which you may effect by the following Proportion, *viz.* As the Denominator *of the given* Vulgar Fraction is to *its Numerator;*

So is an Unit, with ſo many Cyphers as yo intend your Decimal ſhall have Places, to th Decimal required.

So if the Fraction to be reduced were $\frac{3}{4}$, an you would reduce it to a Decimal, conſiſting 4 Places, I ſay, the Proportion is,

As 4 (the Denominator of the given Fraction
Is to 3 (its Numerator)
So is 10000 (the Denominator of the Decim required)
To .7500 (the Decimal required.)

So that I conclude $\frac{3}{4}$ will be reduced to its equivalent Decimal .7500 or .75; for Cyphe on the right hand of a Decimal do neither increaſe nor diminiſh its Value, by the ſevent Definition of the firſt Chapter.

Now, according to the foreſaid Proportio it is evident, that if to the Numerator of an Fraction given to be reduced to a Decimal, yo annex as many Cyphers as you intend its equivalent Decimal ſhall have Places, and then divid it by its Denominator, the Quote will be th Decimal required.

So let there (again) be given $\frac{3}{4}$ to be reduce (as before) to a Decimal of 4 Places; in orde thereunto I annex 4 Cyphers to the Numerator : and it makes 30000, which I divide by the Denominator 4, and it quotes .7500 or .75, for th Decimal equivalent to the Vulgar Fraction $\frac{3}{4}$.

Note, That all Vulgar Fractions cannot be reduced to Decimals, having exactly the ſame Value, altho they may come infinitely near; and th more Places that you make your Decimal to conſiſt of, ſo much the nearer doth it come to th *Truth, but* 4 or 5 Places is exact enough for th *moſt Operations*: ſo if it be required to redu the Vulgar Fraction $\frac{9}{11}$ to a Decimal of [illegible] Places, it will be found to be .818 which is not exact, but yet it wanteth not $\frac{1}{10000}$ part of an Unit of the Truth; and if you make it .8182, it will be ſomewhat more than the Truth.

Again, if you annex 5 Cyphers to the Numerator, and ſo make the Decimal conſiſt of 5 Places, it will then be .81818, yet it will want of the Truth, but not ſo much as when it had but 4 Places, for now it will not want $\frac{1}{100000}$ part of an Unit of the exact Truth; and if you make it to be .81819, it will then exceed the Truth. Thus by increaſing the Number of Places in the Decimal, you may come infinitely near the Truth, but never find a Decimal exactly equivalent in many Caſes.

Note alſo, That if after you have reduced your Vulgar Fraction to a Decimal, according to the foregoing Rule, there be not as many Places in the Decimal, as you annexed Cyphers to the Numerator of the given Vulgar Fraction, then you are to ſupply ſuch Defect by prefixing ſo many Cyphers on the left hand of the ſignificant Figures, as there are Places wanting, according to the fourth Rule of the firſt Chapter.

So if $\frac{11}{941}$ were given to be reduced to a Decimal of any Number of Places, as ſuppoſe 6; in order to it, I annex 6 Cyphers to the Numerator 11, and it makes 11000000 for a Dividend, which divided by 941, it quotes 11689, which conſiſteth but of 5 Places, but it ſhould have 6 Places; wherefore to make it compleat, I prefix a Cypher before it, and it makes .011689 for the true Decimal required; and if it had been required to *conſiſt of 4 Places*, then I annex 4 Cy*phers to the Numerator;* yet after Diviſion is ended, there will be but three Places in the Quotient, *viz.* 116; therefore to make it conſiſt of 4 Places, I prefix a Cypher before it, and it makes .0116 for the Decimal ſought. Again, let there be given $\frac{32}{5642}$ to be reduced to a Decimal of (ſuppoſe) 5 Places, it will be found to be .00567; and $\frac{14}{64857}$ will be reduced to .000215.

To reduce the known Parts of Money, Weight, Meaſure, Time, *&c. to Decimal Fractions.*

II. Hence it is evident, that the known Parts of Money, Weight, Meaſure, Time, and Motion, *&c.* may be reduced to Decimal Fractions of the ſame Value, or infinitely near it; for if (in Money) a Pound Sterling be an Integer, whatſoever is leſs than a Pound, is either a Part or Parts of the ſame; and when you know what part or parts thereof it is, you may reduce it to a Decimal of the ſame Value, by the firſt Rule of this Chapter: ſo if you would know what is the Decimal of a Pound Sterling equal to 7 Shillings, conſider that 7 *s.* is $\frac{7}{20}$ of a Pound, and by the ſaid Rule, the Decimal anſwering thereto is .35 *l.* And if I would know the Decimal equal to 3 *d.* I conſider that 3 *d.* is $\frac{3}{12}$ of $\frac{1}{20}$ of a Pound, or $\frac{3}{240}$ of a Pound, and the Decimal equivalent thereto will be found (by the ſaid Rule) to be .0125. Likewiſe if there were given 7 *s.* 3 *d.* to find the Decimal equal thereunto, firſt I conſider, that 7 *s.* 3 *d.* is 87 Pence, which is $\frac{87}{240}$ of a Pound, and the Decimal equal thereto will be found to be .3625 *l.*

In like manner, if it were required to find *the Decimal* of a Pound Troy-weight equivalent *to 6 oz.—12 pw.* I firſt find that 6 oz.—12 pw.

mak

make 132 *pwts.* which is $\frac{132}{240}$ of a Pound Troy-weight, and the Decimal equivalent thereunto will be found to be .55 by the ſaid firſt Rule of this Chapter. The like is to be underſtood in the reducing of any of the known Parts of Coin, Weight, Meaſure, *&c.* into Decimals.

To find the Value of a Decimal Fraction in the known Parts of Money, Weight, Meaſure, *&c.*

III. When you would find the Value of a Decimal Fraction in the known Parts of Coin, Weight, Meaſure, Time, Motion; or the like, obſerve the following

RULE.

Multiply the given Decimal by the Number of Parts in the next inferior Denomination that are equal to an Integer in the ſame Denomination with the given Decimal, and ſee how many Places are in the Product more than were in the ſaid given Decimal; and cut ſo many off from the left hand with a Daſh of your Pen, and thoſe Figures ſo cut off, are the Value of the ſaid Decimal in the next inferior Denomination to it, and the Figures (if there be any) remaining are the Decimal of an Integer in the ſaid Denomination, and may be reduced as low as you pleaſe by the ſame Rule, as in the following Example.

Let it be required to find the Value of this Decimal of a Pound Sterling, *viz.* .7635.

Firſt, I multiply the given Decimal by 20, and the Product *is 152700, which* is of 6 Places, and

the given Decimal is but of 4 Places, whe
I cut off 2 Figures at the left hand, *viz.* 15,
is ſo many Shillings ; now whe
ſaid 15 is cut off from the reſt,
are yet remaining 2700, which I
tiply by 12, to find out the
thereof in Pence, and the Prod
32400, which conſiſting of 5 F
I cut off one Figure (*viz.* 3) fro
left hand, which is ſo many P
ſo that I conclude the Value o
given Decimal to be 15 *s.*—3*d.* and the re
ing Figures, *viz.* 2400 are the Decimal Pa
a Peny, which, becauſe they do not amou
the Value of a Farthing, I do not reduc
lower. See the Work in the Margin.

7635
20
———
15|2700
12
———
3|2400

So if .6847 *l.* be given, and it be requir
find its Value, if you work as is before dir
you will find it to be 13 *s.*—08 *d.*—1.312 *qu*
And 374 *l.* being ſo reduced, you will find
make 7 *s.*—05 *d.*—3.040 *quarters.*

In like manner, if it were required to r
this Decimal of a Pound Troy-weight,
.84576 *l.* into known Parts, firſt, I multi
by 12, and it produceth 101
from which I cut off the tw
Figures of the left hand, (*viz*
for Ounces, and the remainin
gures, which are 14912 do I
ply by 20, and the Product is 29
from which I cut off the firſt F
(*viz.* 2.) which is two-peny-w
and 98240 remaineth, which I
tiply by 24, and the Produ
2357760, which is 23.57760 Gr
ſo that I conclude the Value

.84576
12
———
10|14912
20
———
2|98240
24
———
392960
196480
———
2357760

given Decimal .84576 Pound Troy-weight to be 10 *oz.*—02 *pw*—23 *gr.* .57760. The ſame is to be obſerved in finding the Value of any other Decimal whatſoever, whether of Coin, Weight, Meaſure, Time or Motion.

I might here have added Tables of Reduction, ſhewing the Decimal Fractions of any of the Parts of Money, Weight, *&c.* as divers Authors have already done; but becauſe they are, tho uſeful, ſeldom made uſe of, and partly by reaſon of the Eaſe in finding the equivalent Decimal of any Fraction whatſoever, according to the Rules herein delivered, I ſhall forbear it.

IV. There is a briefer way of diſcovering the Value of a Decimal of a Pound Sterling, *viz.* the Figure which ſtandeth in the firſt place of Decimal, (*viz.* in the Place of Primes) being doubled gives you the number of Shillings; then let the Figure poſſeſſing the ſecond Place of the Decimal, (*viz.* the Place of Seconds) be eſteem'd ſo many Tens, and the Figure in the third Place account ſo many Units, which ſaid Tens and Units being accounted one intire Number, and made leſs by one, will be ſo many Farthings, which ſaid Shillings and Farthings are the Value of the given Decimal; but if the Figure in the ſecond place be 5, or elſe exceed 5, then reckon one Shilling for that, and for the Exceſs above 5, eſteem every Unit 10, as before.

Example 1.

What is the Value of .7365 *l.*

The *Figure 7 (ſtanding* in the place of Primes) *being doubled, gives 14,* which is ſo many Shillings in the place of Seconds, is placed under 8 and which are likewiſe Seconds, or Hundredths, an the Figure 3 in the place of Thirds or Thou ſandths is placed under 7, which is alſo ſo man Thirds, *&c.* The ſame order is to be obſerv' in placing of the Decimals of mixed Numbers t be added; as ſuppoſe there were given theſe fo lowing mixed Numbers to be added together, *vi* 168.3572 and 36.864 and 7.42 and .6: Now i order to the finding out their Sum, I diſpoſe them in order one under the other as followet Where you may obſerve that the whole Num bers themſelves, or integral Parts of the give mix'd Numbers are placed one under the othe as is directed in Addition of whole Number without any reſpect at all to the Decimals a nexed to them; and the Decimals are placed u der each other, according to the Directions g ven in the laſt Rule, without any reſpect had the Integers properly belonging to them.

168.3572
36.864
7.42
.6

III. Having placed your given Decimals in c der, according to this Rule, draw a Line und them, as in Addition of whole Numbers, und which Line you are to place their Sum; then pr ceed in your Work in every reſpect, as in t Addition of Integers, beginning at the rig hand, and ſo proceeding thro' the Decimals wit out any regard to them as Decimals, but as *they were all* whole Numbers: As for Examp *let us take* the Decimals given in the firſt Exa ple of the laſt Rule foregoing: and firſt I put down *6* under the Line, becauſe there is no other Figure or Number to add to it; then I proceed to the next, ſaying 2 and 4 make *6*, which I *alſo* ſet down in order under the Line: then I ſay 3 and 7 make 10, ſo I ſet down 0, and carry 1 to the next, ſaying 1 that I carry, and 2 and 8 make 11, for which I ſet down 1, and carry 1 to the next, ſaying 1 that I carry and 8 and *6* make 15, which I put in its *place* under the Line, becauſe it is the laſt. And becauſe the Figure 5 ſtandeth under the Place of Primes, I put a Point before it, that is to ſay, between 1 and 5, and the Work is finiſhed; the Number 1 being an Integer, and the reſt a Decimal, whereby I find the Sum to be 1.51066; that is 1 Integer, and .51066 parts of an Integer.

.00746
.0832
.62
.8
———
1.51066

After the ſame manner, if the mixed Numbers n the ſecond Example of the foregoing Rule were given to be added, their Sum will be found o be 213.2413, that is, 213 Integers, and .2412 lecimal Parts of an Integer, as you may ſee by he following Work.

168.3572
36.864
7.42
.6
———
213.2412

Other Examples for the Learner's Practice may be ſuch as *follow.*

42.608	4.368	748
16.07	7.573	36.72
26.009	.724	9.564
42.8	.56	.7358
——	——	——
127.487	13.225	795.0198

CHAP. IV.

Subſtraction of Decimal Fractions.

I. WHEN two Decimal Fractions are given, and their Difference or Exceſs is required, you muſt place them (in order to the Work) as you were taught in the foregoing Chapter of Addition, and the Operation is the very ſame in every reſpect as in Subſtraction of whole Numbers of one Denomination; beginning at the right hand, as in the following Example.

Let it be required to ſubſtract the Decimal .634 from the Decimal .728; in order to the Work I put them one under the other, *viz.* the biggeſt uppermoſt, and take each Figure in the lowermoſt out of its correſpondent Figure in the uppermoſt, putting their reſpective Differences in order below the Line; and I find, when I have finiſhed the Operation, the Remainder, or Difference, *to be* .094 as by the Work appeareth.

.728
.634
——
.094

In like manner if the mix'd Number 42.347 were given to be ſubſtracted from the mixed Number 76.123 I place them in the ſame Order as is directed in Addition before going, only with this Caution, be ſure to place the biggeſt uppermoſt; then proceed to take each Figure in the lowermoſt out of its correſpondent Figure in the uppermoſt, as if they were whole Numbers: and having finiſhed the Work, the Remainder, or Difference, will be found to be 33.776, as *you* ſee it done in the Margin.

76.123
42.347
———
33.776

When the Decimal given to be ſubſtracted does not conſiſt of an equal Number of Places, ſuch Defect muſt be ſupplied by annexing Cyphers, or ſuppoſing as many Cyphers to be annexed (as are wanting) on the right hand, and then the Work will be as in the former Examples.

Example.

Let it be requir'd to ſubſtract .037486 from .84; now becauſe .84 hath but 2 Places, and the other hath 6, I ſupply that defect by annexing 4 Cyphers thereto, as in the Margin; and the Work being finiſhed, I find the Remainder or Difference to be .802514.

.840000
.037486
———
.802514

The ſame is to be obſerved when a Decimal Fraction or mixed Number is given to be ſubſtracted from a whole Number; as ſuppoſe 15.486 were given to be ſubſtracted from 64, becauſe there is no Decimal annexed to 64; you are to ſupply the decimal *Places with* Cyphers, *and then proceed in the Work* as before is directed;

64.000
15.486
———
48.514

that is, 252 Integers, and .7932 decimal Parts o an Integer.

A ſecond Example may be of a mix'd Numbe given to be multiplied by a decimal Fraction as thus, Let it be required to multiply 38.574 by .00463; I prepare the given Numbers for Operation, as is before directed, and having finiſhed the Work, I find the Product to amount t .178600398. Then to find the true Value of th Product, I conſider the Number of decimal Place in both the Factors, which I find to be 9, *viz.* 4 in the Multiplicand and 5 in the Multiplier; therefore I mark out 9 Places towards the right han of the Product for a decimal Fraction, which indeed is the whole Product; and therefore I conclude the true Value of the Product to be .178600398; as by the following Operation appeareth, *viz.*

```
    38.5746
     .00463
    -------
    1157238
   2314476
  1542984
  ---------
  .178600398
```

A third Example ſhall be of 2 decimal Fractions, the one being given to be multiplied by the other; as, let there be given .63478 to be multiplied by .8264; having diſpoſed of the given Numbers according to order, and finiſhed the Work of Multiplication, as is before directed, I find the Product to amount to .524582192: which being done, to find the true Value thereof, I conſider that there are 9 decimal Places in both th

Factors, *viz.* 5 in the Multiplicand, and 4 in the Multiplier; wherefore I note out 9 Places in the Product for a Decimal Fraction, and ſo I find the true Value of the Product to be .524582192; as by the following Operation appeareth.

$$\begin{array}{r}
.63478\\
.8264\\
\hline
253912\\
380868\\
126956\\
507824\\
\hline
.524582192
\end{array}$$

The like is to be underſtood in any of the like Caſes whatſoever.

II. If it ſo happen (as ſometimes it may) that after your Multiplication is finiſhed, the Figures in the Product do not conſiſt of ſo many Places as there are decimal Figures in the Multiplicand and Multiplier, ſuch Defect muſt be ſupplied by prefixing as many Cyphers before it towards the left hand, as it wanteth Places, and then mark ſuch Product with the ſaid Cyphers for a decimal Fraction, and the true Product required; as in the following Example.

Let it be required to multiply .0476 by .0642, after the Multiplication is finiſhed, I find the Product to be 305692, conſiſting but of 6 Places; but the Number of decimal Places in the Multiplicand and Multiplier is 8: wherefore to make the Product to conſiſt of 8 Places, I prefix 2 Cyphers before it, and then *the true* Product will be .00305592. *The Work followeth.*

.0476
.0642

952
1904
2859

.00305592

In like manner, if .376523 were giv multiplied by .1346, you will find the Pr be 506799958, conſiſting of 9 Places, b are 10 decimal Places in both the given wherefore the Product muſt be increaſe Places, by prefixing a Cypher, which will .0506799958, as by the following Work.

.376523
.1346

2259138
1506092
1129569
376523

.0506799958

By this time I doubt not but the dilige ner is well acquainted with Multiplicatio cimal Fractions, the Work being as plai ſy as in whole Numbers. The next we is Diviſion.

CH

CHAP. VI.

Diviſion of Decimal Fractions.

HAving gone thro' Addition, Subſtraction, and Multiplication, (the Operation being, as *you* ſee, in every reſpect the very ſame as in whole Numbers) we come now to Diviſion; and altho in Decimals (as well as in whole Numbers) Diviſion may ſeem ſomewhat difficult to the young Practitioner, yet we ſhall endeavour to render it as plain and eaſy as poſſibly may be.

I. The Operation in Diviſion of Decimals is in every reſpect the ſame with that of whole Numbers; therefore the Difficulty in Diviſion of Decimals lieth not in the Operation, but in finding out the Value of the Quotient after the Work of Diviſion is ended, a general Rule for finding of which ſhall be given by and by.

II. It is neceſſary many times to annex a Cypher or Cyphers to the Dividend, whether it be a whole Number, or a mix'd Number, or a Decimal Fraction, for many times the Diviſor conſiſteth of more Places than the Dividend, and in that Caſe there muſt be a competent Number of Cyphers annexed to the Dividend: As, ſuppoſe it were requir'd to divide 73.564 by 46.24897; here you cannot conveniently proceed in the Work till you have annexed Cyphers to the Dividend, to increaſe the Number of Places in the decimal Part thereof, *and you may* annex as many as you *pleaſe; for by the ſeventh* Rule of the firſt Chapter, Cyphers annexed to a Decimal Fractio neither augment nor diminiſh its Value.

III. When a Queſtion to be wrought by I ſion of Decimals is propoſed, conſider wh there are as many decimal Figures in the I dend, as there are in the Diviſor; if there b wanting, make them full as many, or rather r by annexing Cyphers thereto, according to Rule foregoing: But in ſome Caſes there m neceſſity be more; for when there is an e Number of decimal Places in the Dividend, in the Diviſor, and a Diviſion can be made, the Quotient will infallibly be a whole Nu without any Fraction, except what is in the mainder.

IV. In Multiplication of decimal Fractions Product containeth as many decimal Figur there are decimal Places in the Multiplicand Multiplier; and in Diviſion, if you multipl Quotient by the Diviſor, the Product will b qual to the Dividend: upon which conſidera the true Value of the Quotient of any Div may infallibly be known by this

General Rule.

After the Work of Diviſion is ended, con how many decimal Places are in the Divi more than there are in the Diviſor; and how ny ſoever the Exceſs is, let ſo many in the Q tient be ſeparated from the reſt, for a Deci But if there are not ſo many Figures in the Q tient as the ſaid Exceſs is, ſuch Defect muſt ſupplied by prefixing as many Cyphers on *left hand*, putting a Point before them, as *been taught* already; then ſhall ſuch Dec

as aforeſaid, be the true Value of the Quotient ſought.

I ſhall explain this Rule by Examples of the ſeveral Caſes that may happen in the Diviſion of Decimals, which are 9, as followeth:

1	a whole Number	*being given to be divided by*	a whole Number,
2			a mix'd Number,
3			a Decimal Fraction.
4	a mix'd Number		a whole Number,
5			a mix'd Number,
6			a Decimal Fraction.
7	a Decimal Fraction		a whole Number,
8			a mix'd Number,
9			a Decimal Fraction.

Caſe 1.

A whole Number given to be divided by a whole Number.

V. When you are to divide one whole Number by another, and they are not commenſurable, tho there are no Decimals in either the Dividend or the Diviſor, yet if you annex a competent Number of Cyphers to the Dividend, there will be a Decimal in the Quotient, conſiſting of as many Places as you annexed Cyphers to the Dividend.

Example 1.

Let there be given 5729 to be divided by 438; according to the foregoing Rule, I annex a Number of Cyphers (ſuppoſe 4) to the given Dividend, which will ſupply 4 decimal Places, and it *will be 57290000: and a*fter the Work of Divi*ſion is finiſhed, I find the* Quotient to be 130799.

428)

ter, Cyphers annexed to a Decimal Fraction do neither augment nor diminiſh its Value.

III. When a Queſtion to be wrought by Diviſion of Decimals is propoſed, conſider whether there are as many decimal Figures in the Dividend, as there are in the Diviſor; if there be any wanting, make them full as many, or rather more, by annexing Cyphers thereto, according to the Rule foregoing: But in ſome Caſes there muſt of neceſſity be more; for when there is an equal Number of decimal Places in the Dividend, and in the Diviſor, and a Diviſion can be made, then the Quotient will infallibly be a whole Number without any Fraction, except what is in the Remainder.

IV. In Multiplication of decimal Fractions, the Product containeth as many decimal Figures as there are decimal Places in the Multiplicand and Multiplier; and in Diviſion, if you multiply the Quotient by the Diviſor, the Product will be equal to the Dividend: upon which conſideration the true Value of the Quotient of any Diviſion may infallibly be known by this

General Rule.

After the Work of Diviſion is ended, conſider how many decimal Places are in the Dividend more than there are in the Diviſor; and how manyſoever the Exceſs is, let ſo many in the Quotient be ſeparated from the reſt, for a Decimal. But if there are not ſo many Figures in the Quotient as the ſaid Exceſs is, ſuch Defect muſt be ſupplied by prefixing as many Cyphers on the *left hand*, putting a Point before them, as hath *been taught* already; then ſhall ſuch Decimal

as aforeſaid, be the true Value of the Quotient ſought.

I ſhall explain this Rule by Examples of the ſeveral Caſes that may happen in the Diviſion of Decimals, which are 9, as followeth:

1	a whole Number	*being given to be divided by*	a whole Number,
2			a mix'd Number,
3			a Decimal Fraction.
4	a mix'd Number		a whole Number,
5			a mix'd Number,
6			a Decimal Fraction.
7	a Decimal Fraction		a whole Number,
8			a mix'd Number,
9			a Decimal Fraction.

Caſe 1.

A whole Number given to be divided by a whole Number.

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7	a Decimal Fraction		a whole Number,
8			a mix'd Number,
9			a Decimal Fraction.

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After the Work of Diviſion is ended, conſider how many decimal Places are in the Dividend more than there are in the Diviſor; and how many ſoever the Exceſs is, let ſo many in the Quotient be ſeparated from the reſt, for a Decimal. But if there are not ſo many Figures in the Quotient as the ſaid Exceſs is, ſuch Defect muſt be ſupplied by prefixing as many Cyphers on the *left hand*, putting a Point before them, as hath *been taught* already; then ſhall ſuch Decimal

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Example 1.

Let there be given 5729 to be divided by 438; according to the foregoing Rule, I annex a Number of Cyphers (ſuppoſe 4) to the given Dividend, which will ſupply 4 decimal Places, and it *will be 57290000: and after* the Work of Divi*ſion is finiſhed, I find the* Quotient to be 130799.

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438) 5729.0000 (13.0799, *&c.*

Now to find out the Value of the Quotient by the general Rule before-going, I conſider that there are no Decimals in the Diviſor, but there are 4 in the Dividend, and conſequently by the ſaid Rule there muſt be 4 decimal Places noted out in the Quotient, by ſetting a Point before them, and then the true Value of the Quotient will be found to be 13.0799.

Example 2.

Let there be given 48 to be divided by 4376; you cannot here make any Work till you have annexed Cyphers to the Dividend, becauſe the Diviſor is bigger than the Dividend, and therefore annex as many as you think convenient; ſuppoſe 6, and having finiſh'd the Work of Diviſion, you will find the Quotient to be 1096: Now to find out its true Value, conſider that there are no decimal Places in the Diviſor, but there are 6 in the Dividend, therefore there muſt be 6 decimal Places in the Quotient; but the Quotient as yet poſſeſſeth but 4 Places, therefore to make them up 6, according to the ſaid general Rule, I prefix two Cyphers before the other Figures, on the left hand of the ſame, ſo as they may take place in the Decimal, by putting a Point before them, ſo will the true Quotient be .001096, *&c.*

4376) 48.000000 (.001096, *&c.*

This firſt Caſe may very well ſerve for a farther Illuſtration of the firſt Rule of the ſecond Chapter of this Book.

Caſe 2.

Example 3.

A whole Number given to be divided by a mix'd Number.

Let the whole Number 586 be given to be divided by the mix'd Number 36.4865; here you may obſerve that altho the Dividend be greater than the Diviſor, yet there can be no Operation until the Dividend is prepared by annexing a competent Number of Cyphers to it, and, according to the third Rule of this Chapter, I muſt annex at leaſt 4, but here I ſhall take 6 (or more at pleaſure) and then the Dividend will be 586.000000; and the Work being finiſhed, as in Diviſion of whole Numbers, the Quotient will be found to be 16.06, &c.

36.4856) 586.000000 (16.06, &c.

Now to diſcover the Value of this Quotient, according to the general Rule foregoing, I conſider that there are 4 decimal Figures in the Diviſor, and 6 decimal Places in the Dividend, the Exceſs being 2, and conſequently there muſt be 2 decimal Places noted in the Quotient, by putting a Point before 'em; and then the true Quotient will be 16.06, as you may prove at your leiſure.

Example 4.

Another Example of the ſecond Caſe may be this: Let there be given the Number 2 to be divided by the mix'd Number 28.74, having prepared the Dividend, by annexing 6 Cyphers it, (or more at pleaſure) and finiſhed the Wo of Diviſion, as in whole Numbers, I find the Qu tient to be 695, *&c*.

28.74) 2.000000 (.0695, *&c*.

Now to find out the true Value of this Qu tient, I conſider, according to the general Rul that there are but 2 decimal Places in the Diviſo and 6 in the Dividend; therefore (the Exceſs be ing 4) there muſt be four decimal Places in th Quotient, but there are but 3 Places, wherefo I make 'em up 4 by prefixing a Cypher before 'en according to the latter part of the ſaid gener Rule.

Caſe 3.

Example 5.

A whole Number given to be divided by a Decim Fraction.

Let there be given the whole Number 48 to b divided by the Decimal .0675; after the Divi dend is prepar'd by annexing a competent Num ber of Cyphers, as ſuppoſe 7, after the Work c Diviſion is ended, I find the Quotient to amou to 711.111, as followeth:

.0675) 48.0000000 (711.111, *&c*.

Now to find out the Value of the ſaid Quotien by the foregoing general Rule, I conſider tha there are 4 decimal Places in the Diviſor, and in the Dividend, the Exceſs being 3; wherefo I conclude that, according to the ſaid Rule, ther muſt be three decimal Figures in the Quote, cu off or ſeparated from the reſt by a Point; an then the true Value of the Quotient will b 711.111, that is, 711 Integers, and 111 decim Parts of an Integer, or very near.

Caſe 4.

Example 6.

A mix'd Number given to be divided by a whole Number.

Let there be given the mix'd Number 743.574, to be divided by the whole Number 75.

After the Dividend is prepared, by annexing Cyphers at pleaſure, and the Operation (according to Diviſion of whole Numbers) finiſhed, you will find this Quotient, *viz.* 9.91432.

75) 743.57400 (9.91432.

Now to find out the true Value of the ſaid Quotient, I conſider that after there are 2 Cyphers annexed to the Dividend, that the decimal Part thereof will poſſeſs 5 Places; and becauſe there are none in the Diviſor, therefore the Exceſs is 5, and conſequently (according to the ſaid general Rule) I note 5 Places in the Quotient for the decimal Part, which being done, I find the true Value of it to be 9.91432.

Example 7.

Again, let the Dividend in the laſt Example, *viz.* 743.574 be given to be divided by the whole Number 43576, and the Quotient will be found to be 17063, if there be 3 Cyphers annexed to the Dividend; and there will be 6 decimal Places in it, and not one in the Diviſor, wherefore there muſt be 6 decimal Places in the Quotient, but there are but 5: therefore to make them 6, according to the ſaid general Rule, I prefix a Cypher, and then the true Value of the Quotient will be .017063, as upon Proof you will eaſily find.

43567) 743.574000 (.017063, *&c.*

Caſe 5.

Example 8.

A mix'd Number given to be divided by a mix'd Number.

Let the following mix'd Number, *viz.* 3.748 be given to be divided by the mix'd Number 46.375. Here, according to former Directions, I annex Cyphers (at pleaſure) to the Dividend; ſuppoſe 5, then will the Dividend be 3.74800000: and having finiſh'd the Work of Diviſion as if they were whole Numbers, I find the Quote to be 8084, *&c.* But the true Value of this Quotient thus found I as yet know not; therefore to make a diſcovery of its Value, I conſider that in the Dividend there are 8 decimal Places, and in the Diviſor there are but 3 ſuch Places, therefore the Number of decimal Places in the Dividend exceeds the Number of Places in the Diviſor by 5; ſo that by the foregoing general Rule I know that there muſt be 5 decimal Places in the Quotient, but there are only 4 Figures, *viz.* 8084; but to make them 5 according to the general Rule, I prefix a Cypher before the other Figures, and it makes .08084, which is the true Quotient ſought.

46.375) 3.74800000 (.08084, *&c.*

Caſe 6.

A mix'd Number given to be divided by a Decimal Fraction.

Example 9.

Let there be given the mix'd Number 54.379 to be divided by the Decimal Fraction .34687 : a competent Number of Cyphers, being to be added, ſo that there may be 3, or 4, or 5 decimal Places in *the* Dividend more than there are in the Diviſor, therefore I annex 6 Cyphers, and then the Dividend will be 54.379000000 ; and when the Work of Diviſion is ended, the Quotient will be found to be 1567705.

Which being done, the next thing in order to the compleating of the Work, is to find out the true Value of the ſaid Quotient, which is eaſily done by the ſaid general Rule ; for I conſider that in the Diviſor there are 5 decimal Places, but in the Dividend there are 9 (*viz.* 3 given ſignificant Figures, and 6 Cyphers annexed) ſo that the Exceſs is 4 ; therefore I conclude that there muſt be 4 decimal Places in the Quotient, and the reſt are of the integral Part, ſo that I find the true Quotient is 156.7705, that is, 156 Integers and .7705 or $\frac{7705}{10000}$ parts of an Integer ; which you may eaſily prove at your leiſure.

.34687) 54.379000000 (156.7705, &c.

Example 10.

If there were given the mix'd Number 45.384 to be *divided by* .000247 ; here are not ſo many *decimal Places in the* Dividend as there are in the Diviſor, therefore do I increaſe their Nur ber by annexing 5 Cyphers thereto, and then tl Dividend will be 45.38400000; then do I procei to the Operation, taking no notice at all of tl Cyphers which are before the Diviſor, but wo as if there were none at all; and when the Wo of Diviſion is finiſhed, I find the Quotient to l 183740.89, &c.

.000247) 45.38400000 (183740.89, &c.

Now the Quotient being found, I come next find out its Value; which to do, I conſider th: there are 6 decimal Places in the Diviſor, and in the Dividend, ſo that the Exceſs is two Place: therefore I conclude, according to the ſaid gı neral Rule, that there muſt be 2 decimal Placı noted in the Quotient, ſo that then its true V: lue will be found to be 183740.89, &c.

Caſe 7.

A Decimal Fraction given to be divided by a whı Number.

Example 11.

Let it be required to divide the Decimal Fra tion .07864 by the whole Number 25. Here i this Example, there is no need of annexing an Cyphers to the Dividend to prepare it for Opı ration, but yet you may at your pleaſure; onl becauſe there is no neceſſity, I ſhall forbear i and proceed to the Work according to the Ru of Diviſion in whole Numbers; and the Woı *being* finiſhed, I find the Quotient to be 314 *then I proceed* to find out the Value of this Quo

tie

tient by the general Rule foregoing; and becauſe there is no Decimal in the Diviſor, and 5 in the Dividend, therefore there muſt be 5 decimal Places in the Quotient, and there are but 3 Places as yet, therefore do I prefix two Cyphers before the Quotient thus found, and note them for the true Quotient ſought, which is .00314, as by the Operation appeareth.

25) .07864 (.00314

Caſe 8.

A Decimal Fraction given to be divided by a mix'd Number.

Example 12.

Let there be given the Decimal Fraction .846 to be divided by the mix'd Number 3.476; here I firſt prepare the Dividend for the Work by annexing 4 Cyphers thereto, and having finiſhed the Work of Diviſion, I find the Quotient to be 2433; and to diſcover its Value according to the general Rule, I conſider that there are in the Dividend (after the 4 Cyphers are thereto annexed) 7 decimal Places, and in the Diviſor there are but 3, ſo that the Exceſs is 4, therefore I conclude that there muſt be 4 decimal Places in the Quotient, ſo that the true Quotient is the decimal Number .2433, *&c.* as followeth.

3.476) .8460000 (.2433, *&c.*

But if to the ſaid Dividend .846 there had be
annex'd 5 Cyphers, then the true Quotient wou
have been .24338, *&c.* and if there had been
Cyphers annex'd thereto, then had the Quotie
been .243383, *&c.*

Example 13.

Let it be requir'd to divide this Decimal Fra
tion; *viz.* .846, by the mix'd Number 34.7
after I have annexed Cyphers to the Dividen
to prepare it for the Work, and the Work
Diviſion being finiſh'd, I find (as before) t
Quotient to be 2433 ; but the Value of it bei
found out by the general Rule, will be differe
from the former Quote: for having taken t
Number of decimal Places in the Dividend a
the Diviſor, I find the Exceſs to be 5 in the D
vidend, ſo that there ſhould be 5 decimal Plac
in the Quotient, but there are now but 4 Place
wherefore to ſupply that Defect, I prefix a Cyph
before the ſaid Quote, and put the Point befo
it, ſo as it may take place in the Decimal, a
then the true Value of the Quotient will
.02433 *&c.* as followeth.

34.76) .8460000 (.02433, *&c.*

And if the Diviſor had been 347.6, then t
Quote would have been .002433, *&c.*

Caſe 9.

A Decimal Fraction given to be divided by a Decimal Fraction.

Example 14.

Let there be given the decimal Fraction .835796 to be divided by .243: Here I may annex Cyphers at pleaſure to the Dividend to prepare it for Operation; but becauſe there is no neceſſity for it, I ſhall forbear, and proceed to the Work, as in Diviſion of whole Numbers, which being finiſhed, I find in the Quotient the Number 3439: and now I have nothing to do but to find out the true Value of this Quotient, and in order thereunto, I conſider that in the Dividend are 6 decimal Places, and in the Diviſor but 3, wherefore the Exceſs is 3, which is the number of decimal Places in the Quotient, which being ſeparated from the reſt by a Point, according to former Directions, the true Value of the Quotient will be found to be 3.439, *&c.*

.243) .835796 (3.439, *&c.*

But if the Dividend had had a Cypher annex'd to it, then the Quotient would have been 3.4394, *&c.* and if two Cyphers had been annexed to it, then the Quotient would have been 3.43948, *&c.* But if the Dividend had been .0835796, and the Diviſor the ſame as before, the Operation would have been ſtill the ſame, and the ſame Figures would be in the Quotient, but not of the ſame Value, *for they would* have been all Decimals,

viz. .3439, *&c.* But if the Dividend had
(as before .835796) and the Diviſor had
(.0243) the ſame as before, with a Cypher
fixed before it to depreſs its Value, tho the
ration be the very ſame, yet the Value o
Quotient would have been 34.39, *&c.* A
the Diviſor had had two Cyphers prefixed b
it thus, .00243, then the Quotient would
been 343.9, *&c.* And if the Diviſor had
(.000243) the ſame as before, with three Cy
prefixed before it, then the Quotient would
been 3439, conſiſting intirely of Integers
cept you have annexed Cyphers to the Divi
Thus have I largely gone over all the Caſe
can happen in Diviſion of Decimals, and
given one or more Examples in every Caſ
that I hope by this time the diligent Rea
made capable of performing any Opera
either in Addition, Subſtraction, Multi
tion, or Diviſion of Decimals; and if he
perfected, perhaps he may be deſirous to l
ſomething of the Uſe and Application i
practical Parts of Arithmetick, before he c
to the more difficult part of the Extractio
Roots; and becauſe I would not dull the
of his Appetite, I ſhall give him a Taſte of
excellent Uſe in the Rule of Proportion, a
the Menſuration of ſome Superficies and S
and then come to ſhew their Uſe in the ext
ing the Cube and Square Roots, and the cal
ting of Intereſt, *&c.*

CH.

CHAP. VII.

The Rule of Three in Decimals.

I Shall not here meddle with the Rule of Three in its diſtinct kinds, *viz.* ſingle, double, direct, or inverſe, ſuppoſing the Learner to be acquainted with that already in the Practice of Vulgar Arithmetick.

I. In the Rule of Three in Decimals, the Operation is in every reſpect the ſame as in whole Numbers, ſo is it in all the Parts or Rules of Arithmetick; only when you work in Decimals, you muſt have reſpect to the decimal Rules before taught; for in Decimals you muſt add, ſubtract, multiply, and divide, when, and after the ſame manner as you do in whole Numbers. A few Examples will make you perfect in the Knowledge thereof.

Example 1.

If $1\frac{1}{4}$ Pound of Tobacco coſt 3 *s.* 6 *d.* how much will $326\frac{1}{4}$ Pound coſt at that rate?

When the fractional Parts of the Numbers in this Queſtion are turned into Decimals, then it will be read thus, *viz.*

If 1.75 Pounds of Tobacco coſt 3.5 *s.* how much will 326.25 Pounds of the ſame coſt at that rate?

The Numbers being orderly placed, as is directed in the ſixth Rule of the tenth Chapter of my Vulgar Arithmetick, will ſtand as followeth, *viz.*

l. *s.* *l.*
1·75 : 3·5 : 326.25.

And if you multiply the third Number by the ſecond, or the ſecond by the third, which is all one, and divide the Product thereof by the firſt, as is directed in the tenth Rule of the ſeventh Chapter of my Vulgar Arithmetick, (only in multiplying and dividing you muſt have regard to Multiplication and Diviſion of Decimals delivered in the two Chapters foregoing) when the Work is finiſhed, the Anſwer will be found to be *652,5s.* or 32 *l.* 12 *s.* 6 *d.* See the following Work.

l. *s.* *l.* *s.*
.75 : 3.5 :: 326.25 : 652.5
3.5

163125
97875

s.
1.75) 1141.875 (652.5
1050

918
875

437
350

875
875

(0)

52.5 *s.* or
12 *s.* 6 *d.*

Example 2.

C. of Tobacco coſt 25 *l.* 7 *s.* what will
Price of 17 C Weight of the ſame at that

given Numbers being rightly ſtated, to-
with the whole Operation, take as fol-

```
C.      l.        C.      l.
9  :  25.35  ::  17  :  47.883
        17
      ------
      17745
      2535
      ------   l.
9)  430.950 (47.883
    36
    ----
     70
     63
     ----
     79
     72
     ----      Facit 47.883 l. or
      75        47 l. 17 s. 8d. fere.
      72
      ----
       30
       27.
       ----
       (3)
```

Here you ſee that the Anſwer in Decimals 47.883 l. Now the Value of this Decimal Fr tion may be diſcovered at firſt ſight (by th brief way of finding the Value of a decimal P of a Pound Sterling, delivered in the fourth R of the ſecond Chapter foregoing) to be 17 *s.* 8 *fere*; for 8 Primes is 16 Shillings, and 8 Secon is 1 Shilling more, and 3 Seconds over, whi with the 3 in the Place of Thirds, makes 3 *from which* abating 1, becauſe it is above *there remains* 32 Farthings, or 8 Pence.

Exa

Example 3.

If an Ounce of Gold be worth 2 *l.* 19 *s.* 4 *d.* I demand the Price of 19 *oz.* 3 *pw.* 5 *gr.* at that rate.

By the fourth Rule of the ſecond Chapter, 3 *pw.* 5 *gr.* may be reduced to this Decimal of an Ounce, *viz.* .160416, ſo that 19.160416 is a mix'd Number equal in Value to 19 *oz.* 3 *pw.* 5 *gr.* And by the ſame Rule .9666 is found to be the Decimal Part of a Pound Sterling equal in Value to 19 *s.* 4 *d.* ſo that the Decimals being found out, and the Numbers given in the Queſtion being ſtated in order, will be as followeth, *viz.*

oz.		*l.*		*oz.*		*l.*
1	:	2.9666	: :	19.160416	:	56.841

19.160416
2.9666
———————
114962496
114962496
114962496
172443744
38320832
———————
56.8412901056

l.
Facit 56.841, *&c.*
l. *s.* *d.*
or 56—16—10 *fere.*

So that I find the Anſwer to the Queſtion to be 56.84129, *&c.* or 56 *l.* 16 *s.* 10 *d.* very near, as it may be diſcovered by the brief way of finding the Value of the Decimal of a Pound Sterling, delivered before in the thirteenth Page.

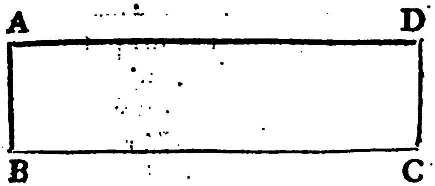
CHAP. VIII.

The Uſe of Decimals in the Menſu tion of Superficies and Solids.

PROP. I.

To meaſure a long Square.

THIS Figure by Geometricians is calle Rectangular Parallelogram, and it may ry fitly be repreſented by a long ſquare Ta



or a long Board, or the like, as the Fig ABCD; and to find out its Content, the R is,

Multiply the Length of it in Feet or Inches, the breadth of it in Feet, or in Inches, and Product will give you the true ſuperficial Cont of it in Feet or Inches.

Example.

There is a Table whoſe length is 18.75 Feet, and its Breadth 3.5 Feet, I demand its Content in Feet.

To anſwer this Queſtion, I take 18.75 Feet, (the length of it) and multiply it by 3.5 Feet, (the breadth of it) and the Product is 65.625 Feet, which is the Content of the Table, as was required. See the Work.

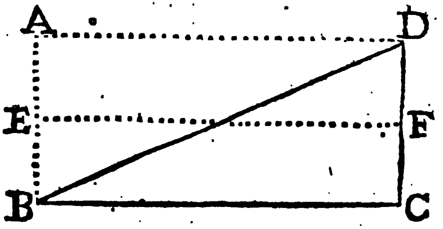
```
18.75
  3.5
-----
 9375
5625
-----
```

Facit 65.625 Feet.

Here by the way take notice, that altho amongſt Artificers the two Foot Rule is generally divided, each Foot into 12 Inches, *&c.* yet for him that is at any time employ'd in the Practice of Meaſuring, it would be moſt neceſſary for him to have his two Foot Rule, each Foot divided into 10 equal Parts, and each of thoſe Parts, divided again into ten other equal Parts: ſo would the whole Foot be divided into 100 equal Parts, and thereby would it be made fit to take the Dimenſions of any thing whatſoever, in Feet and decimal Parts of a Foot; and thereby the Content of any thing may be found as exactly, if not more exactly and near, than if the Foot were divided into Inches, Quarters, and half Quarters; and thereby many times would there be much La-

bour

the prick'd Line B A parallel to the Perpendicular, till it meet the Line A D in A, then will there be made the Parallelogram, or long Square A B C D, of which the Triangle B C D is half, the Diagonal B D, dividing the whole Parallelogram into two equal parts.



Now it is plain from the firſt Propoſition, that if you multiply the Side B C by the Side C D, then the Product will be the Content of the whole Parallelogram A B C D, and then the half of that Content will be the Content of the given Triangle B C D; or if you take half C D, which is C F, and half of B A, which is B E, and draw the Line E F, then will E F divide the Parallelogram A B C D, into two equal Parallelograms, and either of them is equal to the given Triangle B C D: now if, by the firſt Propoſition, I can find the Content of the Parallelogram E B C F, I find alſo the Content of the Triangle B C D, becauſe they are equal; whence it comes to paſs that if you multiply the Baſe by half the Perpendicular, or the Perpendicular by half the Baſe of a Rectangular Triangle (which is all one) the Product will be the true Content thereof.

Example.

Example.

the former Triangle the Baſe B C is 18.28 , and the Perpendicular C D is 12.26 Feet, I nd its Content in Feet.

ere I firſt find the Content of the whole llelogram, by multiplying the Sides toge- , and the Product is 224.1128 Feet, and the of that Product is the Content of the Trian- B C D, which is 112.564 Feet. See the fol- ing Work:

```
      18.28 the Side B C,
      12.26 the Side C D.
     ------
      10968
      3656
     3656
    1828
   -------
 2) 224.1128 (112.0564 Feet.
```

he Content would have been the ſame, if I multiplied the one Side by half of the other, :h indeed is the ſhorteſt way, and moſt prac- . See the Work.

The whole Side B C, 18.28
Half the Side C D, 6.13

5484
1828
10968

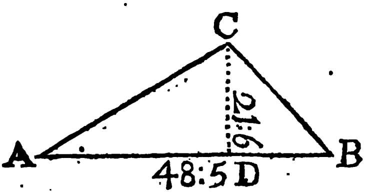
Facit 112.0564 Feet.

The Anſwer would have been the ſame, if had taken the whole Side C D, and multiplied by half the Side B C.

PROP. III.

To find the Content of any Plain Triang not Rectangular.

THE beſt and eaſieſt way is to let fall a Pe pendicular upon the longeſt Side, from t Angle that is oppoſite to it, which will divide into two right-angled plain Triangles; as ſu poſe there were given the plain Triangle A B as followeth:



Here the Side A B being the longeſt Side, I let fall a Perpendicular from its oppoſite Angle at C, which falleth upon D, in the Line A B, ſo is the Line C D the neareſt Diſtance between the angular Point C, and the Line A B, and divideth the given Triangle A B C into two right-angled Triangles, *viz.* A D C, and C D B; and if you find the Content of theſe two right-angled Triangles (according to the Directions in the ſecond Propoſition) and add them together, their Sum will be the Content of the given Triangle A B C. But it may be more artificially found out thus,

Multiply half the Line C D, into the whole Line A B, the Product will give the Content of the Triangle which was ſought; or if you multiply the whole Line C D, into half the Line A B, the Product will be the Content of the given Triangle, which is very plain from a due Conſideration of the Method uſed in ſolving the Second Propoſition.

Example.

Let the Baſe or longeſt Side A B be 48.5 Feet long, and let the length of the Perpendicular C D be 21.6 Feet, I deſire to know how many ſquare or ſuperficial Feet are contained in the ſaid Triangle.

To reſolve this Queſtion, according to the foregoing Rule, I firſt Bi-part the Baſe A B 48.5, which is 24.25, which I multiply by the length of the Perpendicular C D 21.6; and the Product is 523.8: See the following Work.

24.25	or thus,	48.5
21.6		10.8
14550		3880
2425		4850
4850		523.80
523.800 *facit.*		

So that you ſee the Content is the ſame, which of the foreſaid ways ſoever you work; obſerve the ſame Method in finding the Content of any oblique Triangle given.

PROP. IV.

To find the Content of a Trapezium.

A Trapezium is a plain Figure, having four unequal Sides, and as many unequal Angles; it matters not how unequal they are: and to find out its Content obſerve the following Directions, *viz.*

Divide it into two oblique Triangles, by drawing a Line from any one of the Angles to the *Angle that is* oppoſite thereto, which Line ſhall be *a common Baſe* to both the Triangles.

The

Then if you find out the Content of both theſe Triangles, according to the Method preſcrib'd in the third Propoſition, the Sum of their Contents is the Content of the given Trapezium.

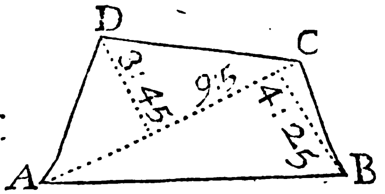
Or it may be more artificially found out thus, *viz.*

Multiply the Length of the common Baſe, by half the Sum of the Perpendiculars let fall from the Angles oppoſite to the ſaid common Baſe, and the Product will be the Content of the whole Trapezium. Or elſe,

Multiply the Sum of the ſaid Perpendiculars by half the ſaid common Baſe, and it will produce the ſame Effect.

Example.

In the following Trapezium ABCD, draw the Baſe AC, which ſuppoſe to be 9.5 Feet, then let fall the Perpendicular at D, which let be 3.45 Feet, and that at B 4.25 Feet, the Sum of the ſaid Perpendiculars is 7.7, half of which is 3.85; by which, if the common Baſe AC be multiplied, the Product (which is) 36.575 is the Content of the Trapezium required. Or if you multiply 7.7, the Sum of the Perpendiculars, by half of the common Baſe 9.5, which is 4.75, the Product will be the ſame. See the following Work.



3.45

3.45 the Perpendicular at D,
4.25 the Perpendicular at B,

7.70 their Sum,

3.85 their half Sum,
9.5 A C

1925
3465

Facit 36.575

4.75 half A C
7.7

3325
3325

Facit 36.575

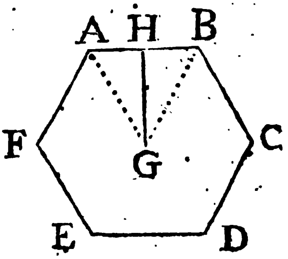
PROP. V.

To find the Content of any Regular Polygon.

A Regular Polygon is a plain Figure conſiſting of equal Sides, and equal Angles, *viz.* a Pentagon, conſiſting of five equal Sides, and five equal Angles; a Hexagon, conſiſting of ſix equal Sides, and ſix equal Angles; an Heptagon of ſeven equal Sides, an Octagon of eight, *&c.* And to meaſure any one of theſe regular Planes do thus; draw a Line from the Center of the Figure to the middle of any one of the Sides, and multiply that Line into half the Sum of the Sides, and the Product thence ariſing is the Content of the given Plane.

I ſhall give you an Example of this in the Menſuration of an Hexagon, or Plane of ſix equal Sides.

Let there be given the Hexagon A B C D E F, having the length of each Side 30, then will the length of the Perpendicular G H be 26 *fere*; now there being in all ſix Sides, and each of them in length 30, the Sum of them all is 180, the half of which is 90, which being multiplied by 26,



the Product will be 2340, which is the Content of the given Hexagon.

The reaſon of this manner of working is very plain, if from the Center you draw the Lines G A and G B, thereby making the Triangle G A B, whoſe Content (by the third Propoſition) is found by multiplying the Perpendicular G H into half the Side A B, *viz.* into H A, or H B, but there are ſix ſuch Triangles in the given Polygon; therefore G H, multiplied into ſix times H B, produceth the Content required.

PROP. VI.

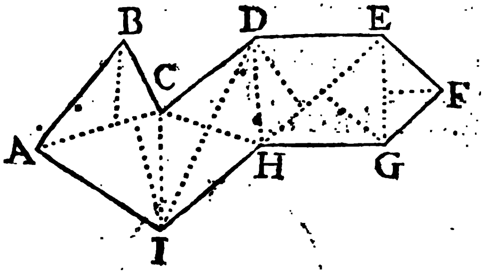
To find the Content of any Irregular Polygon.

LET it be required to meaſure the following Figure A B C D E F G H I.

Firſt take care that the whole Plane be divided into Trapeziums and Triangles, according to your own Fancy, and as the Nature of the Plane will bear, and then meaſure thoſe Trapeziums and Triangles as is directed in the third and fourth Propoſitions beforegoing, and add the ſeveral Contents together, ſo will the Sum give you the whole Content of that irregular Polygon.

As in this Example, firſt I draw the Lines A C, and D H, and E G, ſo is the whole Figure divided into the Trapeziums A B C I, and C I H D, and D H G E, and the Triangle E G F; *the Contents* of which being ſeverally found out

by



by the third and fourth Propoſition, the Sum of them will be the Content of the whole Figure.

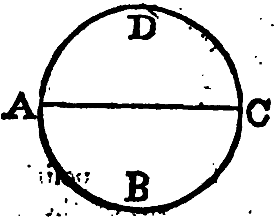
P R O P. VII.

To find the Circumference of a Circle having the Diameter given.

A Circle is a Geometrical Figure exactly round, ſo that if from a Point in the middle of it, called the Centre, there be ever ſo many Lines drawn to the Circumference, they will all be of equal length. But between the Diameter and Circumference of a Circle there cannot be found a true and exact Proportion. *Archimedes* hath demonſtrated the Proportion to be near as 7 is to 22; but that of *Van Ceulen* is the moſt exact, who makes it to be as 1 is to 3. 14159265358979323846, *&c.* but for Practice, *this following Proportion* is ſufficient, *viz.*

As 1 is to 3.1416
So is the Diameter of any Circle
To its Circumference.

In the Circle deſcribed in the Margin, the Diameter A C is 28, I demand what is the Circumference A B C D.



To anſwer which, I ſay by the Proportion foregoing, as 1 is to 3.1416, ſo is 28 the Diameter to 87.9648, which is the Circumference A B C D. The Work followeth.

As 1 is to 3.1416, ſo is 28 to 87.9648.
28
251328
62832
87.9648

PROP.

P R O P. VIII.

find the Content of a Circle, having the Diameter given.

I R S T find out the Circumference by the
laſt Propoſition, then multiply half the Cir-
ference by half the Diameter, and that Pro-
: is the Content.

Example.

here is a Circle, whoſe Diameter is 14 Inches,
mand how many ſquare Inches are the Con-
of that Circle.
y the foregoing Propoſition, I find the Cir-
ference to be 43.9824 Inches, the half of
ch is 21.9912; which being multiplied by
half the given Diameter) the Product is
9384; which is the Content required. See
Vork.

```
   21.9912
         7
   -------
Facit 153.9384 Square Inches.
```

* * *

P R O P.

PROP. IX.

To find the ſolid Content of a ſquare Piec of Timber, Stone, &c. whoſe Baſes ar equal; that is, whoſe Ends are of th ſame Bigneſs.

SUCH a ſolid Piece, by Geometricians, i called a Parallelopipedon, and its Conten is thus found out, *viz.*

First find out the Superficial Content of th Baſe or End, (by the firſt Propoſition) then multiply that Content by the whole Length, an that Product is the ſolid Content of the whol Piece.

Example.

There is a ſquare Piece of Timber, the tw contiguous Sides at the end of which, are, 2. Feet, and 1.8 Feet, and its length is 22 Feet, demand how many ſolid Feet are in that Piec of Timber.

Firſt I multiply 2.5 by 1.8, the Sides of th Baſe, and that produceth 4.5 for the Content o the Baſe or End, and that Product I multiply b 22, the length, and that produceth 99 Feet and ſo many is there contained in that Piec of Timber, as you may ſee in the followin Work.

2.5 } The 2 Sides at the end.
1.8 }

200
25

450 The ſuperficial Content of the end.
22

900
900

acit 99.00 Feet for the Content required.

Here note, If the Sides of the End, or Baſe,
e given in Inches, and its length in Feet, then
duce the Sides of the Baſe into the decimal
arts of a Foot, and proceed as before; or you
ay find out the Content of the Baſe in Inches,
d multiply that Content by the length in Feet,
d that Product divided by 144, will give you
e Content in Feet, or elſe reduce the length in-
Inches, and multiply the Content of the Baſe
ereby, and divide that Product by 1728 (for
ere are ſo many cubical Inches in a Foot) and
e Quotient will give you the ſolid Content in
et. But the decimal way is preferred.

* * *

PROP.

PROP. X.

To find the ſolid Content of a Cylinder, having the Diameter of its Baſe given.

A Cylinder is a Solid whoſe Baſes are circular, equal, and parallel, and may fitly be repreſented by a round Pillar, or a Rolling-Stone of a Garden; and to find the ſolid Content of ſuch a Body, this is

The Rule.

Firſt find the Plane of the Baſe, by the ſeventh and eighth Propoſitions foregoing, and then multiply that by the length thereof, which Product will give you the ſolid Content of the given Cylinder.

Example.

There is a Cylinder (ſuppoſe a Rolling-Stone) whoſe length is 8.75 Feet, and the Diameter of its Baſe 2.8 Feet, I demand the ſolid Content thereof.

As 1
Is to 3.1416,
So is 2.8, the given Diameter,
To 8.79648, the Circumference of the Baſe, half of which (*viz.* 4.39824) being multiplied by 1.4 the Semi-diameter, will produce 6.157536 for the Content of the Baſe, which being multiplied

plied by 8.75 (the length) it produceth 53.87844 for the ſolid Content required.

If there had been given the Circumference of the Cylinder, then the Diameter of the Baſe muſt have been found out by the Converſe of the ſeventh Propoſition; as ſuppoſe there had been given 8.75 the length of the Cylinder, and 8.79648 its Circumference, to find the Solidity thereof, firſt I find out the Diameter by the following Proportion, *viz.*

As 3.1416
Is to 1,
So is 8.79648, the given Circumference,
To 2.8, the requir'd Diameter.

And then the reſt of the Work is the ſame with that before.

PROP. XI.

To find the ſolid Content of a Cone.

A Cone is a ſolid Body, having a Circle for its Baſe, and its Superficies Circular, decreaſing its equidiſtant Diameters from the Baſe proportionably, till it remaineth in a Point over the Centre of its Baſe, and may fitly be repreſented by a Sugar-loaf, or a round Spire-Steeple; and to find its ſolid Content, this is

The Rule.

By the 7th and 8th Propoſitions foregoing, find out the Plain of its Baſe, and multiply that by $\frac{1}{3}$

of its height, and that Product is the ſolid Content of the Cone required.

Example.

There is a Cone, the Circumference of whoſe Baſe is 22.5, and its Height is 16; I demand the ſolid Content of ſuch a Cone.

As 3.1416
Is to 1,
So is 22.5, the Circumference of the Baſe,
To 7.162, the Diameter of the Baſe.

Then I multiply half 22.5, which is 11.25, by half 7.162, which is 3.581, and it produceth 40.28625, which is the ſuperficial Content of the Baſe; then I take $\frac{1}{3}$ of the height of the Cone (16) which is 5.333 very near, by which I multiply 40.28625 (the ſuperficial Content of the Baſe) and it produceth 214.8467125. See the Work as follows.

```
   40.28625
      5.333
   --------
   12085875
  12085875
 12085875
20143125
-----------
```

Solid Content is 214.84657125.

The ſame is to be obſerved in the Menſuration of any other Cone.

But here you are to obſerve, that the ſlanting ſide of the Cone, (*viz.*) the length from the

Vertex to the Extremity of its Baſe) is not to be taken for its true Height, but a Perpendicular let fall from its Vertex, or Centre of its Baſe, is its true Height; and how you may find out that Perpendicular's Length ſhall be ſhown you in the Work of the fourth Propoſition of the eleventh Chapter.

PROP. XII.

To find the ſolid Content of a Pyramid.

BEtween the Cone and Pyramid this is the difference: as the Cone hath a circular Baſe and Superficies, the Pyramid hath a Polygon for its Baſe, ſo that its Baſe and Superficies are angular, its Vertex terminating in a Point juſt over the Centre of its Baſe; and to find out its Solidity, here followeth

The Rule.

Find out the ſuperficial Content of the Baſe by the fifth Propoſition foregoing, and multiply that by $\frac{1}{3}$ of its Height, and it produceth the ſolid Content of the Pyramid.

Example 1.

There is a Pyramid whoſe Baſe is a Hexagon, the Side of which is 30, and its perpendicular Height is 54, I demand the ſolid Content of ſuch a Pyramid.

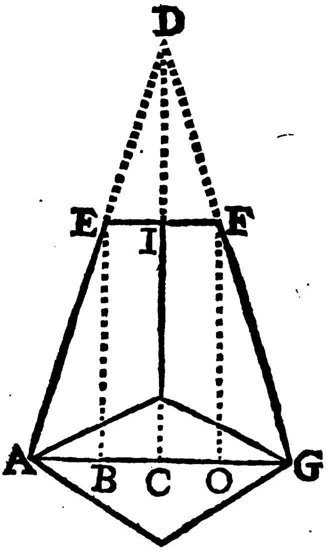
Here, by the fifth Propoſition, I find the ſuperficial Content of *the Baſe to be* 2340; then do I take ⅓ of the perpendicular Height of the Py mid, which is 18, and thereby do I multi 2340 (the Plane of the Baſe) and the Product 42120, which is the ſolid Content of the gi Pyramid.

Here note by the way, that a Line drawn fr the Point at the top of the Pyramid to the I tremity of any part of the Baſis, is not the t Height of any Pyramid; but a Perpendicular fall from the Cuſpis (or Top) to the Centre the Baſe, is the true height: And to find ſuch perpendicular Heights, ſhall be ſhewn in fourth Propoſition of the eleventh Chapter.

PRO

PROP. XIII.

To meaſure the Fruſtum of a Pyramid or Cone.



THE Fruſtum here given to be meaſured is AGEF, the Side of the greater Baſe at A being 24 Inches, and the Side of the leſſer Baſe at E being 8 Inches, and the Length of it IC, 20 Foot, equal to EB, or FO.

It is evident that if I find the Solidity of the whole Pyramid AGD, and alſo the Solidity of the leſſer Pyramid EFD, and then ſubſtract the Content of EFD, from the Content of AGD, that there will remain the Solidity of the Fruſtum AGEF; and certainly this way of meaſuring the Fruſtum of a Pyramid, or Cone, is the moſt exact of any: And it may be eaſily meaſured thus, firſt of all find out the Height of the whole Pyramid CD, which you may do by the following Proportion, *viz.*

As the Semi-difference of the Sides of the Baſes
Is to the Height of the Fruſtum,
So is the half Side of the greater Baſe,
To the Height of the whole Pyramid.

And this Proportion will hold as good, if you work by the Semi-difference of the Diameters of the Baſes, as if by the Semi-difference of the Sides of the Baſes.

As in the foregoing Figure, let A G be the Diameter of the greater Baſe, and E F the Diameter of the leſſer Baſe, from E and F, let fall the Perpendiculars E B and F O, then ſhall B O be equal to E F, and the Sum of A B and O G are the Difference of the Diameters of the Baſes E F and A G; and conſequently A B is the Semi-difference, and B E is the height of the Fruſtum, and A C is half the Side of the greater Baſe, and C D is the height of the whole Pyramid. Then by *Euclid. 6. 4.*

As A B, the Semi-difference of Diameters,
Is to B E, the Height of the Fruſtum;
So is A C, half the greater Diameter,
To C D, the Height of the whole Pyramid.

So the Height of the whole Pyramid A G D, will be found to be 30 Foot; for the greater Diameter A G is 24 Inches, the leſſer 8, the Difference 16, the Semi-difference 8, therefore ſhall C D be 30 Foot; for

8 : 20 : : 12 : 30

Now having found the Height of the whole Pyramid to be 30 Foot, I thereby (according to the twelfth Propoſition foregoing) find the Content of the whole Pyramid to be 40 Foot; then in the leſſer Pyramid E F D there is given the Side of its Baſe E F, 8 Inches, and its height I D, 10 Inches; for C D 30—C I 20—I D 10; and by the ſaid twelfth Propoſition I find the ſolid Content of it to be 1.48 Feet, which being ſubſtracted from 40, (the Content of the greater Pyramid) there will remain 38.52 Feet, for the true ſolid Content of the given Fruſtum A G E F.

After the ſame manner is found the Solidity of the Fruſtum of a Cone, the Height of the whole Cone being found out by the difference of the Diameters of its Baſes; and by the eleventh Propoſition find the Solidity of the whole Cone, and alſo the Solidity of the leſſer Cone, that is cut off from the Fruſtum, then ſubſtract the Content of the leſſer from the Content of the greater, and the Remainder will be the ſolid Content of the Fruſtum.

This laſt Propoſition is uſeful in the meaſuring of tapering Timber round or ſquared, and for finding the liquid Capacity of Brewers Conical, or Pyramidal Tuns.

Thus have I ſhew'd the Uſe of Decimals in the Menſuration of the moſt uſeful Planes and Solids: I might proceed farther to ſhew their Application in the particular Menſuration of Board, Glaſs, Pavement, Plaiſtering, Painting, Wainſcot, Tiling, Flooring, Tapeſtry, Brick-work, Timber, and Stone; but it requireth (rather) a particular Treatiſe, than the narrow Bounds here allowed for ſuch a Work.

* * *

CHAP. IX.

The Extraction of the Square Root.

IN the Solution of any Queſtion, or in the working of any Sum whatſoever belonging to any of the Rules of Vulgar or Decimal Arithmetick, there have been (at leaſt) two Things or Numbers given, whereby the Anſwer might be found; but in the Extraction of the ſquare, cube, and all other Roots, there is but one Number given to find out the Number ſought, *viz.* there is a ſquare Number given to find its Root, a cube Number to find its Root, *&c.* And,

I. A ſquare Number is that which is produced by multiplying any Number by (or into) itſelf, which Number given to be ſo multiplied, is call'd the Root; as if the Number 8 were given to be multiplied by it ſelf, it produceth *64*, then is 8 called the Root, and *64* is its Square: ſo the Root 12 hath for its Square 144.

II. When a ſquare Number is given, and its Root is required, the Operation itſelf is called the Extraction of the ſquare Root.

III. Square Numbers are of two kinds, *viz.* either ſingle or compound.

IV. A ſingle ſquare Number is that which is produced by the Multiplication of a Digit, or ſingle Number into itſelf; and conſequently ſuch a ſquare Number muſt be under 100, which is the ſquare of 10; ſo 25 being given for a ſquare Number, it is a ſingle ſquare, having for its Root 5: And 81 is a ſingle ſquare Number, having for its Root the Digit 9. All the ſingle ſquare Numbers, with their Roots, are contained in the following Table.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81

V. When the Root of any ſquare Number is required, it being leſſer than 100, and yet is not exactly a ſingle ſquare, expreſſed in the Table above, then you are to take the Root of that ſingle ſquare Number expreſſed in the ſaid Table, which (being leſs) is neareſt to the given ſquare; as if it were 74 whoſe Root is requir'd, I find that 81 (the ſquare of 9) is too much, and 64 (the ſquare of 8) is too little, but yet it is the neareſt ſquare Number that is leſſer than 74, and therefore I take 8 to be the ſquare Root of 74; but yet it is plain, that 8 is too little for the Root of 74. And to find out the fractional part of this Root, you ſhall be plainly taught by and by.

VI. A compound ſquare Number is that which hath above 9 for its Root.

VII. The Root of a ſingle ſquare Number may be diſcovered at the firſt ſight, but the Extraction of a compound ſquare Number is more tedious and difficult, its Root conſiſting of two

Places,

Places, at the leaſt, and the Square itſelf, of a hundred at the leaſt.

VIII. When a compound ſquare Number is given, and it is required to have its ſquare Root extracted, before you can proceed to the Operation, your ſquare Number muſt be prepared, by pointing it at every ſecond Figure, beginning at the Place of Units.

As, ſuppoſe you were to extract the ſquare Root of 2304; firſt, I put a Point over 4 (it ſtanding in the Place of Units) and then paſſing over the ſecond Place (or Place of Tens) which is 0, I put a Point over the Figure ſtanding in the third Place, (or Place of Hundreds) which is 3, and the preparative Work is done, as you may ſee in the Margin. Now if there had been more Places in the given Number, then I muſt have put a Point over the Figure, ſtanding in the fifth Place, and another over that in the ſeventh, *&c.* And here note, that as many Points as you put over the given ſquare Number, ſo many Figures there will be in the Root, that is, the Root will conſiſt of ſo many Places.

$2\dot{3}0\dot{4}$

So if there were given the Number 33016516 to have its ſquare Root extracted, after I have pointed it according to the Direction before given, it will ſtand as in the Margin; and becauſe the Points that are put over it are in Number 4, I conclude the Root itſelf will conſiſt of 4 Places or Figures.

$3\dot{3}0\dot{1}6\dot{5}1\dot{6}$

IX. When you have thus prepared your Number, then draw a crooked Line on the right hand of your Number, behind which to place your Root, as you do for a Quotient in Diviſion.

Note, That when your Number is prepared for Operation, as in the eighth Rule, the Numbers contained between Point and Point, may not unfitly be termed Squares, and in the enſuing Work we ſhall ſo call them; as in the foreſaid Number *33016516*, being pointed as before, I call *33* the firſt Square, 01 the ſecond, *65* the third, and *16* the fourth and laſt Square: every Square (except ſometimes the firſt) conſiſting of two Figures or Places, the laſt of which towards the right hand hath always a Point over it, and if it ſo happen (as often it doth) that the laſt Figure (in any given ſquare Number) toward the left hand hath a Point over it, then that Number alone ſhall be accounted the firſt Square.

As if the Number *676* were given, when it is pointed for the Work, according to Direction, as you ſee in the Margin, I account *6* for the firſt Square, and *76* for the ſecond.

$\dot{6}7\dot{6}$

Theſe things being underſtood, we ſhall lay down thoſe general Rules requiſite for the Management of the Work itſelf.

X. When your Number is prepared, find out the ſquare Root of the firſt Square, according to the fifth Rule foregoing, and place that Root behind the ſaid crooked Line. As,

Let it be required to extract the ſquare Root of the ſaid Number 2304; here the firſt ſquare Number is 23, and (according to the ſaid fifth Rule)

$\dot{2}3\dot{0}4$ (4

its Root is 4, which I place behind the crooked Line, as you ſee in the Margin.

```
 . .
2304 (4
 16
———————
  7
```

XI. Then ſquare the ſaid Root, and place its Square, which is 16, under the ſaid firſt Square 23, and having drawn a Line underneath, ſubſtract the ſaid Square 16 from 23, and place the Remainder, which is 7, underneath the ſaid Line, as you may perceive by the Work in the Margin.

XII. Then to the ſaid Remainder bring down the Figures of the next Square, and annex them thereto on the right hand, ſo that they may make one entire Number, which (for diſtinction's ſake) we ſhall call the Reſolvend.

```
 . .
2304 (4
 16
———————
 704 Re.
```

As in this Example, to the Remainder 7, I bring down the next Square 04, and annex it thereto, and it maketh 704 for a Reſolvend, as you may ſee in the Margin.

XIII. Always let the whole Reſolvend (except the laſt Figure on the right hand) be eſteemed a Dividend, on the left hand of which draw a crooked Line, before which to place a Diviſor, as in Diviſion.

```
 . .
2304 (4.
 16
———————
)704
```

So in this Example, the Reſolvend 704 is to be made a Dividend, all but the laſt Place, which is 4; ſo that the Dividend is 70, before which I draw a crooked Line, as you ſee in the Margin.

XIV. Let the Quotient, expreſſing the Root (or part of the Root ſought) be doubled, or multiplied by 2, and that Double or Product ſhall be a Diviſor, and muſt be placed on the left hand of the Reſolvend, before the ſaid crooked Line.

So in our Example, the Number 4, which was put for part of the Root being doubled, makes 8, which I put before the Reſolvend for a Diviſor, as it appears in the Margin.

```
     . .
    2304 (4
    16
   ------
 8) 704
```

XV. Then (according to the Rule of Diviſion in whole Numbers) ſeek how often the ſaid Diviſor is contained in the ſaid Dividend, and put the Anſwer down in the Quotient, and alſo on the right hand of the Diviſor.

As in our Example, I ſeek how often the Diviſor 8 is contained in the Dividend 70, which I find to be 8 times, therefore I put 8 in the Quotient for part of the Root, and alſo on the right hand of the Diviſor. See the Work in the Margin.

```
     . .
    2304 (48
    16
   ------
88) 704
```

XVI. Then by the Figure laſt put for part of the Root, multiply the ſaid Diviſor, together with the Figure that you annexed to it (accounting them both as one entire Number) and place the Product underneath the ſaid Reſolvend, *drawing a Line* under it, and then ſubſtract it out of the ſaid Reſolvend, placing tl Remainder beneath the Line.

```
  . .
 2304 (48
 16
 ---------
88) 704
    704
 ---------
    (0)
```

As in our Example, havir placed 8 in the Quotient, ar alſo on the right hand of th Diviſor, then in the place o the Diviſor, there ſtands 8 which I multiply by 8, th Number laſt put in the Qu tient, and the Product is 70 which I place in order und the Reſolvend 704, and havin drawn a Line underneath, I ſubſtract the ſai Product 704, from the Reſolvend 704, an there remaineth 0, ſo is the Work finiſhed, an I find the ſquare Root of 2304 to be 48. Se the Work in the Margin.

Here note, That if at any time when yo have multiplied the Number ſtanding in th place of the Diviſor, by the Figure laſt place in the Quotient or Root (as is directed in th laſt Rule) if the Product be greater than th Reſolvend, then conclude the Work to be erro neous, to correct which put a leſſer Figure i the Root, and proceed as is before directed.

Note alſo, That the Work of the 12th, 13th 14th, 15th, and 16th Rules muſt be repeate as often as there are Points over the Figure except for the firſt Square, which is to b wrought according to the Directions given i the tenth and eleventh Rules foregoing, an the Work of thoſe two Rules is to be obſerve but once in the Extraction of a ſquare Roo tho it conſiſts of never ſo many Squares o *Points.*

Th

Theſe things will appear plain and eaſy in the working of one or two more Examples.

Example 2.

Let it be required to extract the ſquare Root of 33016516.

Here, in order to the Work, I firſt prepare my Number, by diſtinguiſhing it into Squares, by pointing it according to the Eighth Rule foregoing, and thereby I find that 33 is the firſt Square, and (according to the tenth Rule) I take the ſquare Root of 33, which is 5, and place for the firſt Figure of the Root, then (according to the eleventh Rule) I ſquare the Root 5, and it makes 25, which I place under the ſaid firſt ſquare Number 33, and ſubſtract it therefrom, and the Remainder 8 I place below the Line, as in the following Work.

```
 .  .  .  .
33016516 (5
25
----
 8
```

Then (according to the twelfth Rule) I annex to the ſaid Remainder (8) the next Square (01) and it makes 801 for a Reſolvend, then muſt 80 (according to the thirteenth Rule) be my Dividend, and (according to the fourteenth Rule) I double the Number (5) in the Root, and it makes 10 for a Diviſor, and thereby I divide the ſaid Dividend (80) and I find that it *quotes 7, which* (according to the fifteenth *Rule) I put in the* place of the Root after 5, and likewiſe before the Diviſor (10) ſo that in the place of the Diviſor, inſtead of 10, there is now 107.

Then, according to the ſixteenth Rule, I multiply the ſaid 107 by 7, the Figure laſt placed in the Root, and the Product is 749, which I place orderly under the ſaid Reſolvend, and ſubſtract it therefrom, and the Remainder is 52, which I put below the Line, as in the following Work.

33016516 (57
25

107) 801 Reſolvend.
749

52

Then I repeat the ſame Work over again, in finding the next Figure of the Root, as I did in finding the laſt, *viz.* to the Remainder 52, according to the twelfth Rule, I bring down, and thereto annex the next (third) Square 65, and it makes 5265 for a new Reſolvend; then, according to the thirteenth Rule, is 526 a new Dividend; and, according to the fourteenth Rule, I take the Root, and double it for a new Diviſor, and it makes 114, which place before the Reſolvend 5265.

Then, according to the fifteenth Rule, I ſeek how often the Diviſor 114 is contained in the Dividend 526, and I find it will bear 4, which I place in the Root orderly, and alſo on the right hand of the Diviſor 114, and then there will be in the Place of the Diviſor the Number 1144 which, according to the Sixteenth Rule, I multiply by the Figure 4, laſt put in the Root, and the Product is 4576, which I place orderly under the Reſolvend 5265, and ſubſtract it therefrom, and the Remainder is 689, which I place under the Line, as is before directed. See the whole Work as followeth.

```
         . . . .
        33016516 (574
        25
       ──────────
   107) 801 Reſolvend.
        749 Product.
       ────
  1144) 5265 Reſolvend.
        4576 Product.
       ─────
         689
```

Then I again repeat the Work of the 12th, 13th, 14th, 15th, and 16th Rules of this Chapter for finding the next Figure of the Root, *viz.* firſt I bring down 16, the next ſquare Number, and annex it to the Remainder 689, according to the 12th Rule, and it makes 68916, for a new Reſolvend, of which by the 13th Rule, 6891 is a new Dividend: then, according to the 14th Rule, I double the Root, and it makes 1148 for a Diviſor, which I place on the left ſide the Reſolvend, and then ſeek how often it is contained in the ſaid Dividend 6891, and the Anſwer is 6, which I place for part of the Root in order, and alſo on the right hand of the ſaid Diviſor, ſo that *in the place of the* Diviſor 1148, will then ſtand the Number 11486, which by the ſixteenth Rule, I multiply by 6 (the Figure laſt placed in the Root) and the Product is 68916, which I place in order under the Reſolvend, and ſubſtract it therefrom, and the Remainder is 0, and ſo the Work is finiſhed, whereby I find the ſquare Root of 33016516 to be 5746, as by the whole Operation appeareth.

. . . .

33016516 (5746

25

107) 801 Reſolvend.

749 Product.

1144) 5265 Reſolvend.

4576 Product.

11486) 68916 Reſolvend.

68916 Product.

(0)

And if the Root had conſiſted of ever ſo many Places, yet for every Figure put therein, except the firſt, for which you are to obſerve the 10th and 11th Rule, the Work of the 12th, 13th, 14th, 15th, and 16th Rules muſt be repeated according to the ſecond Note after the Sixteenth Rule foregoing.

Example.

Example 3.

A third Example may be this; let it be required to extract the ſquare Root of *8328996.*

In the working of this Example you will ſee the Uſe of the firſt Note upon the ſixteenth Rule, for only the Number 8 is the firſt Square, as you may ſee by the pointing of the given Number; and after the whole Work of Extraction is finiſhed, you will find the ſquare Root of the given Number to be *2886*, as in the following Operation.

• • • •
8328996 (*2886* Root.
4

48) 432 Reſolvend.
384 Product ſubſtract.

568) 4889 Reſolvend.
4544 Product ſubſtract.

5766) 34596 Reſolvend.
34596 Product ſubſtract.

(0)

XVII. When there is given a Number that is not a ſquare Number, that is, whoſe Root cannot be exactly found, and you are deſirous to find the fractional Part of the Root as near as *may be, you are to obſerve* the eighth Rule in *preparing your Number* for Extraction, and then to annex thereto an even Number of Cypher pleaſure; and note, That as many Pairs of C phers as you annex thereto, ſo many Decim will there be in the Root expreſſed (which it come not to be the exact Root, yet will it co ſo near the Truth, that if the laſt Decimal Fig placed in the Root be increaſed by an Unit, will be too much) and as many Points as th are over the given integral ſquare Number, many Places will there always be in the integ Part of the Root, as in the following Examp where it is required to extract the ſquare R of 129596.

Firſt, I proceed to the Work of Extracti (according to the former Rules) as if it w an exact ſquare Number, and find the integ Root to be 359, as followeth.

```
        .  .  .
       129596 (359
       9
       ----
  65)  395
       325
       ----
 709)  7096
       6381
       ----
        715 Remainder.
```

But becauſe (when the Work is finiſhed) th is a Remainder of 715, I annex a competent e Number of Cyphers to the given Number, as *4, or 6, or 8,* and point them out in the ſa *manner,* as if they were ſignificant Figures in *Integer;* then bring two of them down t ſaid Remainder 715, and annex them thereto, ſo have you 71500 for a new Reſolvend; then find out a new Diviſor, by doubling the Root, as is before directed, and proceed as if the annexed Cyphers were ſignificant Figures, or whole Numbers, as far as you pleaſe, as in this Example, where the Work is carried on till there are three decimal Figures in the Root; and the Work being finiſhed, I find the Root to be 359.994, and there *is* a Remainder of 319964. See the Work.

```
 .  .  .  .  .  .
129596.000000 (359.994
9
-----------
65) 395
    325
    ---
709) 7096
     6381
     ----
7189) 71500
      64701
      -----
71989) 679900
       647901
       ------
719984) 3199900
        2879936
        -------
         319964 remains.
```

But if you proceed to put another Decimal in the Root, you will find it to be 359.9944, and the *Remainder will be* 3196864. Now you may *perceive that the ſaid* Root is too little, becauſe *there is a Remainder,* but yet it is ſo near the truth, that if the laſt Figure thereof were increaſed by an Unit, and ſo made 359.9945 it would then be too much; as you may prove at your leiſure.

XVIII. The ſquare Root of a vulgar Fraction that is commenſurable to its Root, is thus found, *viz.* extract the ſquare Root of the Numerator, for a new Numerator, and likewiſe the ſquare Root of the Denominator, for a new Denominator; ſo ſhall that new Fraction be the ſquare Root of the given Fraction. As for

Example.

Let it be required to extract the ſquare Root of $\frac{25}{36}$; firſt I take the ſquare Root of 25, which is 5, and place it for a new Numerator, then I take the ſquare Root of the Denominator 36, which is 6, and place it for a new Denominator, ſo is $\frac{5}{6}$ the ſquare Root of $\frac{25}{36}$, which was required: In like manner if $\frac{4}{9}$ were given to have its ſquare Root extracted, its Root would be found to be $\frac{2}{3}$; and $\frac{1}{4}$ is the ſquare Root of $\frac{1}{16}$ the ſame is to be obſerved for any other.

Note. But here Note diligently; before you proceed to extract the ſquare Root of any Fraction, that you reduce it to its loweſt Terms: for it may happen, that in its given Terms, it may be incommenſurable to its Root, but being reduced to its loweſt Terms, it may be commenſurable, and its Root exactly found out. So $\frac{50}{72}$ is incommenſurable to its Root, *but being* reduced to $\frac{25}{36}$, its ſquare Root will be *found to* be $\frac{5}{6}$ as before.

XIX. The ſquare Root of a mix'd Number that is commenſurable to its Root, is thus found out, *viz.* reduce the mix'd Number to an improper Fraction, and then extract the ſquare Root of the Numerator, and the ſquare Root of the Denominator, for a new Numerator, and a new Denominator, as in the laſt Rule.

So if it were required to extract the ſquare Root of $1\frac{11}{25}$; firſt I reduce it to an improper Fraction, and ſo it is $\frac{36}{25}$, whoſe ſquare Root is $\frac{7}{5}=1\frac{4}{5}$: ſo if it were required to extract the ſquare Root of $3\frac{13}{81}$; firſt I reduce the given mixed Number to the improper Fraction $\frac{256}{81}$; and then extract the ſquare Root of the Numerator 256, and I find it to be 16, for a new Numerator: and likewiſe the ſquare Root of 81, the Denominator, which I find to be 9, for a new Denominator, ſo is $\frac{16}{9}=1\frac{7}{9}$ the ſquare Root of the given mix'd Number $3\frac{13}{81}$, which was requir'd.

XX. When you are to extract the ſquare Root of a Fraction that is incommenſurable to its Root, prefix before the given Fraction this Character $\surd$ or $\surd q.$ ſignifying the ſquare Root of that before which it is prefixed; ſo the ſquare Root of $\frac{24}{29}$ is thus expreſſed, $\surd\frac{24}{29}$, or $\surd\ q.\ \frac{24}{29}$, the like of any other. But if you would know as near as may be the ſquare Root of any ſuch Fraction, reduce it to a Decimal of the ſame Value by the firſt Rule of the ſecond Chapter; but let the Decimal conſiſt of an even Number of Places, *viz.* either of two, four, ſix, eight, or more Places; and the more Places it conſiſteth of, ſo much *the nearer* the Truth will the Root *be; then extract the ſquare* Root of that Decimal *(according to the* Rules before delivered) in every reſpect as if it were a whole Numbe ſo ſhall this Root ſo found be very near the tru Root; and ſo near that if it conſiſt of three Places it ſhall not want $\frac{1}{1000}$ part of an Unit of th true Root, and if of four Places, it ſhall no want $\frac{1}{10000}$ part of an Unit of the Truth.

So if I would extract the ſquare Root of $\frac{3}{4}$ firſt I reduce it to a Decimal, which I find to b .75, and becauſe I would have the Root to conſi of four Places, I annex ſix Cyphers thereto, an it makes .75000000; then extracting the ſquar Root thereof, as if it were a whole Number, I find it to be .8660, and there is a Remainder o 4400; but if I would have it conſiſt of five Places, then I annex 2 more Cyphers to the ſaid Remainder, and make it 440000, and proceed, an then I find the Root to be .86602, and the Remainder to be 93596.

XXI. In like manner, if it were required t extract the ſquare Root of a mix'd Number incommenſurable to its Root, as near as may be firſt reduce the fractional Part to a Decimal, bu let it conſiſt of an even Number of Places, *viz* of 2, 4, 6, 8, or more Places, then proceed t extract its ſquare Root, according to the Rule formerly delivered in this Chapter, in every reſpect, as if it were a whole Number, ſo ſhall th Root ſo found be very near the Truth, and th more Places it conſiſteth of, ſo much the neare will it be to the true Root. And note, That i the Root there will be ſo many Decimal Places as you placed Points over the decimal Part of th ſquare Number.

So if it were required to extract the ſquar Root of 28 $\frac{6}{13}$, firſt I reduce the fractional par

$\frac{6}{13}$ to a Decimal, and it makes .461538, ſo then the mixed Number, whoſe ſquare Root I am to extract is 28.461538, which being pointed, and the Work of Extraction finiſhed, according to the former Rules, I find its ſquare Root to be very near 5.334, and there is a Remainder of 9982. But if I had proceeded yet farther, and made the decimal Part to have conſiſted of ſeven Places, it would have had for its ſquare Root 5.3349, which doth not want $\frac{1}{10000}$ part of an Unit of the true Root.

But if you would not extract the ſquare Root of ſuch a mix'd Number, then prefix before it this Character, $\surd$ or $\surd q$. ſo if the ſaid mix'd Number 28 $\frac{6}{13}$ were given, I would expreſs its ſquare Root thus, *viz.* $\surd\, 28 \frac{6}{13}$, or $\surd\, q.\, 28 \frac{6}{13}$: The like is to be underſtood of any other.

XXII. When you are to extract the ſquare Root of a Decimal Fraction, which hath 2 or 3 Cyphers poſſeſſing the two or three firſt Places on the left hand of the given Decimal, then cut off 2 of them with a Daſh of the Pen, and put a Cypher to poſſeſs the firſt place of the Root, and proceed to extract the ſquare Root of the remaining Figures, according to the former Rules, as if there had been no ſuch Cyphers before the given Decimal; and if the given Decimal have 4 Cyphers before it, cut them off with a Daſh of the Pen, and put 2 Cyphers in the Root, and then proceed as before.

So if it were required to extract the ſquare Root of $\frac{1}{169}$; firſt I reduce it to a Decimal Fraction, and it makes .005910, then I cut off the two firſt Cyphers, and place one Cypher in the Root, then I proceed to extract the ſquare Root of the remaining Figures, *viz.* 5910, as if had been no ſuch Cyphers before them, find the Root to be very near .077, as yo try at your leiſure.

The Proof of the Extraction of the Square Root.

XXIII. The Operation in the Extracti the ſquare Root is thus proved, *viz.* Multip Root into itſelf, and (if be no Remainder afte Work of Extraction is fin the Product (if the Wor truly done) will be equal Number firſt given. As in the firſt Exa where it is required to extract the ſquare of 2304, which is there found to be 48. if I multiply 48 by itſelf it produceth which is the given Number, and therefore clude the Operation to be true. But if aft Work of Extraction is finiſhed, there i Remainder, then when you have multiplie Root by itſelf, to the Product add the ſai mainder, and if the Sum be equal to the Number, the Operation is right, otherwiſ As in the Example of the Seventeenth where it is required to extract the ſquare of 129596, and there is found to be 35 and the Remainder is 319964. Now to the Work, I multiply the Root 359.994 ſelf, and it produceth 129595.680036, ſhould be 129596, therefore to the ſaid P I add the ſaid Remainder 319964, and th is 129596, and therefore I conclude the to be truly wrought.

CH

CHAP. X.

The Extraction of the Cube Root.

I. A Cube Number is that which is produced by multiplying any Number into itſelf, and again into that Product, which ſaid given Number is called the Cube-Root.

As ſuppoſe 5 were given to find its Cube; firſt I multiply 5 into itſelf, and it produceth 25, which is called the Square of 5; then I again multiply 25, the ſaid Square, by 5, and it produceth 125, which is called the Cube of 5. And here note, That as 125 is called the Cube of 5, ſo is 5 called the Cube Root of 125.

II. The Extraction of the Cube Root is nothing elſe than when by having a Cube Number given we find out its Cube Root; which ſaid Cube Number given is always ſuppoſed to be a certain Number of little Cubes, comprehended within one intire great Cube, which ſaid Cube may very well be repreſented by a Dye, or any other ſolid Body, having its Length, Breadth, and Depth equal; this being ſuppoſed, let there be laid 9 Dyes, conſiſting of a Square, whoſe Side ſhall be 3, and upon them let there be laid 9 more, then will there be in all 27 Dyes, which will conſtitute one greater Cube, whoſe length, breadth and depth will be 3 Dyes, and this greater Cube comprehendeth 27 leſſer Cubes. Now the Extraction of the Cube Root is, by having the Number of little Cubes (27) comprehended in the greater given Cube, to find out how many of the leſſer Cubes make up the Side of the greater.

III. A Cube Number is either Simple or Compound.

IV. A Simple Cube Number is that which hath for its Root or Side, one of the 9 Digits, and it is therefore always leſſer than 1000; ſo ſhall you find that 343 is a Simple Cube Number, whoſe Side or Root is 7, for $7 \times 7 \times 7 = 343$, all which ſaid Simple Cubes and Squares, as alſo their Roots, are expreſſed in the Tablet following.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81
Cubes	1	8	27	64	125	216	343	512	729

V. A Compound Cube Number is that which is produced by the Multiplication of a Number conſiſting of two places (at the leaſt three times) into it ſelf continually, and is therefore never leſs than 1000, ſo 1728 is a Compound Cube Number, produced by the Multiplication of 12 into its ſelf 3 times, for $12 \times 12 \times 12 = 1728$.

VI. When a Compound Cube Number is given to have its Cube Root extracted, before you can go about it, you muſt prepare it for the Work by pointing it; which is thus done, *viz.* put a point over the firſt Figure towards the right hand, *viz.* over the place of Units, then (paſſing the two next places) put a point over the fourth Figure, or place of Thouſands, and ſo proceed by putting a point over every third Figure, as you did over every ſecond Figure in the Extraction of the ſquare Root, till you have finiſhed your pointing; that being done, on the right hand of the ſaid Cube Number draw a crooked Line, behind which to place its Cube Root, as you do to place the Quotient in Diviſion, as in the following Example.

Example 1.

Let it be required to extract the Cube Root of 262144.

$26\dot{2}14\dot{4}$ (

In order to prepare this Cube Number for the Extraction of its Cube Root, I firſt put a point over the firſt Figure (4) towards the right hand, and then over-paſſing the two next Figures (14) I put another point over the fourth Figure (2) and then is the given Number diſtributed into ſeveral parts not unfitly called Cubes, *viz.* 262 (as far as the firſt point goeth) is the firſt Cube, and the 144 (from thence to the ſecond point) is the ſecond Cube, and then I draw a crooked Line behind it, as you ſee in the Margin.

VII. Having proceeded thus far, find out
Cube Root of the firſt Cube 262, but becauſe
is not an exact Cube Number, take the Cube R
of that Number in the foregoing Tablet, wh
being leſſer than it is, yet is neareſt to it, wh
I here find to be 6, and place it behind
crooked Line for the firſt Figure in the Root,
you ſee in the following Work.

$$26\dot{2}14\dot{4}\ (6$$

VIII. This being done, Cube the ſaid Num
which is placed in the Root, and ſubſcribe
Cube under the firſt Cube of the given Numb
So in this Example 216, being the Cube of
place it under 262, the firſt Cube of the gi
Number 262144, as followeth.

$$\begin{array}{l} 26\dot{2}14\dot{4}\ (6 \\ 216 \end{array}$$

IX. Draw a Line under the Cube thus ſubſc
bed, and ſubſtract it from the firſt Cube of
given Number, placing the Remainder ord
underneath the ſaid Line. So 216, the Cube o
being ſubſtracted from 262, the Remander is
which I place underneath the Line, as follow

262144 (6
216

46

X. Bring down the next Cube Number, and annex it to the ſaid Remainder on the right hand thereof. So 144 being the next Cube, I bring it down and annex it to the Remainder 46, and it makes 46144, which by Artiſts is uſually called the Reſolvend.

262144 (6
216

46144 Reſolvend.

XI. Draw a Line underneath the Reſolvend, then triple the Root, that is, multiply it by 3, and place its Triple under the Reſolvend in ſuch order, that the place of Units in the ſaid Triple may ſtand under the place of Tens in the Reſolvend. So the Triple of 6 is 18, which I place under the Reſolvend, ſo that 8 (the place of Units in the ſaid Triple) may ſtand under 4 in the place of Tens in the Reſolvend, as you may ſee in the following Work.

262144 (6
216

46144 Reſolvend.

18

XII. Square the ſaid Root, and then triple th ſaid ſquare of the Root, and place the ſaid tripl ſquare under the ſaid triple Root, in ſuch order that the place of Units in the triple ſquare o the Root may ſtand underneath the place o Tens in the triple Root: ſo in this Example, the ſquare of the Root 6 is 36, and the triple thereof is 108, which I place under 18, the triple Root; ſo that 8, the place of Units in the ſaid triple ſquare of the Root, may ſtand under 1, the place of Tens in 18, the ſaid triple Root, as followeth.

262144 (6
216

46144 Reſolvend.

18
108

XIII. Draw a Line underneath the ſaid tripl *Root, and* triple ſquare of the Root, as they ar *placed,* and add them together in the ſame ord they ſtand, ſo ſhall their Sum be a Diviſor. n our Example, a Line being drawn under 18 108, and they added together in the ſame ler as they ſtand, the Sum is 1098 for a Divi-, as in the following Work.

262144 (6
216

46144 Reſolvend.

18
108

1098 Diviſor.

XIV. Draw a crooked Line on the left hand the Reſolvend, before which to place the ſaid viſor, and let the whole Reſolvend (except place of Units therein) be eſteemed a Divi-ld; then ſeek how often the ſaid Diviſor is tained in the Dividend, and put the Anſwer the next Figure in the Root. So in our Ex-ole, ſeek how often 1098, the Diviſor, is con-ıed in 4614, the Dividend, obſerving here the al Rules of Diviſion, and the Anſwer I find to 4, which I place for the next Figure in the ıot, as in the Example.

262144 (64
216

1098) 46144 Reſolvend.

18 The Triple Root.
108 The Triple Square of the

1098 Diviſor.

XV. Draw a Line underneath the whole
and then Cube the Figure laſt placed in th
and place its Cube underneath the Reſol
ſuch ſort that the place of Units in the o
ſtand under the place of Units in the ot
in our Example 64 being the Cube of 4 (
gure laſt placed in the Root) I place it un
Reſolvend in ſuch manner that the Figu
the place of Units of the Cube 64, may ſt
der 4, the place of Units in the Reſolven
then the Work will ſtand as followeth.

262144 (64
216

1098) 46144 Reſolvend.
18 The triple Root.
108 The triple Square of the Root.

1098 Diviſor.

64 The Cube of 4.

XVI. Square the Figure laſt placed in the Root, and multiply its Square by the triple Root ſubſcribed underneath the Reſolvend, as is directed in the eleventh Rule of this Chapter, and ſubſcribe the Product under the Cube laſt put down, in ſuch order, that the Place of Units in the ſaid Product may ſtand under the Place of Tens in the ſaid Cube. So in our Example, the Figure laſt plac'd in the Root is 4, which ſquared is 16, and 16 multiplied by 18, the triple Root before ſet down, the Product is 288, which I place under 64, the Cube of 4, in ſuch ſort that 8, in the place of Units of the ſaid Product, may ſtand under 6, the Place of Tens, in the ſaid Cube of 4. View the Work.

262144 (64
216 Cube of 6.

1098) 46144 Reſolvend.

18 Triple Root.
108 Triple ſquare of the R

1098 Diviſor.

64 Cube of 4.
288 The ſquare of 4 in
triple Root.

XVII. Multiply the triple Square of the R
ſubſcribed as is before directed in the twelfth
of this Chapter, by the Figure laſt placed in
Root, and place the Product under the Nu
laſt ſubſcribed, which is the Product of
ſquare of the Figure laſt placed in the Root,
tiplied by the ſaid triple Root, in ſuch ma
that the Place of Units of this may ſtand u
the Place of Tens in that; as in this Exam
the triple Square of the Root is 108, w
multiplied by 4, the Figure laſt placed in
Root, the Product is 432, which I place u
288, the Number laſt ſubſcribed, in ſuch
that the Figure 2, in the place of Units of
ſaid laſt Product, may ſtand under 8, whi
in the place of Tens of the ſaid Number
ſubſcribed, and then the Work will ſtand as
loweth.

262144 (64
216

46144 Reſolvend.

18 Triple Root.
108 Triple Square of the Root.

1098 Diviſor.

64 Cube of 4.
288 Square of 4 in the triple Root.
432 Triple Square of the Root in 4.

VIII. Draw another Line under the Work, l add the three Numbers together, that were placed under the Diviſor, in the ſame order hey there ſtand, and let their Sum be called Subtrahend, which let be ſubtracted out of Reſolvend, nothing the Remainder; ſo in Example I add 64, 288, 432, together in ſame order as they ſtand, and their Sum is 44, the Subtrahend, which I ſubtract out of 44, the Reſolvend, and there is nothing aineth, ſo the whole Work is finiſhed, and nd the Cube Root of 262144 to be 64, hout any Remainder. See the whole Work olloweth.

262144 (64
216

1098) 46144 Reſolvend.

18 Triple Root.
108 Triple Square of the Root.

1098 Diviſor.

64 Cube of 4.
288 Square of 4 in the triple Ro
432 Triple Square of the Root

46144 Subtrahend.

(0)

Now the Learner is to obſerve three thin
general from the Rules before deliver'd,
cerning the Extraction of the Cube-Root.

Obſerve 1. That the Work contain'd i
ſeventh, eighth, and ninth Rules for findin
the firſt Figure of the Root, is not again
repeated throughout the whole Work of E
tion, altho the Root conſiſt of ever ſo many
ces; but the Work of all the Rules follow
to be repeated as often as a new Figure is p
the Root.

Obſerve 2. For every particular Cube in
Number given, diſtinguiſhed by the Points,
cept the firſt) there is to be found out a new
ſolvend, by annexing the next Cube to th
m:

mainder, according to the Tenth Rule; and as often as there is a Reſolvend, ſo often muſt there be found a new Diviſor, by the eleventh, twelfth, and thirteenth Rules; and as often as there is a new Diviſor, ſo often muſt there be found a new Subtrahend, according to the fifteenth, ſixteenth, ſeventeenth, and eighteenth Rules before-going.

Obſerve 3. When the Subtrahend chanceth to be greater than the Reſolvend, then you may conclude there is an Error in your Work, which muſt be corrected by putting a leſſer Figure in the Root.

Example 2.

Let it be required to extract the Cube Root of 48627125.

Having prepared the given Number for the Work of Extraction, according to the ſixth Rule of this Chapter, I find it to be diſtributed into 3 Cubes, *viz.* 48 the firſt, 627 the ſecond, and 125 the third. Then I proceed to the Work; and firſt I find the Cube Root of 48 the firſt Cube, which is 3, then do I cube 3, and place its Cube, which is 27, under 48 the firſt Cube, and ſubſtract it therefrom, and the Remainder is 21, according to the ſeventh, eighth and ninth Rules of this Chapter, and then will the Work ſtand as you ſee in the Margin.

```
 .  .  .
48627125 (3
27
————————
21
```

Then, to the ſaid Remainder 21 do I bring *down, and thereto annex* the next Cube, which is 627, and it makes 21627 for a Reſolvend, a cording to the 10th Rule foregoing, Then dc find out a Diviſor, according to the 11th, 12t and 13th Rules of this Cha ter; and firſt I triple the Ro 3, and it makes 9, which place under 2, the Place Tens in the Reſolvend; th do I ſquare the ſaid Root and that makes 9; then I triple its ſquare 9, and th makes 27, which I place und the ſaid Triple in ſuch ord as is directed in the 12th Rul then drawing a Line unde neath the Work, I add the two ſaid Numbe together (*viz.* the triple Root, and the trip ſquare Root) in ſuch order as they are there pl ced, and their Sum is 279 for a Diviſor, as y may ſee in the Margin.

```
. . .
48627125 (3
27
-----------
21627 Reſolv.
-----------
 09
 27
------
 279
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Then, according to the 14th Rule, I ſeek ho often the ſaid Diviſor 279 is contained in 21 the Dividend, and I find the Anſwer to be which I place for the ſecond Figure in the Roo then do I in the next place go about to find o a Subtrahend; and in order thereunto, firſt, a cording to the fifteenth Rule of this Chapter, cube the Figure 6, laſt placed in the Root, a it maketh 216, which I place under the Reſo vend in ſuch order as is directed in the ſaid fi teenth Rule, that the Place of Units of the o may ſtand under the Place of Units of the ther; then according to the ſixteenth Rule this Chapter, I ſquare the Figure (*viz.* 6) l placed in the Root, which makes 36, and th multiply it by 9, the ſaid triple Root, and

Pro

Product is 324, which I place under 216 the ſaid Cube of 6, in ſuch order as is directed in the ſaid ſixteenth Rule; then do I multiply the ſaid triple Square (*viz.* 27) by the Figure 6 laſt placed in the Root, and it Produceth 162, which I place under the laſt Product 324, in ſuch manner as is directed in the ſeventeenth Rule; then I add theſe three ſeveral Numbers together in the ſame order as they ſtand, and their Sum is 19656 for a Subtrahend, which ſubtracted out of 21627 the Reſolvend, the Remainder is 1971, as you may ſee by the following Work.

$\dot{4}8\dot{6}27\dot{1}25$ (36
27

279) 21627 Reſolvend.

09 Triple Root
27

279 Diviſor.

216 Cube of 6.
324 Square of 6 by the triple Root.
162 Triple Square of the Root by 6.

19656 Subtrahend.

1971 Remainder.

Then to the ſaid Remainder 1971, do I an*nex the Cube Number* 125, according to the tenth Rule, and it makes 1971125 for a Reſolvend.

Then I proceed, according to the 11th, 12th and 13th Rules, to find a Diviſor, and therefore firſt triple the whole Quotient 36, and it is 108 then do I ſquare the who Quote 36, and it mak 1296, which being tripled 3888, which being order placed under the triple Quot and added thereto in that o der, the Sum is 38988 for a new Diviſor. S the Work in the Margin.

108
3888
38988

Then do I ſeek how often the ſaid Diviſor contained in the Dividend 197112, and I find to be 5 times contained therein; and accordin ly, I place 5 in the Root, and proceed accordi to the 15th, 16th, 17th, and 18th Rules, to fi out a Subtrahend; and therefore firſt, I cube t Number 5, laſt plac'd in the Root, and it mak 125; then do I ſquare t ſaid 5, and it makes 25, which I multiply 108, t ſaid triple Root, and pla the Product 2700 und the ſaid Cube, as is bef directed; then do I by t ſaid 5, multiply 3888 the triple Square of t Root, and the Product 19440 do I place u der the former Product 2700, according to f mer Directions, and add the three Numbers t gether, in the ſame order as they ſtand, and th Sum is 1971125, as appears *per* Margin, a Subtrahend, which taken out of the ſaid R *ſolvend*, there remaineth 0, and ſo the Work *finiſhed*, and I find the Cube Root of the gi

125
2700
19440
1971125

Number 48627125 to be 365. View the whole Work laid down as followeth.

· · ·
48627125 (365
27

279) 21627 Reſolvend.

9 The triple Root.
27 The triple Square of the Root.

279 Diviſor.

216 The Cube of 6.
324 The Square of 6 in the triple Root.
162 The triple Square of the Root in 6.

19656 The Subtrahend.

38988) 1971125 Reſolvend.

108 The triple Root.
3888 The triple Square of the Root.

38988 Diviſor.

125 The Cube of 5.
2700 The Square of 5 in the triple Root.
19440 The triple Square of the Root in 5.

1971125 The Subtrahend.

(0)

XIX. When *it is required* to extract the Cube *Root of a Number* that is incommenſurable to its Root, and you are deſirous to know the fractional part of the Root as near as may be, yo are to annex to the given Number a competen Number of Cyphers, which Number of Cypher muſt be always a Multiple of 3, *viz.* either 3, 6, 9, 12, &c. Cyphers; that is, 000, 000000, or 000000000, &c. And having obſerved the 6th Rule for the Punctation of the given Number, likewiſe point the annexed Cyphers, in the ſame manner as if they were ſignificant Figures or Integers; and obſerve, that as many points as you put over the integral part, ſo many places will the integral part of the Root conſiſt of; and ſo many points as are put over the Cyphers, o Decimals, ſo many Decimal places will there be i the Root: This being obſerv'd, the Work itſel in the extracting the Cube Root of a Decima Fraction, or of a mix'd number of Integers an Decimals, is the ſame in every reſpect as if th Number given were an Integral Cube Number according to the Rules before delivered in thi Chapter, as in the following

Example.

Let it be required to extract the Cube Roo of 13798, which is a Number incommenſurabl to its Cube Root; and to find out its Root as nea as may be, I annex to it 9 Cyphers; ſo by tha means I ſhall have 3 Decimals in the Root, an prepare it for Extraction, by pointing it as i before directed, and as you ſee following.

· · · · ·
13798.000000000 (

A

d having performed the Work of Extrac-
according to the former Rules, I find its
Root to be 23.984, which, as by the Re-
ler you may perceive, is ſomewhat too little,
t ſo near the Truth, that, if the decimal
were increaſed by an Unit, and ſo made
5, it would then be too much, and ſo con-
tly it cannot want (*as it is*) $\frac{1}{1000}$ part of
it of the truth; and if the Root had had
er Figure plac'd in it, it would then have
ſo near the truth, that it would not have
d $\frac{1}{10000}$ part of an Unit. For your farther
action, ſee the whole Work performed, as
eth.

* * *

	13	798	000	000	000	(23.984 R
	8					
126)	5	798				Reſolvend
		6				
	1	2				
	1	26				
		27				
		54				
	3	6				
	4	167				Subtrahend
15939)	1	631	000			Reſolvend
			69			
		158	7			
		15	39			Diviſor
			729			
		55	89			
	1	428	3			
	1	484	919			Subtrahend
1714347)		146	081	000		Reſolvend
			7	17		
		17	136	3		
		17	143	47		Diviſor
				512		
			458	88		
		137	090	4		
		137	549	792		Subtrahend
172519314)		8	531	208	000	Reſolvend
				71	94	
		1	725	121	2	
		1	725	193	14	Diviſor
					64	
			1	151	04	
		6	000	484	8	
		6	001	635	904	Subtrahend
		2	529	572	096	Remains.

X

XX. If at any time it is required to extract the Cube Root of a Vulgar Fraction, let ſuch Fraction be firſt reduced to its loweſt Terms; becauſe it may not be commenſurable to its Root in the given Terms; but being reduced to its loweſt Terms, it may: and having ſo done, to perform the Work, this is

To extract the Cube Root of a Vulgar Fraction.

The Rule.

Extract the Cube Root of the Numerator, (by the former Rules) and place that for a new Numerator; then extract the Cube Root of the Denominator, and place that Root for a new Denominator, ſo ſhall this new Fraction be the Cube Root of the given Fraction.

As for Example; let it be required to extract the Cube Root of $\frac{27}{64}$; firſt I take the Cube Root of 27 the Numerator, which is 3, and place it for a new Numerator. Then I take the Cube Root of 64 the Denominator, which is 4, and place it for a new Denominator; ſo ſhall this new Fraction $\frac{3}{4}$ be the Cube Root of the given Fraction $\frac{27}{64}$.

In like manner, if there were given $\frac{105}{56}$ to have its Cube Root extracted, I can eaſily diſcover that there cannot be found any Cube Root exactly, either for the Numerator or Denominator, in the terms they are given in; but being reduced to their loweſt terms, they are $\frac{27}{64}$, whoſe Cube Root is $\frac{3}{4}$, as before.

In like manner the Cube Root of $\frac{16}{1024}$ will be *found to be* $\frac{1}{4} = \sqrt{q}.\ \frac{16}{1024}$.

XXI. But when there is given a Vulgar Fr to have its Cube Ro tracted, it being incon ſurable to its Root, yo find its Cube Root ver if you reduce the give gar Fraction to a De and then extract the Root of that Decimal (by the Rules before vered) in every reſpect as if it were a whole ber; and then ſhall that be a Decimal Cube leſs than the truth; and yet ſo near the that if you add an Unit to the laſt decim gure, it will then be greater than the truth.

To extract the Cube Root of a Vulgar Fraction that is incommenſurable to its Root.

Here take Notice by the way, that you gar Fraction being reduced to a De in order to have its Cube Root ext its equivalent Decimal muſt con ſuch a Number of places as may be a Mult 3; that is, it muſt conſiſt of 3, 6, 9, 12, 1 places: and the more places there are in th cimal, the nearer the truth will the Root be

Note.

Example.

Let it be required to extract the Cube R $\frac{5}{8}$; in order whereunto I reduce it to this mal, *viz.* .625, which becauſe it conſiſte of 3 places, and ſo conſequently can hav 1 Figure in its Root, I increaſe to 9 places b nexing 6 Cyphers thereto, thus, .625000000 then the Root will conſiſt of 3 places; ther proceed to extract its Cube Root, accord the former Rules, and find it to be .85

there will be a Remainder of 2164136, as
may prove at your leiſure.

XII. When your given Vulgar Fraction is re-
d to a Decimal of the ſame Value, and the 3
firſt places toward the left hand are poſſeſ-
y Cyphers, then in this caſe you are to cut
; of them with a daſh of the Pen, and for
ı place a Cypher to poſſeſs the firſt place in
Root, and then proceed to extract the Cube
t of the remaining Figures, according to the
er Rules, as if there had been no ſuch Cy-
s at all.

Example.

et there be given $\frac{4}{8237}$ to have its Cube Root
cted: firſt reduce it to a Decimal Fraction
he firſt Rule of the ſecond Chapter of this
k, and it makes .000485613, &c. Now to
ct the Cube Root of this Fraction, I firſt pre-
it, by pointing it in every reſpect as if it
; a whole Number; then with a daſh of my
I cut off the three firſt Cyphers and put a
o poſſeſs the firſt place in the Root, then I
eed to extract the Cube Root of the remain-
igures 485613, as if there had been no Cy-
s at all before them; and having finiſhed the
k, I find its Cube Root to be .078, as by the
wing Work.

000 485613

000|485613 (078
343

1491) 142613 Reſolvend.

21
147

1491 Diviſor.

512
1344
1176

131552 Subtrahend.

11061 Remainder.

In like manner, if the Decimal which is g to have its Cube Root extracted, have 6 Cyp placed before the ſignificant Figures on the hand, then cut off thoſe 6 Cyphers with a of the Pen, and for them put two Cyphe poſſeſs the two firſt places in the Root; proceed to extract the Cube Root of the rem ing Figures, as if there had been no ſuch phers, &c.

To extract the Cube Root of a mixt Number.

XXIII. When it is requ to extract the Cube Roo a mix'd Number, reduc to an improper Fraction, if it hath a perfect (*Root, then* extract the Cube Root of the N rator, and place it for a new Numerator, and alſo extract the Cube Root of the Denominator, and place it for a new Denominator, ſo ſhall this new Fraction be the Cube Root of the given mix'd Number.

Example.

Let it be required to extract the Cube Root of $5\frac{13}{343}$; having reduced it to an improper Fraction, I find it to be $\frac{1728}{343}$, and having extracted the Cube Root of the Numerator 1728, I find its Root to be 12 for a Numerator, and the Cube Root of 343 the Denominator, is 7 for a Denominator: ſo that I conclude $\frac{12}{7}$ or $1\frac{5}{7}$ to be the Cube Root of the given mix'd Number $5\frac{13}{343}$, as you may prove at your leiſure.

XXIV. But if the given mix'd Number, whoſe Cube Root is required, have not a perfect Root, then you are to reduce the fractional part into a Decimal of the ſame Value (but let the Number of Decimal Places be always a Multiple of 3) and then proceed to extract the Cube Root of that mix'd Number, as if it were a whole Number, always reſerving ſo many Decimal Places in the Root as there are Points over the Decimal Part of the mix'd Number.

Example.

Let it be requir'd to extract the Cube Root of $28\frac{3}{4}$; firſt reduce $\frac{3}{4}$ into its equivalent Decimal, which is .75; but to make it conſiſt of ſix Places, I annex *thereto four* Cyphers, and then the ſaid mix'd Number will be 28.750000, which be done, I proceed to the Work as followeth.

28.750000 (3.06, &c.

27

279) 1750 Reſolvend.

9

27

279 Diviſor.

27090) 1750000 Reſolvend.

90

2700

27090 Diviſor.

216

3240

16200

1652616 Subtrahend.

97384 Remains.

So that I find by the Work, the Cube Ro
28.750000 to be 3.06, &c.

XXV. It is uſual amongſt Artiſts to expreſ
Cube Root of a whole Number, mix'd Num
or Fraction; either Vulgar or Decimal, th

ımenſurable to its Root, by prefixing this ıcter, (*viz.* √*c.*) before the incommenſura- Iumber or Quantity: So the Cube Root of may be thus expreſſed, √*c.* 328, and the Root of 24¾ thus, √*c.* 24¾, or in a Deci- ıix'd Number thus, √*c.* 24.75, and of the on ¾ thus, √*c.* ¾, &c.

XVI. The Operation in the Extraction of Cube Root is proved thus,

Cube the Root found out, s, multiply it three times itſelf; and if any thing n after the Work is done t to the laſt Product, and if that Sum be e- to the given Number, then the Work is performed, otherwiſe not.

The Proof of the Extraction of the Cube Root.

in our firſt Example, where it is requir'd tract the Cube Root of 110592, which is to be 48; and to prove the Work mul- 48 by itſelf, whoſe Product is 2304, which again multiplied by 48, produceth 110592, ı is equal to the given Number, and there- [conclude the Work to be right.

ewiſe to prove the Example of the nine- ı Rule, where it is required to extract the Root of 13798, which is found to be the Number 23.984: Now to prove the Work, the Root, as before directed, and find it 13796.370427904, to which add the Re- ler 1.629572096, and their Sum maketh the Number 13798, which proves the Work right.

CHAP. XI.

The Uſe of the Square and Cube in ſolving ſome Queſtions Arit tical and Geometrical.

PROP. I.

To find a mean Proportional betwee given Numbers.

MUltiply the given Numbers, the c the other, and extract the ſquare R the Product, ſo ſhall that ſquare Root b mean Proportional ſought.

Example.

Let the given Numbers be 12 and 48, a it be required to find a mean Proportiona tween them; firſt multiply the given Nu 12 and 48, the one into the other, and Product is 576, the ſquare Root of which i ſo that I conclude 24 to be a mean Propor between 12 and 48: for

12 : 24 :: 24 : 48

The Square of the Mean being equal to the Product of the Extremes.

This Propoſition is uſeful in finding the Side of the Square that ſhall be equal to any given Parallelogram; for (according to the firſt Propoſition of the eighth Chapter of this Book) if you multiply the contiguous Sides of a rectangular Parallelogram, the one by the other, that Product will be its Content; and if you extract the ſquare Root of that Content, it will give you the Side of a Square (in the ſame meaſure your Parallelogram was) which will be equal to the given Parallelogram.

PROP. II.

To find the Side of a Square that ſhall be equal to the Content of any given Superficies.

FIND out the Content of the given Superficies, by the Rules laid down in the eighth Chapter, and then extract the ſquare Root of the Content, ſo will that Root be the Side of a Square equal to the given Superficies.

Example.

There is a rectangled Triangle, whoſe Baſe and Perpendicular are 6 and 8, I demand the Side of a Square that will be equal to the given Triangle.

According to the ſecond Propoſition of the eighth Chapter, I find the Content of this Triangle to be 144, the ſquare Root of which is 12, and is the Side of a Square equal to the ſaid Triangle.

In like manner, if you extract the ſqua of the Content of a Circle, Pentagon, H *&c.* or of any other Figure, regular or ir it will give the Side of a Square equal to t perficies.

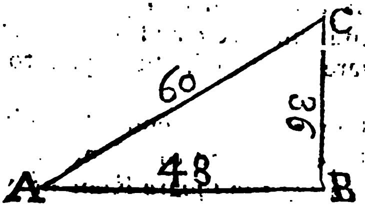
PROP. III.

Having any two of the Sides of a rig gled plain Triangle, given, to fin third Side.

THIS moſt excellent and uſeful Prop is generally called *Pythagoras's* The and in the forty ſeventh Propoſition of Elements of Geometry it is demonſtrated proved, that the Square made of the Hy nuſe, or ſlant Side of a right-angled plai angle is equal to the Sum of the Squares of the Baſe and Perpendicular.

As for Example.

In the Triangle A B C, the Baſe A B and the Perpendicular B C is 36; now mand the length of the Hypothenuſe A C.



find out an Anſwer to this, firſt I ſquare
ſe AB (48) which is 2304, then ſquare
rpendicular (36) and its Square is 1296,
n of which two Squares is 3600, which is
o the Square of the Hypothenuſe AC,
re the ſquare Root of 3600 will give the
of AC, which is 60.

PROP. IV.

[ERE is a Tower, about which there is
Moat that is 48 Foot wide, and a Sca-
adder that is 60 Foot long, which will
rom the outſide of the Moat, to the top
'all that is within the ſaid Moat; now I
d the Height of the ſaid Wall above the

the Baſe AB, in the foregoing Triangle, be
adth of the Moat, and let the Hypothenuſe
the Scaling-Ladder, then is the Perpendi-
C the Height of the Wall above the Wa-
Now it is plain, that (becauſe the Square
is equal to the Sum of the Squares of
d BC) if from the Square of AC, which
, you ſubſtract the Square of AB, which
, there will remain 1296, which is the
of CB, therefore I extract the ſquare
f 1296, and find it to be 36, which is
ight of the ſaid Wall above the Water, as
quired.

he help of this Propoſition may be found
e perpendicular Height of a Cone, or of
mid; for, in a Cone, if you ſquare the
Height, (*which is* the Length of a Line
from its vertical Point to the Circumference

of its Baſe) and from the Square of tha ſtract the Square of the Semi-diameter Baſe, there will remain the Square of t pendicular Height of that Cone.

Alſo (in a Pyramid) if from the Squar ſlant Height of it, you ſubſtract the Sq that Line, which being drawn from the Ce its Baſe, ſhould touch the end of the ſa Line, (whether they meet at an Angle the Remainder will be the Square of tl pendicular Height of that Pyramid, a ſquare Root will give the Height itſelf.

P

PROP. V.

By the Content of a Circle to find its Diameter.

The Proportion is,

As 22
Is to 28,
So is the given Content
To the Square of the Diameter.

Example.

There is a Circle whoſe Superficial Content is 153.9385, I demand its Diameter.

22 : 28 : : 153.9385 : 195.9217.

The Square Root of which is 13.99 (very near 14) for the Diameter required.

PROP. VI.

By the Content of a Circle to find its Circumference.

The Proportion is,

As 7
Is to 88,
So is the given Content
To the Square of the Circumference.

The ſquare Root of which is the Circumference required.

Example.

There is a Circle whoſe Superficial Content is 153.9385; I demand the Circumference of that Circle.

7 : 88 :: 153.9385 : 1935.2268.

The ſquare Root of which is 44 *fere*, which is the Circumference required.

The Cube Root is that by help of which we reſolve all Queſtions Mathematical that concern Solidity, and by which we increaſe ſolid Bodies according to any given Proportion; by it we diſcover the Solidity of a Body that is capable of length, breadth, and depth (or thickneſs) and by having the Solidity given, we diſcover the Side or Diameter of ſuch a Body.

Some Queſtions pertinent thereto may be ſuch as follow.

PROP. VII.

There is a Cube whoſe Side is 4; I demand what ſhall be the Side of a Cube whoſe Solidity is double to the Solidity of that Cube.

To anſwer this Propoſition, find out the Cube of 4 (the Side of the given Cube) which is 64, and double it, which is 128, then extract the Cube Root of 128, and it makes 5.0397 *fere*, and *that is the* Side of the Cube which is double to *the Cube* whoſe Side is 4.

PROP.

PROP. VIII.

Here is a Cube whoſe Solidity is 128 Foot;
I demand the Side of a Cube whoſe Soli-
is half as much.
ke ½ of 128=64, the Cube Root of which
4) anſwers the Queſtion.

PROP. IX.

Aving the ſolid Content of a Globe, to find the Side of a Cube whoſe Solidity ſhall be
to the given Globe.
tract the Cube Root of the given ſolid Con-
f the Globe, and it will give you the Side
Cube required.

Example.

ere is a Globe whoſe ſolid Content is 1728
s; I demand the Side of the Cube equal
o.
ving extracted the Cube Root of 1728, I
t to be 12, which is the Side of the Cube re-
d.

PROP. X.

Aving the Diameter and Weight of a Bullet, to find the Weight of another Bullet
Diameter is given.

As the Cube of the given Bullet's Diameter
Is to its Weight or Solidity,
So is the Cube of the Diameter of any other
To its Weight or Solidity.

Example.

There is a Bullet whoſe Diameter is 4 In and its Weight 9 Pound; I demand the W of another Bullet, whoſe Diameter is 6 $\frac{1}{4}$, or Inches.

The Cube of 4 is 64.
The Cube of 6.25 is 244.140625.

Then I ſay,

64 : 9 :: 224.140625 : 34.3322

So that the Weight required is 34.33227 Po and if you reduce the Decimal to the known of *Averdupois* Weight, you will find the An to be 34 *lb.* 5 *oz.* 5 *dr.*

This kind of Proportion is by Artiſts te triplicate Proportion.

In like manner, the Diameters of two Bu or Globes, being given, and the Solidity of o them, to find out the Solidity of the other, it be done by the ſame Proportion, only chan the middlemoſt Term.

PR

PROP. XI.

TO find the Side of a Cube equal to a given Parallelepipedon.

Find out the Solidity of the given Parallelepipedon by the 8th Propoſition of the 8th Chapter, then is the Cube Root thereof the required Side.

Example.

There is a Parallelepipedon, having the Sides of its Baſe 10 Foot 4 Inches, and 5 Foot 2 Inches, and its length is 20 Foot 8 Inches; I deſire to know what is the Side of a Cube whoſe Content ſhall be equal to the given Parallelepipedon.

The ſuperficial Content of the Baſe is 7688 Inches, which drawn into 248, the length in Inches, the Product is 1906624 Inches for its ſolid Content, the Cube Root of which is 124 Inches, for the Side of a Cube equal to the given Parallelepipedon.

In like manner, if you would find at any time the Side of the Cube equal to any ſolid Body, whether regular or irregular; firſt find the ſolid Content of that Body, and then extracting the Cube Root of its ſolid Content, you have your deſire.

PROP. XII.

BEtween two given Numbers to find two mean Proportionals.

Divide the greater Extreme by the leſſer, and *extract the Cube Root* of the Quotient, and by the ſaid Cube Root multiply the leſſer Extreme, t will the Product give you the leſſer mean Pro tional; then multiply the ſaid leſſer Mean by ſaid Cubick Root, and that Product will give the greater mean Proportional.

Example.

Let the two given Extremes be 6 and 48, tween which it is required to find two n Proportionals

Firſt I divide 48 the greater Extreme, b the leſſer Extreme, and the Quotient is 8; Cube Root of which is 2; then by (the C Root) 2 I multiply 6 (the leſſer Extreme) the Product is 12 for the leſſer mean Propoı nal; and 12 being multiplied by 2 (the C Root) the Product is 24 for the greater r Proportional ſought. Thus have I found 12 24 to be two mean Proportionals between 6 48; for

6 : 12 :: 24 : 48.

In like manner between 3 and 81 will be f 9 and 27 for two mean Proportionals.

PROP. XIII.

THE Concave Diameter of two Guns b known, and the quantity of Gunpo that will charge one of them, to find out much will be ſufficient to charge the other.

The Capacities are one to another, as are *Cubes of their* Diameters; and alſo the Pro *tion is direct.*

I E

Example.

f 25 Pounds of Gunpowder be ſufficient to
rge a Gun whoſe Concave Diameter is 1 ½
ı, or 1.5 Inch, how much Powder will be
icient to charge a Gun whoſe Concave Diame-
is 7 Inches? Anſwer, 25.47.
he Cube of 1.5 is 3.375, and the Cube of 7 is
; wherefore the Proportion is as followeth.

3.375 : .25 :: 343 : 25.47.

Or thus,

3.375 : 343 :: .25 : 25.47.

PROP. XIV.

HE Concave Diameters of two Guns being
given, and the quantity of a weaker ſort of
ıpowder ſufficient to charge one of them, to
how much Gunpowder of a ſtronger ſort (the
portion of the Strength and Weakneſs of the
ıpowder being alſo given) will be ſufficient
harge the other Gun.
his is ſolved by two Operations in the Rule
roportion: Firſt, to find out how much of the
nger ſort of Gunpowder will be of equivalent
ngth with the given quantity of the weaker
, and this Proportion is reciprocal. The ſe-
d is the ſame with that in the foregoing Pro-
tion.

Example.

There is a Gun whoſe Concave Diameter is $1\frac{1}{2}$ Inch, and it requireth .25 Pounds of Powder to charge it; now there is another ſort of Gunpowder which is much ſtronger than the former, and the Proportion between their ſtrength is as 5 to 2; now I demand how much of the ſtrongeſt Powder is ſufficient to charge a Gun whoſe Concave Diameter is 7 Inches.

To anſwer this, firſt I find out how much of the ſtrongeſt Powder will charge that Gun which is $1\frac{1}{2}$ Inch in its Concave Diameter; which is done by the following Proportion, *viz.*

5 : 2 :: .25 : .10.

Thus have I found that $\frac{1}{10}$ of a Pound of the ſtrongeſt Powder will charge a Gun whoſe Concave Diameter is $1\frac{1}{2}$ Inch. And according to the laſt Propoſition, I find by a direct Proportion that 10.16 Pounds of the ſame will be ſufficient to charge a Gun whoſe Concave Diameter is 7 Inches, *viz.*

3.375 : 343 :: .10 : 10.16.

* * *

CHAP.

CHAP. XII.

Concerning Simple Intereſt.

WHEN Money pertaining or belonging to one Perſon is in the Hands, Poſſeſſion, eeping, or is lent to another, and the Debtor th or alloweth to the Creditor a certain Sum, nſideration of Forbearance for certain time, Conſideration for Forbearance is called Inte- Loan, or Uſe-Money; and the Money ſo and forborn is called the Principal.

Intereſt is either Simple or Compound.

I. When, for a Sum of Money lent, there is or Intereſt allowed, and the ſame is not when it becomes due; and if ſuch Intereſt not then become a part of the Principal, it lled Simple Intereſt.

I. In the taking of Intereſt for the Continu- or Forbearance of Money, reſpect muſt be to the Rate limited by Act of Parliament; h Act now in force, forbiddeth or reſtraineth erſons whatſoever from taking more than *6 l.* the Intereſt of 100 *l.* for a Year, and accor- g to the ſame Proportion for a greater or leſſer , not confining the Lender or Borrower to the e of one Year *or more,* any more than it con- *th him or them to the* Limitation of the Sum *e lent or borrowed,* but that the Sum may be either more or leſs than 100 *l.* and may continue in the hands of the Debtor, either a longer or a ſhorter time than one Year, according as the Lender and Borrower do agree, and oblige each other. Now for any time greater than one Year the Rate or Proportion of Intereſt is by Act of Parliament limited, but the Act doth not ſay what part of 6 *l.* ſhall be the Intereſt of 100 *l.* for half a Year, a Quarter of a Year, a Month, a Day, or for any time leſſer than one Year: and in this Caſe ſeveral Artiſts do differ in their Opinions; ſome would have the true proportional Intereſt for any time leſs than a Year, to be diſcover'd by mean Proportionals: as ſuppoſe it were requir'd to know the Intereſt of 100 *l.* for half a Year, at 6 *per Cent. per Annum*, they would have the Intereſt to be reckoned after the Rule of Compound Intereſt, and ſo 30 *l.* is not the Intereſt of 100 *l.* for half a Year, but is too much: But ſay they, to find out the true Intereſt thereof, you are to find a mean Proportional between 100 and 106, and that made leſs by a 100, will give you the Intereſt of 100 *l.* for half a Year and ſo by extracting of Roots they find out the Intereſt for any leſs time than one Year, but this is ſufficiently laborious and painful if it be done without the help of Logarithms; but to perform this Work to the 12th Power for a Month, or to the 52d for a Week, is very tedious, and to the 365th Power for one Day, is ſcarcely poſſible to be effected by natural Numbers; but Cuſtom and daily Practice tell us, that the Intereſt of Money for any time leſs than one Year, ought to be computed according to the Rules of Simple Intereſt, and ſo 3 *l.* is the undoubted Intereſt of 100 *l.* for 6 Months, and 30 Shillings is the Intereſt

10

or a quarter of a Year; but here note by
, that by 6 Months is not meant 6 times 4
or 6 times 28 Days; but ſix Months, or
ear, is to be underſtood the half of 365
nd a quarter of a Year is ¼ of 365 Days,
1 Month is underſtood $\frac{1}{12}$ of 365 Days,
Month conſiſteth of 30 $\frac{5}{12}$ Days.

the aforeſaid Cuſtom of computing the
of Mony for time leſs than one Year,
owing Analogy ſeems
ſumed for a ſafe Expo-
f the Statute, (and
is indeed the ground
on itſelf of Simple In-
viz. that ſuch Propor-
365 (or one Year) hath
ntereſt of any Sum for
ſuch Proportion hath any part of one
r any Number of Days propounded to
reſt of the ſame Sum, for that Time pro-
l. And this (as was ſaid before) is the
Ground-work, and very Foundation of
ner of computing of Simple Intereſt.

Vid. *Sect.* VI. *of the 5th Chapter of Mr.* Kerſey's *Appendix to* Wingate's *Arithmetick.*

ebate or Diſcount, is when there is an Al-
of ſo much *per Cent.*
ney paid before it be
nd as the Increaſe of
at Intereſt is found out
inual Proportionals, arithmetically or geo-
ly increaſing, ſo is the Rebate or Diſcount
ey found out by continual Proportionals,
ng arithmetically or geometrically, that is
g as the Allowance is, either after Simple
pound Intereſt: Now the Nature of Re-
Diſcount is thus, when there is a Sum of

Of Rebate, *what it is.*

Money (ſuppoſe 100 *l.*) to become due at
end of a certain time to come (*viz.* at the
of 12 Months) and it is agreed upon by
Debtor and Creditor that there ſhall be made
ſent Payment of the whole Debt, and it is
wiſe agreed that in conſideration of this pr
Payment, the Creditor ſhall allow the De
after the rate of *6 per Cent. per Ann.* Now
this Agreement the Creditor ought to recei
much Money as being put out to Intereſt fo
ſame time it was paid before it was due,
at the ſame rate of Intereſt that the Diſc
was reckoned at, then would it amount or b
creaſed to the Sum that was firſt due.

The manner of working Queſtions in Re
at ſimple Intereſt, ſhall be ſhewn in the
Rule of this Chapter, and that of working Q
tions in Rebate at compound Intereſt in
fourth Rule of the next Chapter.

VI. When the Intereſt of 100 *l.* for a Ye
known, the Intereſt of any other Sum, for the
time, is alſo found out by a ſingle Rule of d
Proportion, *viz.* the Intereſt of 100 *l.* for a
by the Statute, is *6 l.* I demand what is th
tereſt of 75 *l.* for the ſame time, and at the
rate of Intereſt. The Proportion is as follo

l.	*l.*		*l.*		*l.*	*l.*	*s.*
100	: *6*	: :	75	:	4.5	=4—	10

Or if you would have the Anſwer to pro
both Principal and Intereſt, then make th
cond Number to be the Sum of the given Pr
pal and Intereſt, and the fourth Proport
will anſwer your Deſire. Thus,

l. *l.* *l.* *l.* *l.* *s.*
100 : 106 :: 75 : 79.5=79—10.

VII. When the Intereſt of 100 *l.* for a Year is 'en, and the Intereſt of any other Sum of Pounds, llings and Pence is required for a Year, the An- er may be eaſily found, after the practical Me- d delivered in the following Example.

Let it be required to find the Intereſt of 148 *l.* *s.* 4 *d.* for one Year after the rate of 6 *per Cent.* *Annum*, Simple Intereſt.

Firſt I place the given Numbers, according to direction given for the *Rule of Three*, which ll ſtand thus:

l. *l.* *l.* *s.* *d.*
100 : 6 :: 148—13—04.

Now it is evident, that, if I multiply 148 *l.* 13 *s.* (which is the third Number) by 6 (which is ſecond Number) and divide the Product by (which is the firſt Number) the Quotient ll be the Anſwer: therefore I proceed thus, firſt I multiply the Pence by 6, which makes Pence, or 2 Shillings; therefore I ſet down 0 der the Pence and carry 2 to the next; then I to 13 *s.* ſaying 6 times 13 is 78, and 2 that I rried is 80 *s.* which is 4 *l.* therefore I ſet down nder the Shillings, and carry 4 to the Pounds; n I proceed, ſaying 6 times 8 is 48, and 4 that arry is 52, then I ſet down 2 and carry 5, &c. ceeding thus till the Work be finiſhed, and n will the *Product be* 892 *l.* 00 *s.* 00 *d.* which *duct ſhould be divided by* 100 (the firſt Num- *but it being* an Unit with two Cyphers, I cut off two Figures from the right hand of the Pounds with a daſh of the Pen, and the Figures on the left hand of the ſaid daſh are ſo many Pounds, and thoſe on the right hand of it are the decimal parts of a Pound, whoſe Value may be found out by the third Rule of the ſecond Chapter: but remember, that if there be any Shillings or Pence in the Product, you are to add them to their reſpective Products in your Reduction.

The Work of the foregoing Example is as followeth.

```
 l.     l.        l.    s.    d.
100  :  6  : :  148—13—04
                           6
                ——————————————
                 8|92—00—00
                  |20
                ——|——
                18|40
                  |12
                ——|——
                 4|80
                  | 4
                ——|——
                 3|20
                ——|——
```

So that by the Work I find the Intereſt of 148 *l.* 13 *s.* 4 *d.* for one Year, after the rate of *6 per Cent. per Ann.* to be 8 *l.* 18 *s.* 4 *d.* 3 *qu.*

Another Example may be this, *viz.* I demand the Intereſt of 368 *l.* 15 *s.* 3 *d.* for one Year, at *6 per Cent. per Annum.* Anſwer, 22 l. 2 s. 6 d. as *by the* Work following.

```
                 l.   s.  d.
100 : 6  : :  368—15—03
                        6
              ─────────────
           22|12—11—06
             |20
           ──|──
            2|51
             |12
           ──|──
            6|18
```

II. The Intereſt of 100 *l.* being known for
r, or 365 Days, the Intereſt of any other
nay be known for any other time, or Num-
Days, more or leſs than a Year, by two
Rules of Three direct, *viz.* firſt find out
is the Intereſt of the given Sum for one
or 365 Days, according to the laſt Rule,
having found out that, you may (by ano-
ingle Rule of Three direct) find out its
ſt for any other time more or leſs.

Example.

at is the Intereſt of 322 *l.* for 6 Years af-
e rate of 6 *per Cent. per. Annum*, Simple
ſt?
ſt I find what is the Intereſt of 322 *l.* for
r by the following Proportion.

```
 l.      l.        l.        l.
100  :   6  : :   322   :   19.32
                    6
            ──────────
      100) 19|32 (19.32
```

Thus having found the Intereſt of 322 *l.* for a Year to be 19.32 *l.* at 6 *per Cent.* by the following Proportion I find out its Intereſt for 6 Years to be 115 *l.* 18 *s.* 4¾ *d.* and that added to the Principal, makes 437 *l.* 18 *s.* 4¾ *d.* for the Sum due to the Creditor at the end of the ſaid time.

```
Year.     l.       Years.    l.   s.   d.
  1  :  19.32  : :  6  :   115—18—04 ¾
            6              322
        ------             ------------
       115|92              437—18—04 ¾
          |20
        ------
        18|40
          |12
        ------
         4|80
          | 4
        ------
         3|20
```

And here take notice, that the ſecond Number in this laſt Proportion muſt always be only the Intereſt of the Sum propoſed, and not the Sum of the Principal and Intereſt, as in the ſecond Proportion under the ſixth Rule.

After the ſame manner is the Intereſt of 1 *l.* (at the rate of 6 *per Cent. per Annum*, or any other rate of Intereſt) diſcovered for a Day, by the help of which the Intereſt of any Sum whatſoever may be diſcovered for any Number of Days, as ſhall be ſhewn by and by.

l. *l.* *l.* *l.*
t, 100 : 6 :: 1 : .06

days. *l.* *day.*
y, 365 : .06 : 1 : .0001643835

ıat by the foregoing Proportions I have that the Intereſt of 1 *l.* at 6 *per Cent. per* for a Day, is .0001643835 *l.*

r if you would know the Intereſt of any Sum for any number of Days more or leſs 5, you may do it by the help of the ſaid r, after this manner, *viz.*

iply the Sum whoſe Intereſt is required by l Number, and that Product will give you ereſt of the ſaid Sum for one Day; then y that Product by the number of Days gi- d the laſt Product will give you the In- f the ſaid Sum for the number of Days in eſtion. Take the following Queſtion for ımple, *viz.*

t is the Intereſt of 568 *l.* for 213 Days; ıe rate of 6 *per Cent. per Annum?*

.0001643835

.0001643835
568

13150680
9863010
8219175

.0933698280
213

2801094840
933698280
1867396560

19.8877733640

l. *s.* *d.*
Facit 19——17——09

Having finiſhed the Work, as you ſee, I the Anſwer to be 19.8877, *&c.* which upon I diſcover to be 19 *l.* 17 *s.* 09 *d.* by the brie of valuing a Decimal Fraction of Coin, laid in the fourth Rule of the ſecond Chapter be going.

But when the Intereſt of any Sum of Mo required for any number of Days, as afore at any other rate of Intereſt than at *6 per per Annum*, the aforeſaid Number will not ſerve for the Work, but you are to find out ticular Multipliers for the ſeveral Rates o tereſt, as is before directed. All which I *expreſſed* from 4 to 10 *per Cent.* in the follo *Table.*

When you would find the Intereſt of any Sum for any number of Days at the rate of	4	*per Cent. per Ann. the Multiplier is*	.0001095890
	5		.0001369863
	6		.0001643835
	7		.0001917808
	8		.0002191780
	9		.0002465753
	10		.0002739726

So that when you would find out the Intereſt of any Sum of Money for any number of Days, according to the direction before given, at any rate from 4 to 10 *per Cent. per Annum*, Simple Intereſt, you may perform the Work by the Multiplier in the foregoing Table, which is placed againſt each reſpective rate of Intereſt.

IX. When the preſent Worth of a Sum of Money due at the end of any time to come is required, Rebate being allowed at any rate of Simple Intereſt, it may be found out by the following Method, *viz.* firſt find out the Intereſt of 100 *l.* for the time that the Rebate is to be allowed for, and at the ſame rate of Intereſt propounded, then make the Sum of 100 *l.* and its Intereſt for the propoſed time, to be the firſt Number in the Rule of Three, and 100 *l.* the ſecond Number, and the given Sum whoſe preſent Worth is required let be the third Number, and the fourth Number in a direct Proportion ſhall anſwer the Queſtion, as in the following Example, *viz.*

What preſent Money will ſatisfy a Debt of 100*l.* that is due to the end of a Year yet to come, Diſcount or Rebate being allowed at the rate of 6 *per Cent. per Annum ?*

According to the foregoing Directions, I ſtate *the Numbers as followeth,* and the fourth Pro-

portion:

portional Number or Anſwer to the Queſt
94.33962 l.=94 l. 6 s 09 ½ d. fere.

l.		*l.*		*l.*		*l.*
106	:	100	: :	100	:	*94.33962*

The reaſon of the ſaid Analogy will appear conſider, that there ought to be ſo much ready ney paid, that if it were put out to Intereſt a ſame rate of Intereſt that Rebate was allowe and for the ſame time, the ſame would th augmented to the Sum that was at firſt due : the laſt Queſtion there is given 100 *l.* whi due at the end of 12 Months ; now I ſay there ought to be ſo much Money paid do ſatisfy this Debt, as being put out to Inter *6 per Cent.* for 12 Months, would then be incr to 100 *l.* which is the firſt Sum due : and aga is as evident, that if there were 106 *l.* d the end of 12 Months, or a Year, and pr Payment is agreed upon, allowing Rebate at *Cent. per Annum*, that then there ought to be the Sum of 100 *l.* in full diſcharge of the Debt of 106 *l.* for if, when I have receive ſaid Sum of 100 *l.* I put it out to Intereſt fo Year at the rate of *6 per Cent.* it will then b creaſed to 106 *l.*

Therefore to ſolve the ſaid Queſtion, the portion here uſed is no more than if I ſhoul if 106 *l.* be decreaſed to 100 *l.* what will 100 decreaſed to ? The Anſwer is, to 94 *l.* 6 *s.* and for Proof, if you will ſeek what that Sum be increaſed to at the end of 12 Months, a rate of *6 per Cent.* you will find it to be 100 *l.*

Example 2.

How much preſent Money will ſatisfy a *of 82 l. 15 s.* due at the end of 126 Days

allowing Rebate after the rate of 6 *per*
er Annum?

t I find the Intereſt of 100 *l.* at the ſame
f Intereſt for 126 Days by the following
rtion.

days. *l.* *days.* *l.*
365 : 6 :: 126 : 2.0712

n do I add 2.0712 *l.* (the Intereſt of 100 *l.*)
l. and the Sum is 102.0712, which I make
ſt Number in the Rule of Three, and 100 *l.*
cond, and 82.75 *l.* (the Sum given to be re-
) the third Number, and the fourth Num-
a direct Proportion is the Anſwer to the
on. See the Work as followeth.

l. *l.* *l.*
.0712 : 100 :: 82.75 : 81.0708
100

102.0712) 8275.00 (81.0708

hat by the Work it appears, that 82 *l.* 15 *s.*
: the end of 126 Days yet to come, will be
d with the preſent Payment of 81 *l.* 01 *s.*
Rebate being allowed after the rate of
ent. per Annum.

The Proof of the Rule.

d out (by the Eighth Rule foregoing) how
the preſent Money that is paid upon Re-
vill amount to, being put out to Intereſt
e ſame time, and at the ſame rate of Inte-
at *Rebate was allowed* for, and if the A-
be equal to the Sum that was due at the
end

end of that time, then you may conclude the Work to be rightly performed, otherwiſe not.

Example.

In the foregoing Queſtion it was found that 81.0708 *l.* being paid preſently, would ſatisfy a Debt of 82.75 *l.* due at the end of 126 Days to come; and to prove it, let us ſee whether 81.0708 *l.* being put out to Intereſt for 126 Days, at the rate of 6 *per Cent. per Ann.* will be increaſed to 88.77 *l.* (the Sum which was ſaid to be due at the end of 126 Days to come) which I do by theſe two Proportions following, according to the 8th Rule.

	days.		*l.*		*days.*		*l.*
Firſt,	365	:	6	: :	126	:	2.0712

	l.		*l.*		*l.*		*l.*
Secondly,	100	:	2.0712	: :	81.0708	:	1.6791, *&c.*

So you ſee that I have found the Intereſt o 81.0708 *l.* for 126 Days to be 1.6791 *l.* *&c.* which added to the Principal 81.0708 *l.* the Sum i 82.7499 *l.* which by the brief way of valuing th Decimal of a Pound Sterling, is 82 *l.* 15 *s.* and indeed it doth not want $\frac{1}{10}$ part of a Farthing o the exact Sum, which is occaſioned by the defective Decimal; wherefore I conclude the Worl to be rightly performed.

Upon the foregoing Ninth Rule is grounde the manner of calculating the enſuing Table o Multipliers, which ſheweth in decimal parts o a Pound, the preſent worth of a Pound Sterling *due at the* end of any number of Years to come *not exceeding* 30, Simple Intereſt being comp *at* 6 *per Cent. per Annum.*

T

he firſt Number in the Table being found by this following Proportion, *viz.* s 106 *l.* is to 100 *l.* ſo is 1 *l.* to .*943396*, and ſecond Number in the Table being the preſent rth of 1 *l.* due at the end of two Years to e, is thus found out, *viz.* firſt I conſider that . is the Simple Intereſt of 100 *l.* for two Years, ich added to 100 *l.* makes 112 *l.* wherefore I , as 112 *l.* is to 100 *l.* ſo is 1 *l.* to .*892857 l.* ich is the preſent Worth of 1 *l.* due at the d of two Years to come.

The ſeveral Proportions and Operations for the ole Calculation, being as followeth, *viz.*

1	106	:	100	: :	1	:	.943396
2	112	:	100	: :	1	:	.892857
3	118	:	100	: :	1	:	.847457
4	124	:	100	: :	1	:	.806451
5	130	:	100	: :	1	:	.769230
6	136	:	100	: :	1	:	.735294

And after the ſame manner are all the Numbers the following Table calculated; which being ll underſtood, the way of calculating moſt of enſuing Tables will eaſily be obtained; and Uſe you will find immediately after the Table elf.

TABLE I.

Which ſheweth in decimal Parts of a Pound, the preſent Worth of 1 *l.* due at the end of any number of Years to come under 31, at the rate of 6 *per Cent. per Ann.* Simple Intereſt.

Years.		Years.	
		11	.602409
		12	.581395
		13	.561797
		14	.543478
		15	.526315
		16	.510204
		17	.495049
		18	.480769
		19	.467289
		20	.454545
1	.943396	21	.442477
2	.892856	22	.431034
3	.847457	23	.420168
4	.806451	24	.409836
5	.769230	25	.400000
6	.735294	26	.390625
7	.704225	27	.381679
8	.675075	28	.373134
9	.649350	29	.364963
10	.625000	30	.357143

After the ſame Method might this Table be continued to any number of Years at pleaſure; I might alſo have calculated for other Rates of Intereſt, as thoſe are in the next Chapter concerning Compound Intereſt, but Simple Intereſt being not ſo generally in Practice, I ſhall therefore forbear.

The Uſe of the preceding TABLE.

It is evident (by the Ninth Rule foregoing)

tha

hat if any Sum be paid with an Allowance of ebate, you are to make 100 *l.* with its Intereſt or the ſame time you rebate for) both in one um, to be the firſt Number in the Rule of Three, 00 the ſecond, and the Sum to be rebated the hird; then will the fourth Proportional be the nſwer; and the ſame may be wrought by any ther Number, and its Intereſt, as well as by 00 *l.* and its Intereſt, *mutatis mutandis:* Now in e Table beforegoing there is expreſſed in Decimal Parts of a Pound, the preſent Worth of *l.* due at the end of any Number of Years come under 31, *&c.* that is to ſay, if you ke the Money ſignified by thoſe Decimals, and ut it out to Intereſt at 6 *per Cent. per Annum* imple Intereſt, for ſo many Years as are expreſs'd the Column of Years againſt the ſaid Decimal, en will that Sum at the end of the ſaid Term e augmented to 1 *l.* wherefore if you have any um whatſoever to be rebated for any Number f Years within the Limits of the Table, make *l.* the firſt Number in the Rule of Three, and e Decimal in the Table againſt the Number of ears to be rebated for, make that the ſecond, nd the Sum, whoſe preſent Worth is requir'd e third Number, ſo will the fourth Proportional be the Anſwer; but becauſe the firſt umber (being Unity) neither multiplieth nor ivideth, if you take the Number in the Table, orreſpondent to the Number of Years for which ou would reckon Rebate, and thereby multiply the Sum whoſe preſent Worth is required, the Product will give you the Anſwer.

Example.

There is a Sum of Money, *viz.* 560 *l.* du
the end of 8 Years to come, but the Debto
Creditor agree that preſent Payment ſhal
made, and the Debtor to be allowed Rebate
ter the rate of *6 per Cent. per Annum*, Simpl
tereſt; now I demand how much preſent M
will ſatisfy the ſaid Debt. Anſwer, 378.3
=378 *l.*—07 *s.*—06 $\frac{3}{4}$ *d.* See the following V

l.		*l.*		*l.*		*l.*
1	:	.675675	::	560	:	378.378

```
.675675
    560
-------
40540500
3378375
-------
378.378000
```

First (the Rebate being to be reckoned f
Years) I look for 8 in the Column of Years,
juſt againſt it on the right hand, I find .675
which I multiply by 560 (the Sum whoſe
ſent Worth is required) and the Produ
378.378, which (by the brief way of valuin
Fraction of a Pound Sterling) I find at
ſight to be 378 *l.* 07 *s.* 06 $\frac{3}{4}$ *d.*

This Queſtion, if it had been wrought b
foregoing ninth Rule, would have produce
ſame Anſwer; for the Intereſt of 100 *l.* for
Months is 48 *l.* and 100+48=148, wher
by the Rule of Three, I ſay,

l. : *l.* :: *l.* : *l.*
148 : 100 :: 560 : 378.378

X. When there is an Annuity or yearly Income
Arrears for any number of Years, and you
ld know the Increaſe or Amount of it, allow-
Simple Intereſt at a certain rate *per Cent. per*
for each yearly Payment from the time it firſt
ame due; the Operation will be ſomewhat
re tedious than to find the Amount of one
le Sum, according to the eighth Rule of this
pter, which will clearly appear by ſolving
following Queſtion, *viz.*

here is an Annuity or Income of 100 *l. per*
um, forborn to the end of 6 Years; I de-
d how much is due at the end of the ſaid
n, allowing Intereſt at the rate of *6 per Cent.*
Annum, Simple Intereſt. Anſwer *690 l.*

n order to the Solution of this Queſtion, I
ſider, firſt, that

: is evident, that for the laſt Year, *viz.* the
Year's Payment, there muſt be no Intereſt at
reckoned, becauſe it becomes not due till the
of the ſixth Year. Secondly, there muſt be
oned the Intereſt of 100 *l.* for one Year, *viz.*
which is due at the end of the fifth Year.
rdly, there muſt be reckoned the Intereſt of
l. for two Years, *viz.* that which is due at the
of the fourth Year. Fourthly, there muſt be
oned the Intereſt of 100 *l.* for three Years,
that which is due at the end of the 3d Year.
hly, the Intereſt of 100 *l.* for four Years, *viz.*
which is due at the end of the ſecond Year.
, *ſixthly, the Intereſt* of 100 *l.* for five Years,
that which is due at the end of the firſt Year,

and is forborn the 2d, 3d, 4th, 5th, and 6th Years. All which Intereſts being added together, and their Sum added to the Sum of each Year's Income, the Sum will exhibit the total Sum due at the end of the ſaid ſix Years, which you may perceive by the following Work to be 690 *l.* which is the Anſwer to the foregoing Queſtion.

		l.
The Intereſt of 100 *l.* at 6 *per Cent. per Ann.* Simple Intereſt, for	1 Years, is	06
	2	12
	3	18
	4	24
	5	30
The Sum of the Intereſt is		90
The Sum of the Annuities		600
The Total Amount is		690

The Conſtruction of Table II.

Upon the foregoing reaſon is grounded the Calculation of the following Table, which ſheweth the Amount of 1 *l.* Annuity being forborn to the end of any number of Years under 31, Intereſt being allowed for each yearly Payment, after the rate of 6 *per Cent. per Annum*, Simple Intereſt.

The firſt Number in the Table being 1 *l.* which is that due at the end of the firſt Year, no Intereſt being due for that: The ſecond Number in the Table is 2.06, which is the firſt and ſecond

r's Payment, and the Intereſt of 1 *l.* for one
r, being that which was due at the end of the
Year; the third Number in the Table is
l. being the Increaſe of 1 *l.* for two Years
d to the ſecond Number in that Table, which
06: for the Amount of 1 *l.* at the end of
Years, is 1.12, which added to 2.06 the
nd Number, it makes 3.18 for the third Num-
; the fourth Number is the Amount of 1 *l.*
three Years, which is 1.18, added to the
nber before it, *viz.* the third Number; pro-
ing in the ſame Method till you have compo-
he Table at your pleaſure, each Number in
Table being 1 *l.* and the Amount of 1 *l.* (for
any Years as it ſtandeth againſt in the Ta-
made leſs by one) added to the Number im-
iately preceding it.

TABLE II.

Which ſheweth in Pounds, and Decimal Parts of a Pound, the Amount of 1 *l.* Annuity being forborn to the end of any Number of Years under 31, Simple Intereſt being computed after the rate of *6 per Cent. per Annum.*

Years.		Years.	
1	1.00	11	
2	2.06	12	
3	3.18	13	
4	4.36	14	
5	5.60	15	
6	6.90	16	
7	8.26	17	
8	9.68	18	
9	11.16	19	
10	12.70	20	
		21	
		22	
		23	
		24	
		25	
		26	
		27	
		28	
		29	
		30	

The Uſe of Table II.

In the preceding Table, in the Column the Word Years, are ſet down every Ye ceſſively, from 1 to 30; and the Number *Table* placed againſt each Year is the Amc 1 *l.* Annuity, in Pounds and Decimal P a *Pound,* being forborn ſo many Years

aced againſt. The Uſe of it will plainly ap-
ar by the ſolving of 1 or 2 Queſtions, *viz.*
There is an Annuity of 134 *l.* 10 *s.* 6 *d.* all for-
rn to the end of four Years; I demand how
ich is due to the Creditor at the end of the ſaid
rm, Simple Intereſt being allow'd after the rate
6 per Cent. per Annum. Facit 586 *l.* 10 *s.* 7 *d.*
To anſwer this Queſtion, firſt I look for four
ars in the Column of Years, and the Number
ainſt it is 4.36, which is the Amount of 1 *l.* An-
ity for four Years; therefore having turned the
s. 6 *d.* (in the given Annuity) into a Decimal,
ich is .525, I ſay, by the Rule of Three, thus,

l.		*l.*		*l.*		*l.*
1	:	4.36	: :	134.525	:	586.529.

```
134.525
    436
-------
 807150
403575
53800
-------
586.52900
```

Thus, by the Work, I find the Anſwer to be
6.529 *l.* the Value of which Decimal, by the
ief way of valuing a Decimal, laid down in
e fourth Rule of the ſecond Chapter, I find to
580 *l.* 10 *s.* 7 *d.*
And it is plain, that in ſolving Queſtions by
is Table, (the firſt Number in the Rule of
hree being Unit) if you multiply the given
nnuity by the proper Tabular Number, then
e Product will be the Anſwer.

Example 2.

What is the Amount of an Annuity o 10 *s.* being forborn to the end of 7 Years, ing Simple Intereſt, after the rate of 6 *pe per Annum?* Anſwer, 1243 *l.* 02 *s.* 07 ¼. j

The given Annuity is
The tabular Number for 7 Years is

124

Facit 12

The Rebate of Annuities at Simple Intereſt

XI. When an Annuity or yearly Incom certain number of Years to come, is to be ſo ready Money, and the Sell allow the Buyer Rebate at S Intereſt for his preſent Payı then in this caſe the Buyer to pay ſo much preſent Mon each yearly Payment, as being put out at S Intereſt for ſo many Years as it is rebated would then amount to one yearly Payment the Sum of all thoſe preſent Worths will b preſent Worth of the Annuity required. Th will appear very plain by the following Exa

There is an Annuity or Leaſe of 100 *l. pe* to continue 6 Years yet to come, to be ſol ready Money, the Seller being to allow the Rebate at 6 *per Cent. per Ann.* Simple Int *now I deſire* to know how much preſent *will buy* out the ſaid Leaſe. Facit 499 l.
fore.

It is evident, that if we find out the preſent orth of 100 *l.* due at the end of the firſt Year, d alſo the preſent Worth of 100 *l.* due at the d of the ſecond Year, and the preſent Worth of ɔ *l.* due at the end of the third Year, and likeſe the preſent Worth of 100 *l.* due at the end the fourth, fifth and ſixth Years, and add all ſe preſent Worths together, their Sum will be e preſent Worth of the given Annuity: which veral preſent Worths are found out according the Ninth Rule, by the ſeveral Proportions llowing, *viz.*

Years.	*l.*		*l.*		*l.*		*l.*
1	106	:	100	: :	100	:	94.339622
2	112	:	100	: :	100	:	89.285714
3	118	:	100	: :	100	:	84.745762
4	124	:	100	: :	100	:	80.645169
5	130	:	100	: :	100	:	76.923076
6	136	:	100	: :	100	:	73.529411

The preſent Worth of the ſaid nnuity is } 499.468754

So that you ſee by the foregoing Proportions, e preſent Worth of 100 *l.* *per Annum*, to conue 6 Years, allowing Rebate at 6 *per Cent. per num* Simple Intereſt, is 499.468754 *l.* = 499 *l.* *s.* 4 $\frac{1}{4}$ *d.*

Upon the foregoing eleventh Rule is grounded e Conſtruction and Calcution of the following Tae, which ſheweth the prentWorth of 1 *l.* Annuity *continue any number* of Years under 31, Sim- *Intereſt being* computed after the rate of 6

The Conſtitution of the third Table.

pe

per Cent. per Annum. The firſt Number in th ble is .943396, which is the preſent Worth due at the end of a Year to come. The ſ Number in the Table is 1.836253, which i Sum of the preſent Worths of 1 *l.* due at th of two Years to come, and of 1 *l.* due at th of one Year to come added together. An third Number in the Table is 2.683710, wh the Sum of the preſent Worths of 1 *l.* due end of three, two, and one Years to come. the ſame Method is the whole Table calcul

But the Numbers in the ſaid Table may eaſily be found out thus, *viz.* look in the fir ble, and let the firſt Number of that be th Number of this third Table; and let the S the firſt Number in this, and the ſecond N in that, be the ſecond Number in this Table for the third Number in this Table, take th of the ſecond in this, and the third in tha ble: And in this manner you may procee you have compoſed the whole Table.

TABLE III.

Which ſheweth the preſent Worth of 1 *l.* Annuity to continue any number of Years under 31. Simple Intereſt being computed at *6 per Cent. per Annum.*

Years.		Years.	
1	.943396	11	8.251334
2	1.836253	12	8.832729
3	2.683710	13	9.394526
4	3.490161	14	9.938004
5	4.259391	15	10.464319
6	4.094685	16	10.974523
7	5.698900	17	11.469572
8	6.374575	18	11.950341
9	7.023925	19	12.437630
10	7.648925	20	12.892175
		21	13.334652
		22	13.765686
		23	14.175524
		24	14.585360
		25	14.985360
		26	15.375985
		27	15.757664
		28	16.120798
		29	16.485761
		30	16.842904

The Uſe of Table III.

In the foregoing third Table, in the left hand Column, under the Title of Years, are expreſſed all the integral Numbers, from 1 to 30, which ſignify ſo many Years; and the Numbers in the right hand Column, which are placed againſt *the number of Years*, are Pounds, and decimal

parts of a Pound Sterling; and every one of
are the preſent Worth of 1 *l.* Annuity to con
ſo many Years to come as are placed againſt
in the Column of Years, Rebate being allow
Simple Intereſt, *6 per Cent. per Annum.*

As, ſuppoſe there were a Leaſe of 20 Shi
per Annum, to continue 6 Years, to be ſol
preſent Money, allowing the Buyer Rebat
per Cent. per Annum, Simple Intereſt; I deſ
know how much is its preſent Worth. To a
this, I look in the Column of Years for *6*, a
the next Column on the right hand, juſt agai
you have 4.994685 *l.* 4 *l.* 19 *s.* 10 ¼ *d.*
is the Anſwer to the Queſtion. And by th
of this Table may the preſent worth of any
nuity, to continue any number of Years und
be found, allowing Rebate at *6 per Cent. pe*
num, Simple Intereſt, by one ſingle Rule of T
direct, according to the manner of ſolving th
lowing Queſtion, *viz.*

Queſt. 1.

There is a Leaſe of 18 Years yet to come,
yearly Value of 130 *l.* to be ſold for ready M
and the Purchaſer is to be allowed Rebate,
the rate of *6 per Cent. per Ann.* Simple Int
now I demand how much is the preſent Wo

$$\begin{array}{llll} l. & l. & l. & l. \\ 1 : & 11.950341 :: & 130 : & 1553.544330 \\ & 130 & & \end{array}$$

$$\begin{array}{l} 358510230 \\ 11950341 \end{array}$$

$$1553.544330 = \overset{l.}{1553} - \overset{s.}{10} - \overset{d.}{10\frac{3}{4}}.$$

So that by the Work you find the Anſwer to be 1553.544 *l.* &c. or 1553 *l.* 10 *s.* 10 $\frac{3}{4}$ *d.* very near; which ſaid Anſwer is nothing elſe but the Product of the tabular Number (11.950341 *l.*) multiplied by the given Annuity (130 *l.*) For it s evident, that if the preſent Worth of 1 *l.* Annuity to continue 18 Years be 11.950341 *l.* then he preſent Worth of 130 *l.* *per Annum*, to continue the ſame number of Years (and Rebate being allowed at the ſame rate *per Cent. per Annum*, or the one as for the other) muſt be 130 times as much. But when Rebate is to be allowed after any other rate than 6 *per Cent. per Annum*, then the foregoing Table will not at all be uſeful, but you muſt have recourſe to a Table calculated for the ſame rate of Intereſt; which you may eaſily perform at leiſure by the foregoing Rules.

Queſt. 2.

What Annuity to continue eighteen Years will 1553.544330 purchaſe, allowing the Buyer Simple Intereſt at 6 *per Cent. per Annum?* Facit 130 *l.*

This Queſtion is but the Converſe of the
mer, and may be thus reſolved, *viz.* Take th
bular Number, correſponding to 18 Years, w
is 11.950341, by which divide the given
chaſe-Money, and the Quotient will give yo
Annuity that it will purchaſe, *viz.*

l.	*l.*	*l.*
11.950341)	1553.54433	(130

So that by the Work I find it will purcha
Annuity of 130 *l.* to continue 18 Years.

The reaſon of the Work is plain; for if th
bular Number, correſpondent to 18 Years, b
preſent Worth of 1 *l.* Annuity, to continu
Years to come, then it is certain that ſo
Money as is expreſſed by that tabular Nur
will purchaſe an Annuity of 1 *l.* to con
18 Years; and conſequently we may fin
the help of the ſaid Table what Annuity ar
ther Sum of Money will purchaſe, to continu
number of Years, not exceeding 30, by a
Rule of Three Direct; as in the laſt Que
the Proportion is as followeth, *viz.*

l.		*l.*		*l.*		
11.950341	:	1	: :	1553.54433	:	1

And it is no more in effect than a Sum in
viſion; for the ſecond Number (being 1) ne
multiplieth not divideth, &c.

By what hath been ſaid concerning the U
the foregoing Table, you may perceive tha
preſent Worth of an Annuity is found ou
Multiplication; and to know what Annuity
Sum will purchaſe, is performed by Diviſion.

I might have made Tables for other Rates of Intereſt; but Simple Intereſt being ſeldom allowed in the purchaſing or valuing of Leaſes and Annuities, they being generally purchaſed at Compound Intereſt, or Intereſt upon Intereſt, makes me forbear; and indeed at Simple Intereſt a Leaſe is over-valued.

CHAP. XIII.

Of Compound Intereſt.

I. WHAT hath been ſaid in the laſt Chapter, I judge ſufficient for the underſtanding of the Nature and Uſe of Simple Intereſt; and that being well underſtood, the Nature of Compound Intereſt will not ſeem difficult to the ſtudious Learner: and the better he is acquainted with the Nature of Simple Intereſt, ſo much the eaſier will he come to the Knowledge of the Nature and Uſe of Compound Intereſt.

II. Compound Intereſt is, when a Sum of Money is put out to Intereſt, and the Intereſt thereof becoming due, is ſtill continued in the hands of the Debtor, ſo as to become part of the Principal, Intereſt being reckoned for it from the time it becometh due; for which reaſon it is called Intereſt upon Intereſt: And as Simple Intereſt increaſeth by a Series of Arithmetical Proportionals continued, ſo doth Compound Intereſt increaſe by a Rank or Series of continual Geometrical Proportionals; for when a Sum of Money is put out to Intereſt at any rate *per Cent. per Annum*, (as ſuppoſe 100 *l.* to be put out to receive at the end of one Year 6 *l.* for its Intereſt) it is evident, that if the Intereſt (being 6 *l.*) be continued in the hands of the Debtor, there will be at the end of the ſecond Year the Increaſe of f 106 *l.* which is 112.36 *l.* and at the third ear's end there will be the Increaſe of 112.36 *l.* that every Number proceedeth from that go- ig before it, after the ſame rate or reaſon as o proceedeth from 100; as you ſee following.

100 : 106 :: 106 : 112.36
00 : 106 :: 112.36 : 119.1016
100 : 106 :: 119.1016 : 126.247696

So that by the Augmentation of 100 *l.* in four ars you have this Rank of Geometrical Pro- rtionals continued, *viz.* 100, 106, 112.36, 9.1016, and 126.247696, which is in Num- r 5, *viz.* more by one than is the Number of ars, the laſt of which is the Amount of 100 *l.* at er *Cent.* for four Years, reckoning compound tereſt, or Intereſt upon Intereſt; and each of eſe Proportionals proceedeth from that going fore it, as 106 proceedeth from 100, that is to *l*, every of the ſaid Proportionals is in ſuch pro- rtion to that which goeth before it, as 106 is 100, or as 100 is to 106, ſo is any one of them that which followeth it; or if you take any ree of them which are placed together, there is is Proportion between 'em, *viz.* as the firſt of oſe three is to the ſecond, ſo is the ſecond to the ird, and the third to the fourth, and the fourth the fifth, and the fifth to the ſixth, *&c.* whence is evident that they have amongſt themſelves is following Qualification, *viz.* that the Square any one of them is equal to the Rectangle or oduct, made by that which is placed imme- ately before it, and that immediately after it; d the ſame would it be if there were never ſo ny Terms, and is a peculiar Property of all

Numbers that are Geometrical Proportional tinued.

III. The Intereſt of 100 *l.* for a Year known, the Compound Intereſt of any oth for any Number of Years may be likewiſe out by ſo many ſingle Rules of Three, as are given Years; for,

As 100 *l.* is to its Increaſe for one Y is any other Sum to its Increaſe for the Time; and ſo is the firſt Year's Increaſe t ſecond, and the ſecond Year's Increaſe t third, and ſo is the third Year's Increaſe fourth, *&c.*

Example.

Let it be required to find how much will be increaſed to, being put to Intereſ *per Cent. per Annum*, Compound Intereſt, Years. Anſwer 468 *l.* 7 *s.* 4 ¾ *d. fere.* S following Work.

l. *l.*
100 : 106 : : 350 : 371
371 : 393.26
393.26 : 416.855
416.8556 : 441.866
441.866936 : 468.378

Whereby you ſee that 350 *l.* being put tereſt after the rate of 6 *per Cent.* will at t Year's end be increaſed to 371 *l.* and 371 *l.* put out for the ſecond Year, will be increa

.26 *l.* and 393.26 *l.* being made a Princi-
and put out at the ſame rate for the third
r, will at the end thereof be increaſed to
8556 *l.* and at the end of 5 Years it will be
eaſed to 468.37895216 *l.*

nd upon the aforeſaid Grounds is calculated
following Table I. whoſe Conſtruction and
immediately followeth the ſame.

TABLE I.

Which ſheweth what one Pound will amount to, being forborn to the end of any Number of Years to come, not exceeding 30, Compound Intereſt being computed at 5, 6, 7, 8, 9, or 10 *per Cent. per Annum.*

Years.	5.	6.	7.	8.	9.	10.
1	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000
2	1.10250	1.12360	1.14490	1.16640	1.18810	1.21000
3	1.15762	1.19101	1.22504	1.25971	1.29502	1.33100
4	1.21550	1.26247	1.31079	1.36048	1.41158	1.46410
5	1.27628	1.33822	1.40255	1.46932	1.53862	1.61051
6	1.34009	1.41851	1.50073	1.58687	1.67710	1.77156
7	1.40710	1.50363	1.60578	1.71382	1.82803	1.94871
8	1.47745	1.59384	1.71818	1.85093	1.99256	2.14358
9	1.55132	1.68947	1.83845	1.99900	2.17189	2.35794
10	1.62889	1.79084	1.96715	2.15892	2.36736	2.59374
11	1.71033	1.89829	2.10485	2.33163	2.58042	2.85311
12	1.79585	2.01219	2.25219	2.51817	2.81266	3.13842
13	1.88564	2.13292	2.40984	2.71962	3.06580	3.45227

A Continuation of the preceding TABLE I.

Years.	5.	6.	7.	8.	9.	10.
16	2.18287	2.54035	2.95216	3.42594	3.97030	4.59497
17	2.29201	2.69277	3.15881	3.70001	4.32763	5.05447
18	2.40661	2.85433	3.37993	3.99601	4.71712	5.55991
19	2.52695	3.02559	3.61652	4.31570	5.14166	6.11590
20	2.65329	3.20713	3.86968	4.66095	5.60441	6.72749
21	2.78596	3.39956	4.14056	5.03383	6.10880	7.40024
22	2.92526	3.60353	4.43040	5.43654	6.65860	8.14027
23	3.07152	3.81975	4.74053	5.87146	7.25787	8.95430
24	3.22509	4.04893	5.07236	6.34118	7.91108	9.84973
25	3.38635	4.29187	5.42743	6.84847	8.62308	10.83470
26	3.55567	4.54938	5.80735	7.30934	9.39915	11.91818
27	3.73345	4.82234	6.21386	7.98806	10.24508	13.10999
28	3.92012	5.11178	6.64883	8.62710	11.16713	14.42099
29	4.11613	5.41838	7.11425	9.31727	12.17218	15.86309
30	4.32194	5.74349	7.61225	10.06265	13.26767	17.44940

The

The Conſtruction of the foregoing Table I.

By the third Rule foregoing it is evident that the Intereſt of 100 *l.* for a Year being known, the Compound Intereſt for any other Sum may be found out for any Number of Years; according to which Rule all the Numbers in the ſaid Table are found out, being the Amount of 1 *l.* at Compound Intereſt, for any Number of Years, not exceeding 30, being put out at any of theſe Rates, *viz.* 5, 6, 7, 8, 9, or 10 *per Cent. per Annum*; which Numbers are found out by the Rule of Proportion: Thus,

$$100 : 105 :: \left\{\begin{array}{lcl} 1 & : & 1.05 \\ 1.05 & : & 1.1025 \\ 1.1025 & : & 1.157625 \\ 1.157625 & : & 1.21550625 \end{array}\right.$$

By which means the four firſt Numbers in the ſecond Column of the Table (being placed under the Number 5) are found, and by a Continuation of the ſame Operation are all the reſt of the Numbers in that Column found out; which is indeed nothing elſe than a continual Multiplication of the firſt Number (*viz.* 1.05) into i ſelf 29 times, and ſo the laſt Number in tha Column is the thirtieth Power of 1.05, and th ſame Column may be continued to any othe Number of Years at pleaſure above 30; the Numbers in this Column being the Intereſt of 1 *l.* a

5 pe

5 *per Cent. per Annum*, Compound Intereſt, for 30 Years.

The Numbers in the third Column under the Figure *6*, are the Increaſe of 1 *l.* at *6 per Cent. per Annum*, Compound Intereſt, for 30 Years, and are found out by multiplying 1.06 into itſelf 29 times, according to the Rule of continual Multiplication. The like is to be underſtood of all the reſt.

The Uſe of the foregoing Table.

In the firſt Column of the Table under the Title *Years*, are expreſſed the Number of Years from 1 to 30, and in the ſecond Column, under Figure 5, and againſt every reſpective Year, are expreſſed the Increaſe of 1 *l.* being put out at 5 *per Cent. per Annum*, compound Intereſt.

In the third Column under the Number *6*, is expreſſed the yearly Increaſe of 1 *l.* being put out at *6 per Cent. per Annum*, compound Intereſt; and ſo in the 4th, 5th, 6th, and 7th Columns, are the yearly Amounts of 1 *l.* at 7, 8, 9, and 10 *per Cent. per Annum*, Compound Intereſt.

All which Numbers in the ſaid Table are Multipliers, for the producing of the Amount or Increaſe of any other Sum, being put out at Compound Intereſt, at any rate of Intereſt, and for any Number of Years therein expreſſed, as will appear by the following Examples.

Example 1.

I demand the full Amount of 365 *l.* being put to Intereſt for 9 Years, Intereſt being computed after the rate of 6 *per Cent. per Annun* pound Intereſt. *Facit* 616 *l.* 13 *s.* 10 ½

Here, becauſe the Sum propoſed is pu 6 *per Cent.* and for 9 Years, I look in the of 6 *per Cent.* which is the third Colum Table under the Figure 6, and juſt agai the Column of Years I find 1.68947, the Increaſe of 1 *l.* being forborn the ſa and at the ſame rate of Intereſt; where the Rule of Three I ſay,

$$\begin{array}{ccccccc} l. & & l. & & l. & & l. \\ 1 & : & 1.68947 & :: & 365 & : & 616656 \\ & & 365 & & & & \\ \hline & & 844735 & & & & \\ & & 1013682 & & & & \\ & & 506841 & & & & \\ \hline & & 616.65655 & & & & \end{array}$$

So that by the Work I find that if the 365 *l.* be all forborn to the end of 9 Yea Intereſt be computed for the ſame at 6 *per Annum*, Compound Intereſt, it will increaſed to 616.65655, which is 616 1 ½ *d.*

Example 2.

What will 128 *l.* 16 *s.* 8 *d.* be increaſed utmoſt Improvement thereof being made Years, at 7 *per Cent. per Annum*, Compo tereſt? *Facit* 355 *l.* 9 *s.* 1 *d.*

Firſt turn the 16 *s.* 8 *d.* into the Decimal of a Pound, by the ſecond Rule of the ſecond Chapter foregoing, and you will find it to be .8333, ſo that the given Sum is 128.8333, *&c.*

Now to anſwer this Queſtion, I look into the foregoing Table, and in the Column of 7 *per Cent.* and juſt againſt 15 Years I find 2.75903, which is the uttermoſt Increaſe of 1 *l.* for 15 Years, at 7 *per Cent.* Compound Intereſt; by which if you multiply the given Sum, the Product will be the Anſwer to the Queſtion, as by the following Work will plainly appear.

```
1  :  2.75903  : :  128.8333
                     2.75903
                  ----------
                     3864999
                   115949970
                   6441665
                  9018331
                 2576666
                 -----------
                 355.454939699
```

By the foregoing Work the Anſwer is found to be 355.4549, *&c.*=355 *l.* 9 *s.* 1 *d.*

But if any Sum be put out at Compound Intereſt for Months or Days over and above the given number of Years, then the Work will be ſomewhat different from the former. Firſt you muſt find out the Amount of the given Sum for the given number of Years; and then by the 8th Rule of the foregoing Chapter, find out the Intereſt of that Amount for the odd Time, being either Months *or Days under a Year;* and that Intereſt being *added to the aforeſaid* Amount, that Sum will be

th

the Anſwer to the Queſtion. This is ſo obvious that it needeth no Example.

Of Rebate or Diſcount at Compound Intereſt.

IV. When a Sum of Money, due at the end of any number of Years to come, is to be ſatisfied with preſent Money, allowing Rebate at Compound Intereſt, there muſt be found a Rank or Series of continual Proportionals, more in number by one than the number of Years for which the Diſcount is propoſed; of which Rank or Series of Proportionals the Sum to be ſatisfied by preſent Payment muſt be the firſt, and the ſecond muſt decreaſe from that after the rate or proportion as 100 decreaſeth from the Sum of 100 added to its Intereſt for one Year, after the rate of Intereſt propounded; that is to ſay, as 100 proceedeth from 106, or 108, if the Intereſt be 6 or 8 *per Cent.* and after the ſame rate or reaſon muſt the third decreaſe from the ſecond, and the fourth from the third, *&c.*

When a Queſtion is ſtated for the Rebate of Money at Compound Intereſt, it is ſolvable by as many ſingle Rules of Three, as the number of Years for which the Sum propoſed is to be rebated; and it is nothing elſe but the Inverſe of the third Rule of this Chapter, as may be proved by the working of the following Queſtion taken out of the ſaid Rule, where it is proved that 350 *l* being forborn in the Debtor's hands for 5 Years at 6 *per Cent.* Compound Intereſt, it will then be increaſed to 468.38001216; now let the ſaid Queſtion be inverted thus, *viz.*

There is a Sum of Money, *viz.* 468.38001216 *due at the* end of 5 Years to come; now I deman

h

how much preſent Money will ſatisfy the ſaid Debt, Rebate being allowed after the rate of 6 *per Cent. per Annum*, Compound Intereſt.

Firſt I ſay, as 106 is to 100, ſo is the Sum due at the end of five Years, *viz.* 468.38001216, to 441.867936, which is the Sum due at the fourth Year's end; and ſo is the Sum due at the fourth Year's end, to the Sum due at the third Year's end, *&c.* as by the Work appeareth.

$$106 : 100 :: \begin{cases} 468.38001216 : 441.867936 \\ 441.867936 : 416.8556 \\ 416.8556 : 393.26 \\ 393.26 : 371 \\ 371 : 350 \end{cases}$$

So that by the foregoing Work you ſee, that if 468.38001216 *l.* be due at the end of five Years to come, and is to be ſatisfied by the Payment of preſent Money, Rebate being allowed at 6 *per Cent. per Annum*, Compound Intereſt, 350 *l.* is the Sum required.

And upon this Rule is grounded the Calculation of the following Table, which ſheweth what 1 *l.* due at the end of any number of Years to come, not exceeding 30, is worth in preſent Money, Rebate being reckon'd at any of theſe Rates, *viz.* 5, 6, 7, 8, 9, or 10 *per Cent. per Annum*, Compound Intereſt.

TABLE

TABLE II.

Which ſheweth the preſent Worth of one Pound payable at the end of any number of Years to come, not exceeding 30, Rebate being yearly allowed at 5, 6, 7, 8, 9, or 10 *per Cent. per Annum*, Compound Intereſt.

Years.	5.	6.	7.	8.	9.	10.
1	.952381	.943396	.934579	.925925	.917431	.909090
2	.907029	.889996	.873438	.857338	.841680	.826446
3	.863837	.839619	.816297	.793832	.772183	.751314
4	.822702	.792093	.762895	.735029	.708425	.683013
5	.783526	.747258	.712986	.680583	.649331	.620921
6	.746215	.704960	.666342	.630169	.596267	.564474
7	.710681	.665057	.622749	.583490	.547034	.513158
8	.676839	.627412	.582009	.540268	.501866	.466507
9	.644608	.591898	.543933	.500248	.460427	.424097
10	.613913	.558394	.508349	.463192	.422410	.385543
11	.584679	.526787	.475092	.428882	.387532	.350494
12	.556837	.496969	.444012	.397113	.355534	.318630

A Continuation of the preceding TABLE II.

Years.	5.	6.	7.	8.	9.	10.
16	.458111	.393646	.338734	.291890	.251869	.217629
17	.436296	.371364	.316574	.270269	.231073	.197844
18	.415520	.350343	.295864	.250249	.211993	.179858
19	.395733	.330512	.276508	.231712	.194489	.163508
20	.376889	.311804	.258419	.214548	.178430	.148643
21	.358942	.294155	.241513	.198655	.163698	.135130
22	.341849	.277505	.225713	.183940	.150181	.122846
23	.325571	.261797	.210947	.170315	.137781	.111678
24	.310767	.246908	.197146	.157699	.126405	.101525
25	.295302	.232998	.184249	.146018	.115967	.092296
26	.281240	.219810	.172195	.135201	.106392	.083905
27	.267848	.207367	.160930	.125186	.097607	.076277
28	.255093	.195630	.150402	.115913	.089548	.069343
29	.242946	.184556	.140562	.107327	.082154	.063039
30	.231377	.174110	.131367	.099377	.075371	.057308

The

Compound Intereſt; and after the ſame m
are all the Numbers in the foregoing Table
as you may ſee by the following Example,
the four firſt Numbers in the third Colum
methodically found out by the Rule, that
the Column of Rebate at 6 *per Cent.* *per A*

1 | 106 : 100 :: 1 : .94339622641, *&c.*
2 | 106 : 100 :: 9433962264 : .88999644
3 | 106 : 100 :: 88999644 : .83961928,
4 | 106 : 100 :: 83961928 : .79209460,

So that by the foregoing Proportions,
firſt, if 106 *l.* be decreaſed to 100 *l.* what w
be decreaſed to? Anſwer, to .94339 *l.* *&c*
10½ *d.* the five firſt Figures thereof bein
firſt Number in the third Column of the
going Table; and it ſheweth that the p
Worth of 1 *l.* due at the end of one Year to

d Column of the ſaid Table, and is placed a-
oſt two Years in the firſt Column, and ſheweth
preſent Worth of 1 *l.* due at the end of two
rs to come, Rebate being allowed after the
: of 6 *per Cent. per Annum*, Compound Intereſt.
nd after the ſame manner are all the reſt of
Numbers in the ſaid Column of 6 *per Cent.*
d out, and alſo all the other Decimal Frac-
s in the ſecond, fourth, fifth, ſixth, *&c.* Co-
ns, ſhewing the Rebate of 1 *l.* for any number
Years not exceeding 30, at 5, 6, 7, 8, 9, and
per Cent. (*mutatis mutandis.*)

The Uſe of the foregoing Table II.

The firſt Column is the number of Years for
Rebate of 1 *l.* and the Number in the reſt of
Columns are Decimal Fractions, ſhewing the
ſent Worth of 1 *l.* due at the end of ſo many
rs to come, as they are placed againſt in the
umn of Years, Rebate being allowed at the
e rate of Intereſt under which they are placed,
Figures 5, 6, 7, 8, 9, and 10, placed at the
, denoting the ſame. An Example or two
ll make its Uſe more plain.

Example 1.

I demand how much preſent Money will ſatiſ-
a Debt of 684 *l.* due at the end of 6 Years to
ne, allowing Rebate after the rate of 8 *per*
t. per Annum, Compound Intereſt. To anſwer
s Queſtion, look in the Column of 8 *per Cent.*
l againſt 6 years I find this Number, *viz.* .63017,
ich ſheweth, that if 1 *l.* be due at the end of

6 Years to come, its preſent Worth is .6
Rebate being allowed after the rate of 8 *pe*
per Annum, Compound Intereſt.

Therefore I ſay by the Rule of Three,
be decreaſed to .63017 *l.* what will 684 *l.*
creaſed to at that rate ? Facit 431.0362
431 *l.* 00 *s.* 8 ½ *d.* as by the Work appearet

```
l.      l.          l.      l.
1  :  63017  : :  684  :  431.0362
        684
  ─────────
      252068
     504136
    378102
    ──────────
    431.03628 Facit 431 l. 00 s. 8 ½ d. j
```

So that you ſee the Sum propoſed being
plied by the proper tabular Number, pro
the Anſwer to the Queſtion for the Num
which is here the firſt Number in the R
Three, and doth not either multiply or d
and therefore the Anſwer is found out by the
tiplication only. Obſerve the Work of th
Example.

Example 2.

What is the preſent Worth of 164 *l.* 15 *s*
at the end of *9* Years to come, allowing R
after the rate of *6 per Cent. per Annum*, Com
Intereſt ? Facit 97 *l.* 10 *s.* 3 ¾ *d.*

Look in the Table aforeſaid, in the Colu
6 per Cent. and againſt *9* in the Column of
you will find this Number, *viz.* .59189, w
the preſent Worth of 1 *l.* due at the e

ears to come, and is the proper Multiplier for nding the Anſwer to this Queſtion; as by the Work.

l.		l.		l.		l.
1	:	.59189	::	164.75	:	97.5138775

.59189
16475

295945
414323
236756
355134
59189

97.5138775

The Anſwer found by the foregoing Operation 97.5138775=97 l. 10 s. 3¼ d.

But if the given Time for the Rebate of any m conſiſteth of odd Months or Days, beſides ars, then in ſuch Caſe the Rebate, at the gi- n Rate of Intereſt, for the odd time, muſt be nd by the 9th Rule of the 12th Chapter fore- ing, for the given Sum, and then the preſent orth of the given Sum thus decreaſed, muſt found for the number of Years, as in the two t Examples.

Example 3.

There is 640 *l.* 10 *s.* due at the end of ſix ars and three Months to come, what is its ſent Worth, Rebate being allowed at the rate 7 *per Cent. per Annum*, compound Intereſt?

Firſt, I find the Decreaſe of 640.5 Months thus, *viz.*

mon. *l.* *mon.* *l.*
12 : 7 :: 3 : 1.75.

So that I find the Intereſt of 100 *l.* fo Months, at 7 *per Cent.* to be 1.75 *l.* whic to 100 *l.* makes 101.75; then to find t creaſe of 640.5 *l.* for three Months, I ſay

l. *l.* *l.* *l.*
101.75 : 100 :: 640.5 : 629.4

So that I find by the laſt Proportion, tl the end of 6 Years 3 Months, there w 640.5 *l.* yet at the end of 6 Years there due but 629.484 *l.* whoſe preſent Worth, foregoing Directions, will be found to l 9 *s.* for,

l. *l.* *l.* *l.*
1 : 66634 :: 629.484 : 419.450, &c.=

Read the ninth Rule of the twelfth foregoing, and you will eaſily underſt Method here uſed for ſolving Queſtions nature.

V. Queſtions in Rebate at Compound may be reſolved by the firſt Table of thi ter, which ſheweth the Increaſe of 1 *l.* pound Intereſt, &c. But as in the ſecon *you* make the Tabular Numbers Multip *find out* the preſent Worth of a Sum, would find out the preſent Worth of a Sum by the firſt Table, you muſt then make thoſe Tabular Numbers Diviſors; the Reaſon whereof is plain, for the firſt Table ſheweth the Increaſe of 1 *l.* for 30 Years, *&c.* But they may likewiſe ſerve to ſhew what Sum of Money due at the end of any number of Years to come under 31 (allowing Rebate according to the rates of Intereſt therein mentioned) 1 *l.* preſent Money will ſatisfy. Now to reſolve Queſtions in Rebate by this *Table*, look in the Column of the propoſed Intereſt or Rebate, and againſt the propoſed number of Years is the Tabular Number for your Work, which muſt be according to the following Proportion, *viz.*

As the Tabular Number ſo found
Is to 1,
So is the Sum propoſed to be rebated
To its preſent Worth.

To make this a little more plain, I ſhall anſwer the firſt Queſtion in the Uſe of the ſecond Table, by the help of the firſt Table only, which is as followeth, *viz.*

I demand how much preſent Money will ſatiſfy a Debt of 684 *l.* due at the end of 6 Years to come, allowing Rebate after the rate of 8 *per Cent. per Annum*, Compound Intereſt.

Look in Table I. in the Column of 8 *per Cent.* and againſt 6 Years you will find this Number, *viz.* 1.58687; therefore the Proportion is as followeth:

The manner of valuing Annuities that are in Arrear.

VI. When an Annuity is in Arrear, and i quired to know its Improvement, acco Intereſt upon Inter each particular Sum, the time it becomes the end of the given term of Years; the r how to work ſuch Queſtions will be appa the working of the following Queſtion, *v*

There is an Annuity of 150 *l.* to cont the end of five Years, and the utmoſt Im ment thereof to be made after the rate *Cent. per Annum*, Compound Intereſt; no mand how much will then be due to the tor.

It is evident that there muſt be found ou the Amount of 150 *l.* for one Year, *viz* which is due at the end of the fourth Y

Thirdly, there muſt be accounted the Improvement of 150 *l.* for 3 Years, *viz.* that which is due at the end of the ſecond Year, it lying in the hands of the Debtor the third, fourth, and fifth Years.

And in the fourth place, there muſt be accounted the utmoſt Improvement of 150 *l.* for four Years, *viz.* that which is due at the end of the firſt Year, it lying in the Debtor's hands the ſecond, third, fourth, and fifth Years.

And beſides, there muſt be accounted 150 *l.* due at the end of the fifth Year, no Intereſt being reckoned for that time, becauſe it becometh not due till the expiration of the laſt Year, and then the Sum of all theſe is the utmoſt Amount of that Annuity.

The ſolving of Queſtions concerning Annuities at Compound Intereſt, will not be any thing different in its Operation, from the manner of ſolving a Queſtion concerning a ſingle Sum of Money put out for Years at Compound Intereſt, by the third Rule before-going. As, ſuppoſe that inſtead of an Annuity of 150 *l.* there was a ſingle Sum of 150 *l.* put out for 4 Years at Compound Intereſt, at 6 *per Cent.* what would be its utmoſt Improvement at the end of the ſaid Term?

Here you will eaſily perceive that in ſolving the one, the other is alſo ſolved.

See the Work according to the foregoing Rule:

l.		l.				
100	:	106	: :	150	:	159
				159	:	168.54
				168.54	:	178.6524
				178.6524	:	189.36154
						150
						845.5539

Now if the foregoing Proportions be well ſidered, you will find that

	l.
The Sum due at the end of the fifth Year, being that Year's Rent, is ———	150
And 150 *l.* due at the end of the fourth Year, will at the fifth Year's end be increaſed to ———	159
And 150 *l.* due at the end of the third Year, will at the end of the fifth Year be increaſed to ———	168.54
And 150 *l.* due at the ſecond Year's end, will at the fifth Year's end be increaſed to ———	178.652
And 150 *l.* due at the firſt Year's end, will at the fifth Year's end be increaſed to ———	189.361
The Sum of all theſe being due *at the five* Years end, is ———	845.55

o that if an Annuity of 150 *l.* be all forborn he end of five Years, and it be improved to utmoſt, after the rate of *6 per Cent. per An-* , Compound Intereſt; it will then be increas'd ie Sum of 845.553944=845 *l.* 11 *s.* 0¼ *d.*

low if the particular Numbers in finding out Augmentation of the ſaid Annuity, according he manner before preſcribed, be well view'd, the Method in finding them out be well con- red, *it* will appear that if an Annuity pay- : *by* yearly Payments, be all forborn to the of any number of Years, and the utmoſt Im- rement thereof be made at Compound Inte- , the total then due at the end of the ſaid e, or term of Years, will be the Sum of a ies or Rank of continual Proportionals, as iy in number as the Years of the Annuity, or rly Payment itſelf; and the ſecond proceeding n the firſt after the ſame rate or proportion :00 *l.* and its Intereſt for a Year added toge- r, proceedeth from 100 *l.* And after the ſame doth the third proceed from the ſecond, and fourth from the third, *&c.*

And upon this Rule is grounded the
tion of the following Table, which ſhewe
1 *l.* Annuity (being forborn to the en
Number of Years to come, not exceed
will be increaſed to, Compound Intere
computed after any of the Rates menti
the head of the Table.

But conſidering that as an Annuity in
yearly at compound Intereſt, the Sum due
Year's end is the Sum of a Series of
Proportionals, equal in number to the
Payments, and that the firſt Number is
nual Payment itſelf; therefore may a Ta
ſhew the annual Increaſe of 1 *l.* Annuit
great eaſe be made from the firſt Table,
the yearly Increaſe of 1 *l.* at Compound I
as will plainly appear by what followeth.

Let us pitch upon making the Column
Cent. per Annum in the third Table. I
the firſt Table, and you will find the Col

the ſaid ſecond Number; then add the ſecond Number in the third Table to the ſecond in the firſt, and their Sum is the third Number in the third Table; then add the ſaid third Number to the third Number in the firſt Table, and their Sum is the fourth Number in the third Table, *&c.* And after this manner proceed till you have made all the Numbers in the ſaid Column of *6 per Cent.* And after the ſame Method are the reſt of the Columns made, (the firſt Number in each being 1. or 1.00000) *mutatis mutandis.*

But here note, that the Numbers in the ſaid firſt Table ought to be continued to more places than are there expreſſed, to prevent the Errors that elſe may be found in the third Table, by adding of defective Decimals. The Uſe of the ſaid Table is ſhewn immediately after the ſame.

TABLE III.

Which ſheweth, in Pounds and Decimal Parts of a Pound, the Increaſe or Amount of 1 *l.* Annuity, to continue any number of Years not exceeding 30, Compound Intereſt being computed at 5, 6, 7, 8, 9, and 10 *per Cent. per Annum.*

Years.	5.	6.	7.	8.	9.	10.
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.05000	2.06000	2.07000	2.08000	2.09000	2.10000
3	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000
4	4.31012	4.37461	4.43994	4.50611	4.57312	4.64100
5	5.52563	5.63709	5.75073	5.86660	5.98471	6.10510
6	6.80191	6.97531	7.15329	7.33592	7.52333	7.71561
7	8.14200	8.39383	8.65402	8.92280	9.20043	9.48717
8	9.54910	9.89746	10.25980	10.63662	11.02847	11.43588
9	11.02656	11.49131	11.97798	12.48755	13.02103	13.57947
10	12.57789	13.18079	13.81644	14.48656	15.19292	15.93742
11	14.20678	14.97164	15.78359	16.64548	17.56029	18.53116
12	15.91712	16.86994	17.88845	18.97712	20.14071	21.38428

A Continuation of the preceding TABLE III.

Years.	5.	6.	7.	8.	9.	10.
16	23.65749	25.67252	27.88805	30.32428	33.00339	35.94972
17	25.84036	28.21287	30.84021	33.75022	36.97370	40.54470
18	28.13238	30.90565	33.99903	37.45024	41.30133	45.59917
19	30.53900	33.75999	37.37896	41.44626	46.01845	51.15909
20	33.06595	36.78559	40.99549	45.76196	51.16011	57.27499
21	35.71925	39.99272	44.86517	50.42292	56.76453	64.00249
22	38.50521	43.39228	49.00573	55.45675	62.87333	71.40274
23	41.43047	46.99582	53.43614	60.89329	69.53193	79.54302
24	44.50199	50.81557	58.17667	66.76475	76.78981	88.49732
25	47.72709	54.86451	63.24903	73.10593	84.70089	98.34705
26	51.11345	59.15638	68.67646	79.95441	93.32397	109.18176
27	54.66912	63.70576	74.48382	87.35076	102.72313	121.09994
28	58.40258	68.52810	80.69769	95.33882	112.96821	134.20993
29	62.32271	73.63979	87.34652	103.96593	124.13535	148.63092
30	66.43884	79.05818	94.46078	113.28321	136.30753	164.49402

the Table, are the ſeveral Rates of Intere
100 *l.* for a Year, and the Numbers placed i
ſeveral Columns under thoſe Numbers, ſhe
yearly Increaſe of 1 *l.* Annuity, at the ſam
of Intereſt as it is placed under, and for ſ
ny Years as it is placed againſt in the Colu
Years on the left hand of the Table; an
Uſe of theſe Numbers will be manifeſt b
Method uſed in ſolving the following Que
viz.

There is an Annuity of 34 *l.* 8 *s.* payab
yearly Payment, forborn unto the end
Years; now I demand how much is due a
end of the ſaid Term, Compound Intereſt
allowed at *6 per Cent. per Annum.* Facit 580 *l*
6 d. and ſomewhat more; as will appear b
following Operation.

The increaſe of the ſaid Annuity being
poſed at *6 per Cent.* I look in the Column

l.		l.		l.		l.
1	:	16.86994	::	344	:	580.325936

$$\begin{array}{r} 344 \\ \hline 6747976 \\ 6747976 \\ 5060982 \\ \hline 580.325936 \end{array}$$

Whereby it is apparent, that thoſe tabular Numbers are only Multipliers for the producing of the Amount of any given Annuity, for any number of Years not exceeding 30, any rate of Compound Intereſt being allowed from 5 to 10 *per Cent.* Incluſive, *&c.*

VII. Queſtions concerning the increaſe of Annuities at Compound Intereſt may be likewiſe ſolved by the firſt Table in this Chapter, according to the following Method, *viz.*

When an Annuity is in Arrear, and it is required to know what Sum it is augmented to, Compound Intereſt being computed, *&c.* Find out what Principal will in one Year gain the Annual Rent propoſed, allowing the propoſed rate of Intereſt. Then (as is taught in the Uſe of the firſt Table) find the Increaſe of the ſaid Principal for the number of Years, and at the Rate of Intereſt propoſed, and from the Amount thereof ſubſtract the ſaid Principal; then will that Remainder be the Amount of the given Annuity *for the given* time, as will appear by *ſolving the firſt* Queſtion of the ſixth Rule be-

foregoing, which is this, *viz.* there is an A
ty of 150 *l.* forborn to the end of 5 Years,
is its Amount at 6 *per Cent. per Annum*, Com
Intereſt ?

Now to anſwer this, I find out a Principa
at 6 *per Cent.* will gain 150 *l.* in one Year,
I do by the following Proportion, *viz.*

l.   *l.*   *l.*   *l.*
6 : 100 : : 150 : 2500

So that I find 2500 *l.* to be the Anſwer :
ſuppoſing the ſaid Principal 2500 *l.* to be p
to Intereſt, at 6 *per Cent.* Compound Intere
5 Years, look in the firſt Table in the Colu
6 *per Cent.* and againſt 5 Years you will
1.338225, *&c.* which being multiply'd by
produceth 3345.563944 ; from which if yo
ſtract the ſaid Principal 2500 *l.* there will r
845.563944 for the Anſwer, which is the
with that found before.

VIII. When an Annuity to continue any
ber of Years is to be bought with ready M
there ought to be paid ſo much Money, as
put out at Compound Intereſt, at any Rate
for the time of the Leaſe's continuance, i
tal Amount may be equal to the utmoſ

lowing Example, *viz.* There is an Annuity of 468.38001216 *l.* to continue 5 Years, what is its preſent Worth, allowing Rebate after the rate of 6 *per Cent. per Annum*, Compound Intereſt?

Here it is plain, that there muſt firſt be computed the preſent Worth of the ſaid Annuity, due at the end of the firſt, ſecond, third, fourth, and fifth Years; and the Sum of all theſe preſent Worths will be the preſent Worth of the ſaid Annuity: as will appear by the following Work, which is wrought by the fourth Rule of this Chapter.

	l.
The preſent Worth of 468.38001216 *l.* due at the end of the firſt Year, is —	441.867936
The ſame Sum due at the end of two Years is, in ready Money worth, ———	416.8556
The ſame Sum due at the end of three Years, is worth	393.26
The ſame Sum due at the end of four Years, is worth	371
The ſame due at the end of five Years, is worth in ready Money———	350
The Sum of the ſaid preſent worth is ———	1972.983536

Which is the preſent Worth of an Annuity of *468.38001216 to continue* 5 Years, Rebate being

al

And upon the ſame grounds with the
of the laſt Queſtion, is calculated the fo
fourth Table, which ſheweth the preſent
of 1 *l.* Annuity, to continue any number
not exceeding 30, and payable by year
ments, Rebate being allowed after the
5, 6, 7, 8, 9, and 10 *per Cent. per Annum*
pound Intereſt.

But the Nature of the following Tabl
rightly conſidered, you will find the maki
to be eaſily performed by help of the num
the ſecond Table of this Chapter.

As for Example.

Let us pitch upon the making of the Col
6 per Cent. Firſt, I turn to the ſecond
and by the Numbers in the Column of

n the fourth Table; and after the ſame man-
re all the reſt of the Numbers in that Co-
made, and alſo thoſe in the reſt of the
nns, *mutatis mutandis.*

t remember, whenever you calculate one Ta-
y the help of another, to continue the Ta-
ou make uſe of, to more places than you in-
the Numbers in your Table to conſiſt of,
ear of Errors thro' the Addition of defec-
Decimals.

TABLE IV.

Which ſheweth the preſent Worth of one Pound Annuity payable by yearly Payment, and to continue any Number of Years not exceding 30, Rebate being allowed at 5, 6, 7, 8, 9, or 10 *per Cent. per Annum*, Compound Intereſt.

Years.	5.	6.	7.	8.	9.	10.
1	.95238	.94339	.93457	.92592	.91743	.90909
2	1.85941	1.83339	1.80801	1.78326	1.75911	1.73553
3	2.72324	2.67301	2.62431	2.57709	2.53129	2.48685
4	3.54595	3.46510	3.38721	3.31212	3.23971	3.16986
5	4.32947	4.21236	4.10019	3.99271	3.88965	3.79078
6	5.07569	4.91732	4.76653	4.62287	4.48591	4.35526
7	5.78637	5.58238	5.38928	5.20637	5.03295	4.86841
8	6.46321	6.20979	5.97129	5.74663	5.53481	5.33492
9	7.10782	6.80169	6.51523	6.24688	5.99524	5.75901
10	7.72173	7.36008	7.02358	6.71008	6.41765	6.14456
11	8.30641	7.88687	7.49867	7.13896	6.80519	6.49506
12	8.86325	8.38384	7.94268	7.53607	7.16072	6.81369
13	9.39357	8.85268	8.35765	7.90377	7.48690	7.10335
14	9.89864	9.29498	8.74546	8.24423	7.78614	7.36668
15	10.37965	9.71224	9.10791	8.55947	8.06068	7.60608

A Continuation of the preceding TABLE IV.

Years.	5.	6.	7.	8.	9.	10.
16	10.83776	10.10589	9.44664	8.85136	8.31255	7.82371
17	11.27406	10.47725	9.76322	9.12163	8.54363	8.02155
18	11.68958	10.82760	10.05908	9.37188	8.75562	8.20141
19	12.08531	11.15811	10.33559	9.60359	8.95011	8.36492
20	12.46220	11.46992	10.59401	9.81814	9.12854	8.51356
21	12.82115	11.76407	10.83552	10.01681	9.29224	8.64869
22	13.16300	12.04158	11.06124	10.20074	9.44242	8.77154
23	13.48857	12.30337	11.27218	10.37105	9.58020	8.88329
24	13.79864	12.55035	11.46933	10.52875	9.70661	8.98474
25	14.09394	12.78335	11.65358	10.67477	9.82257	9.07704
26	14.37518	13.00316	11.82577	10.80997	9.92897	9.16094
27	14.64303	13.21053	11.98671	10.93516	10.02657	9.23722
28	14.89812	13.40616	12.13711	11.05107	10.11612	9.30658
29	15.14071	13.59071	12.27767	11.15840	10.19822	9.36960
30	15.39244	13.76482	12.40904	11.25778	10.27365	9.42691

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12	8.86325	8.38384	7.94268	7.53607	7.16072	6.81369
13	9.39357	8.85268	8.35765	7.90377	7.48690	7.10335
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22	13.16300	12.04158	11.06124	10.20074	9.44242	8.77154
23	13.48857	12.30337	11.27218	10.37105	9.58020	8.88329
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30	15.39244	13.76482	12.40904	11.25778	10.27365	9.42691

The

The Uſe of the foregoing Table.

The firſt Column is the Number of Years, from 1 to 30; and the Numbers 5, 6, 7, 8, 9, 10, at the head of the Table, are the Rates of Intereſt of 100 *l.* for a Year, and the Numbers in each of theſe Columns, under the ſaid rates of Intereſt are the preſent Worths of 1 *l.* Annuity to continue for the Number of Years which is placed againſt them, allowing Rebate after the rate of Intereſt at the head of each Column, and are Multipliers ſerving to find the preſent Worth of any other Annuity; as will appear by the following

Example.

There is an Annuity of 48 *l.* to continue 12 Years, and payable by yearly Payments, to be ſold for preſent Money; I demand what it is worth, allowing Rebate at *6 per Cent. per Annum,* Compound Intereſt. *Facit* 402·424=402 *l.* 8 *s.* 5 ¾ *d.* which is thus found out by the foregoing Table, *viz.* look in the ſaid Table, in the Column of *6 per Cent.* and againſt 12 in the Column of Years you have this Number, *viz.* 8.3838, which is the preſent Worth of 1 *l.* Annuity, to continue twelve Years, Rebate being allowed, *&c.* therefore by the Rule of Proportion, I ſay,

1 *l.*

l. l. l.
: 8.38384 : : 48 : 402.42432
48

6707072
3353536

402.42432

that I find the Anſwer to be 402.42432,
is found by multiplying the ſaid Tabular
er by 48, as you ſee by the Work.
herwiſe find a Principal which may bear
roportion to the given Annuity that is to
ated, as 100 beareth to the rate of Intereſt
d in the Rebate. Then find the preſent
of this Principal ſo found, by the Direc-
given in the Uſe of the ſecond Table of
hapter; then ſubſtract the ſaid preſent
from the Principal found, as before, and
emainder will be the preſent Worth of the
Annuity, Rebate being allow'd as propoſed.

Example.

at is the preſent Worth of an Annuity of
o continue 3 Years, allowing Rebate at 8
t. per Annum, Compound Intereſt?
t I find a Principal that ſhall be to the gi-
umber 50, as 100 to 8, which I find to be
by the following Proportion, *viz.*

l. l. l. l.
8 : 100 : : 50 : 625

Then by the ſecond Table I find the pre Worth of 625 *l.* which is 496.145 *l.* whi ſubſtract from the ſaid Principal 625 *l.* and t remaineth 128.855 *l.*=128 *l.* 17 *s.* 1 ½ *d.* which is the preſent Worth of 50 *l. per An* to continue 3 Years, Rebate being allowed *per Cent. per Annum*, Compound Intereſt.

Moreover by the Numbers in the foregoin Table, you may at firſt ſight diſcover how n Years Purchaſe any Leaſe, to continue any N ber of Years not exceeding 30, is worth in dy Money, Compound Intereſt being comp on both ſides, at any of the Rates mention the Head of the Table.

Example.

Suppoſe there were a Leaſe iſſuing ou Lands to continue 16 Years to be ſold for r Money, allowing Rebate at 8 *per Cent. per An* Compound Intereſt, I demand how many Y Purchaſe the ſaid Leaſe is worth.

Look in the Table 4, in the Column of *Cent.* and againſt 16 Years you will find 8.8 which ſheweth that it is worth 8.85136 Y Purchaſe, which is ſomewhat above 8 Year 3 Quarters: But if the ſaid Leaſe had bee Houſes, and 10 *per Cent.* was thought a co nient Allowance for the ſame, then you will it to be worth 7.82371 Years Purchaſe, whi 7 Years and above 3 Quarters Purchaſe.

IX. When there is a Sum of Money propo *ed*, and it is required to know what Annui

ue any given number of Years, it will pur-
according to any given Rate of Intereſt;
ay ſuppoſe any Annuity at
re, then by the Directions*
in the Uſe of the 4th Ta-
r elſe by the 8th Rule of
hapter, find the preſent
of the ſuppoſed Annuity for the number
rs, and at the rate of Intereſt propound-
hich being done, you may find what An-
to continue the ſaid Number of Years, the
ropounded will purchaſe, by the following
rtion, *viz.*

Of the Purchaſe of Annuities at Comp. Intereſt.

the preſent Worth of the ſuppoſed Annuity
o the ſaid Annuity,
s the Sum propounded
the Annuity required.

As for Example.

it be required to find out what Annuity to
ue 4 Years 800 *l.* preſent Money will pur-
Compound Intereſt being computed at *6*
t. per Annum. *Facit* 230.873 *l.*

t, ſuppoſe an Annuity at pleaſure to con-
4 Years, as ſuppoſe 150 *l.* then do I find, by
ghth Rule of this Chapter, the preſent
of the ſaid Annuity to be 519.76584 *l.*
ore by the Rule of Proportion, I ſay,

	l.		*l.*		*l.*
6584, *&c.*	: 150	: :	800	:	230.873

The Conſtruction of the following Table V.

Upon the Reaſon of the foregoing Rule is grounded the Calculation of the following Table for the purchaſing of Annuities; and it may ſomewhat more readily be calculated thus, *viz.*

It is evident by the Conſtruction of the firſt Table of this Chapter, that 1 *l.* preſent is equivalent to 1.06 due at the end of a Year to come; therefore is 1.06 the firſt Number in the Column of 6 *per Cent.* of the following Table; becauſe 1 *l.* will purchaſe 1.06 *l.* Then it is alſo evident by the fourth Table, that the preſent Worth of 1 *l.* Annuity to continue 2 Years at the ſame rate is 1.83339, *&c.* that is, 1.83339, *&c.* will purchaſe a Leaſe of 1 *l. per Annum* to continue two Years, Compound Intereſt being allowed at 6 *per Cent.* therefore by the Rule of Three Direct, I ſay,

l. *l.*
1.83339, *&c.* : 1 :: 1 : .54543, *&c.*

By which I find that 1 *l.* ready Money will buy a Leaſe of .54543 *l. per Annum* to continue two Years, therefore it is the ſecond Number in the following Table. Likewiſe by the fourth Table I find that 2.67301 is the preſent Worth of 1 *l.* Annuity to continue three Years at the ſame rate of Intereſt; wherefore by the Rule of Proportion, I ſay,

l. *l.* *l.* *l.*
2.67301, *&c.* : 1 :: 1 : .37411, *&c.*

Where-

Vhereby I find that 1 *l.* will purchaſe an Annui-
f .37411, to continue 3 Years, Compound In-
ſt being allowed at 6 *per Cent.* wherefore .37411
he third Number in the ſaid Table; whereby
s evident, that if you divide 1, or Unit, by the
eral Numbers in the ſaid Column of 6 *per Cent.*
the 4th Table ſucceſſively, the ſeveral Quoti-
s will give you the Numbers ſucceſſively for the
lumn of 6 *per Cent.* in the fifth Table; and after
ſame manner are all the Numbers in the other
lumns of the ſaid fifth Table found out (ex-
t the firſt Number of each Column, which muſt
the ſame with the firſt Numbers in each Co-
n of the firſt Table) *mutatis mutandis.*

ut it is abſolutely neceſſary that the Numbers
he ſaid 4th Table be continued to more places
n there are expreſſed, to prevent the Errors
t otherwiſe will ariſe, by dividing by defective
cimals.

TABLE

TABLE V.

Which ſheweth what Annuity payable by yearly Payments to continue any number of Years not exceeding 30, one Pound will purchaſe, Compound Intereſt being computed at 5, 6, 7, 8, 9, or 10 *per Cent. per Annum.*

Years.	5.	6.	7.	8.	9.	10.
1	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000
2	.53780	.54543	.55309	.56076	.56846	.57618
3	.36720	.37411	.38105	.38803	.39505	.40211
4	.28201	.28859	.29522	.30192	.30866	.31547
5	.23097	.23739	.24389	.25045	.25709	.26379
6	.19701	.20336	.20979	.21631	.22291	.22960
7	.17281	.17913	.18555	.19207	.19869	.20545
8	.15472	.16103	.16746	.17401	.18067	.18744
9	.14069	.14702	.15348	.16007	.16679	.17364
10	.12950	.13586	.14237	.14902	.15582	.16274
11	.12038	.12679	.13335	.14007	.14694	.15396
12	.11282	.11927	.12590	.13269	.13965	.14676
13	.10645	.11296	.11965	.12652	.13356	.14077
14	.10101	.10758	.11434	.12129	.12843	.13574
15	.09634	.10296	.10979	.11682	.12405	.13147

A Continuation of the preceding TABLE V.

Years,	5.	6.	7.	8.	9.	10.
16	.09226	.09895	.10585	.11298	.12029	.12781
17	.08869	.09544	.10242	.10962	.11704	.12466
18	.08554	.09235	.09941	.10670	.11421	.12192
19	.08274	.08962	.09675	.10412	.11173	.11954
20	.08024	.08718	.09439	.10184	.10954	.11745
21	.07799	.08500	.09228	.09983	.10761	.11562
22	.07597	.08304	.09040	.09803	.10590	.11400
23	.07413	.08127	.08871	.09642	.10438	.11257
24	.07247	.07967	.08718	.09497	.10302	.11129
25	.07095	.07822	.08581	.09367	.10180	.11016
26	.06956	.07690	.08456	.09250	.10071	.10915
27	.06829	.07569	.08342	.09144	.09973	.10825
28	.06712	.07459	.08239	.09048	.09885	.10745
29	.06604	.07357	.08144	.08961	.09805	.10672
30	.06496	.07260	.08058	.08882	.09733	.10607

The

The Uſe of the foregoing Table V.

The Uſe of the foregoing Table will appear in the Solution of the following Queſtion, *viz.*

A Merchant hath 1500 *l.* by him, which he is willing to lay out upon an Annuity, iſſuing out of Lands, to continue 20 Years, beginning preſently, Compound Intereſt being computed on both ſides at *6 per Cent. per Annum.* Now I demand what Annuity the ſaid Sum will buy. Facit 130.77 *l.* =130 *l.* 15 *s.* 5 *d.* very near.

To anſwer this Queſtion, I look in the Column of *6 per Cent.* of the foregoing fifth Table, and againſt 20 in the Column of Years I find .08718, which is the Annuity that 1 *l.* preſent Money will purchaſe to continue 20 Years; wherefore by the Rule of Three Direct, I ſay,

l.		*l.*		*l.*		*l.*
1	:	.08718	: :	1500	:	130.77

X. Queſtions concerning the purchaſing of Leaſes and Annuities may be ſolved very well by the Numbers in the 4th Table, if you make them Diviſors inſtead of Multipliers.

Let the laſt Queſtion be propoſed and ſolved by the 4th Table, *viz.*

What Annuity, to continue 20 Years, will 1500 *l.* ready Money purchaſe, Compound Intereſt being allowed at *6 per Cent.*

To anſwer this, I look in the 4th Table in the Column of *6 per Cent.* againſt 20 Years, and there I find this Number, *viz.* 1146992, which is the preſent worth of 1 *l.* Annuity to continue 20 Years, Compound Intereſt being allowed at *6 per Cent.* And if it be the preſent Worth of 1 l. Annuity, I conclude it will purchaſe 1 l. Annuity to continue the ſame number of Years: wherefore I ſay, by the Rule of Three Direct,

l.		*l.*		*l.*		*l.*
11.46992	:	1	::	1500	:	130.77

So that the Anſwer is the ſame with the former, which was found by the help of the fifth Table.

All the foregoing Tables might have been continued to any greater number of Years at pleaſure. But altho theſe Tables are calculated but for 30 Years, yet they may be made ſerviceable for Years above 30, as ſhall be ſhewed by and by.

Arithmetical Queſtions to exerciſe the Learner in the Precedent Tables.

Queſt. 1. There is a Leaſe of 20 Years to begin preſently, which in ready Money is worth 1200 *l.* But ſuppoſe the ſaid Leaſe were not to begin till the expiration of 8 Years, I demand what would be the preſent Worth of the ſaid Leaſe, Rebate being allowed at 8 *per Cent. per Annum*, Compound Intereſt.

The main Intent of this Queſtion is to ſhew the Uſe of the ſecond Table; for if you find the preſent Worth of 1200 *l.* due at the end of 8 Years at 8 *per Cent.* the Queſtion is anſwered; which according to the Directions given after the ſaid Table, will be found to be 648.3216 *l.*=648 *l.* 6 *s.* 5 *d.*

Queſt. 2. A oweth to B 600 *l.* to be paid in 6 Years, *viz.* 100 *l.* every Year; but being weakned in his Eſtate, is not able to perform; but an Eſtate being to come into his hands at the end of 10 Years: B is willing to forbear it all till then, and to be allowed Compound Intereſt at 8 *per Cent.* for his Forbearance; I demand how much will be due to B at the 10 Years end.

This Queſtion is ſolved by help of the third and firſt Tables: for firſt, 100 *l.* is to be paid in the nature of Annuity for 6 Years; therefore by the third Table I find the Amount of an Annuity of 100 *l.* to continue 6 Years at 8 *per Cent.* which is 733.592 *l.* and will be due at the expiration of 6 Years; and then is that Sum to be forborn to the end of 10 Years; which is 4 Years after the 6 Years; which being a ſingle Sum, its Amount is found by the firſt Table to be 998.037 *l.* *&c.* which is the Anſwer to the Queſtion.

Queſt. 3. There is a Leaſe to continue 21 Years to be ſold for 1000 *l.* but the Leſſee deſireth rather to pay an annual Rent: now the Queſtion is what that annual Rent ought to be, Compound Intereſt being computed at 10 *per Cent. per Annum?*

The Intent of this Queſtion is to find what Annuity to continue 21 Years 1000 *l.* will purchaſe at 10 *per Cent.* which is to be done by the 5th Table, thus:

Becauſe the time is for 21 Years, look in the Column of Years for 21, and juſt againſt it in the Column of 10 *per Cent.* you will find .11562, by which multiply 1000, and the Product is 115.62 *l.* and ſo much will 1000 *l.* purchaſe for 21 Years at 10 *per Cent.* Compound Intereſt.

Queſt. 4. *A* and *B* have each of them a Leaſe to continue 20 Years; *A* hath 80 *l.* per *Annum*, and *B* 120 *l.* *per Annum*, and they agree to make an Exchange, upon this Condition, that *A* ſhall pay in ready Money the Exceſs of his Eſtate, allowing him Compound Intereſt at 8 *per Cent.* Now I demand how much ready Money *A* ought to give *B* upon this Exchange, according to that Condition?

Subſtract 80 *l.* from 120 *l.* and the Remainder is 40 *l.* and ſo much *per Annum*, is the Leaſe of *B* worth more than that of *A*; therefore *A* muſt pay *B* ſo much Money as will purchaſe 40 *l.* *per Annum* to continue 20 Years at 8 *per Cent.* which by the 3d Table will be found to be 392.7256 *l.*

Queſt. 5. There is a Houſe to be let by Leaſe for 21 Years, for which the Leſſor will have 50 *l.* Fine, 70 *l.* *per Annum*; but the Leſſee is willing to pay the greater Fine, that he may have the Rent but 40 *l.* *per Annum*. Now I demand what Fine he ought to pay upon that Condition, Compound Intereſt being allowed at 8 *per Cent.* *per Annum.*

Take the difference between 40 and 70, which is 30, for the Abatement in the yearly Rent for 21 Years; then by the fourth Table find the preſent Worth of 30 *l.* *per Annum* for 21 Years at 8 *per Cent.* which is 300.5043 *l.* = 300 *l.* 10 *s.* 1 *d.* which added to the ſaid 50 *l.* Fine, makes 350 *l.* 10 *s.* 1 *d.* for the Fine to be paid upon the ſaid Condition.

Queſt. 6. There is a Leaſe to be let of 20 *l.* *per Annum*, and 250 *l.* Fine for 24 Years, and the Leſſee is willing to pay the greater Rent, that he may pay but 50 *l.* Fine; now I demand what Rent he ought to pay upon that Condition, Compound Intereſt being computed at 7 *pet Cent.* *per Annum*?

It is manifeſt, that if the Leſſor taketh Fine, he abateth 200 *l.* therefore find, by the Table, what Annuity to continue 24 Years, will purchaſe at 7 *per Cent.* The tabular ber is .08718, which multiplied by 200 p ceth 17.436 = 17 *l.* 8 *s.* 9 *d.* and ſo much the Leſſee raiſe his Rent if he will have 200 bated of his Fine; to which if you add 20 *l.* propoſed Rent, the Sum is 37 *l.* 8 *s.* 9 *d.* fo yearly Rent to be paid to ſatisfy the ſaid dition.

Queſt. 7. What Annuity to continue 20 Y may I grant preſently, for 900 *l.* to be pa Years hence, accounting 6 *per Cent. per An* Compound Intereſt?

Firſt find by the 2d Table the preſent W of 900 *l.* due 6 Years hence, at 6 *per Cent.* w is 634.464 *l.* = 634 *l.* 9 *s.* 3 ½ *d.*

Then by the fifth Table find what Annuit continue 20 Years, 634.464 will purchaſe at *Cent.* and you will find the Anſwer to be 31257152 *l.* = 55 *l.* 6 *s.* 3 *d.* and ſo much I o to grant yearly for 20 Years for 900 *l.* to be me at the end of 6 Years.

Queſt. 8. I have 6 Years of an old Leaſe y come, and would take a new Leaſe in Reve for 21 Years, after the expiration of the old L the annual Rent whereof is 40 *l.* but I w

Rent, wherefore by the 4th Table find the preſent Worth of 25 *l.* *per Annum* for 21 Years at 8 *per Cent.* which is 250.42025 *l.*=250 *l.* 8 *s.* 5 *d.* Then by the ſecond Table find the preſent Worth of 250.42025 *l.* due at the end of *6* Years to come, at 8 *per Cent.* which is 157.807 *l.* *&c.*= 157 *l.* 16 *s.* 1 ¼ *d.* And ſo much ought I to give to ſatisfy the ſaid Conditions.

Queſt. 9. There is a Leaſe to be let for 12 Years, for 20 *l.* *per Annum*, and 20 *l.* Fine; but the Leſſee deſiring to take a Leaſe of the ſame for 21 Years, and to pay the ſame Rent, the Queſtion is, what Fine ought to be paid for the Leaſe of 21 Years, accounting Compound Intereſt at *6 per Cent.* *Facit* 280 *l.* 12 *s.* 5 *d.*

By the fifth Table, ſeek what Annuity to continue 12 Years, 200 *l.* will purchaſe at *6 per Cent.* which you will find to be 23.854 *l.* Then by the fourth Table find the preſent Worth of 23.854 *l.* Annuity to continue 21 Years at *6 per Cent.* which is 280.620 *l.* *&c.*=280 *l.* 12 *s.* 5 *d.* and ſo much ought the Leſſee to pay for a Fine, to have his Leaſe for 21 Years.

Queſt. 10. A Gentleman hath 1000 *l.* which he would lay out to purchaſe an Annuity of 100 *l.* to be paid by yearly Payments: Now the Queſtion is, how many Years muſt the ſaid Annuity continue, Compound Intereſt being allowed on both ſides at 8 *per Cent.* *per Annum.*

Firſt, Divide 1000 by 100, and the Quotient will be 10, which ſheweth that the Buyer giveth 10 Years Purchaſe for the ſaid Annuity.

Then in the fourth Table, and in the Column of 8 *per Cent.* look for the Number 10, which *cannot be exactly* found, but the neareſt to it, *and leſs than it, is 9.81814*, which is placed againſt

20 Years, and the neareſt to it greater than 10.01681, therefore I conclude that the An muſt continue above 20 Years, but not 21 Y and to find out how much it muſt continue than 20 Years, I work thus, *viz.* firſt, I fin Difference between the ſaid Tabular Nur 10.01681 and 9.81814, which is .19867; t find the Difference between the leſſer of th Tabular Numbers, *viz.* 9.81814 and 10, Number that I would find in the Table, wh .18186, then by the Rule of Proportion, I

l.		*Year.*		*l.*		*Yea*
.19867	:	1	: :	.18186	:	.9153

which is as much as to ſay, as the greater Diffe .19867 is to one Year, ſo is the leſſer Differer .9153 parts of a Year, which is 47 Weeks, Days; therefore the Number of Years foug the Queſtion is 20 Years, 47 Weeks, and 5

Qu. 11. A Gentleman bought a Leaſe of *per Annum*, to continue 18 Years, for 960 *l.* I demand what Rate of Compound Intere there implied in ſuch a Bargain ?

To anſwer this, Firſt, I divide 960 by 10 the Quotient is 9.6, which ſheweth how Years Purchaſe it was worth; then, becau Leaſe was to continue 18 Years, I look fourth Table, in the Column of Years, for 1 carry my Eye exactly in the Line againſt it, *Cent.* and to know how much it is more than do thus; take the difference between the two l tabular Numbers, which you will find to be 20, and ſubſtract (9.6) the ſaid Quotient, n 10.05908, (the greater tabular Number) the Remainder is .45908; then by the Rule of portion, I ſay,

l.		*l.*		*l.*		*l.*
.68720	:	.45908	: :	1	:	.6680

it is to ſay, as the Difference between the tabular Numbers is to the leſſer Remainder, s 1 *l.* the difference between 7 and 8 *per Cent.* 6680, the proportional Part to be added to 7 *l.* ch is 13 *s.* 4 ½ *d.* ſo that 7 *l.* 13 *s.* 4½ *d.* is near the Intereſt required.

ow to find out Tabular Numbers for Years exceeding 30.

It may many times fall out, that the number ears propoſed in a Queſtion, may exceed the ber of Years limited in the foregoing firſt, nd, third, fourth, and fifth Tables; and in caſes that Defect may be ſupplied by the hod uſed in the Solution of the following ſtions.

ueſt. 12. Suppoſe 80 *l.* were put out to In- ſt at 5 *l. per Cent.* Compound Intereſt, for 40 rs, I demand how much it will then be a- nted to?

his Queſtion is to be ſolved by the firſt Ta- thus, *viz.* Take any two Numbers in the umn *of Years, which* together will make up *and then take the* tabular Numbers in the

Column of 5 *per Cent.* which ſtand againſt
two Numbers, and multiply them together,
then multiply that Product by 80 *l.* the
Sum, and the laſt Product will be the Anſw

As, ſuppoſe you take 30 and 10, or 21 an
or 31 and 9, or 25 and 15, *&c.*

But we will pitch upon 30 and 10, and t
bular Number againſt 30 in the Column of
Cent. is 4.32194, and againſt 10 is 1.62
which two Numbers being multiplied, pr
7.03996, *&c.* which is the Amount of 1
40 Years at 5 *per Cent.* then I multiply 7.0
&c. by 80 *l.* and the Product is 563.197 *l.*
=563 *l.* 3 *s.* 11 ½ *d. fere.*

The Anſwer would have been the ſame,
had pitched upon any other two Numbers to
made up 40. And for tryal hereof, let us
upon 25 and 15; the tabular Number a
25 is 3.38635, and the tabular Number a
15 Years is 2.07892, and the Product of
two tabular Numbers is 7.0399, *&c.* which
tiplied by 80, produceth 563.197 *l.* as be
and ſo much will 80 *l.* be increaſed to in 40 Y
at 5 *per Cent. per Annum* Compound Intereſt.
like is to be underſtood for any other nu
of Years.

Queſt. 13. Suppoſe 420 *l.* to be payable a nd that againſt 20 is .376889, and the Product f theſe two is .807203, *&c.* which is the preſent Worth of 1 *l.* due 50 Years hence, at 5 *per Cent. per Annum*, wherefore I multiply .807203, *&c.* by 420, and the Product is *36.62544*, *&c.* =*36 l.* 12 *s.* *6 d.* and ſo much is the preſent Worth of 420 *l.* due 50 Years hence, at 5 *per Cent. per Annum*, Compound Intereſt.

Queſt. 14. An Heir being beyond the Sea, did not return till 36 Years after an Eſtate of 30 *l. per Annum*, was fallen to him by the Death of the Proprietor; the Queſtion is, what was then due to him, Compound Intereſt being computed at 6 *per Cent. per Annum?*

This Queſtion is of the nature of thoſe belonging to the third Table, and the manner of ſolving it is thus, *viz.*

Find out (by the ſeventh Rule of this Chapter) what Principal will in one Year gain 30 *l.* at 6 *per Cent.* by the following Proportion.

l. *l.* *l.* *l.*
6 : 100 :: 30 : 500

Having found 500 *l.* to be the Principal, ſeek (after the manner of the 12th Queſtion) by the firſt Table, the Amount or Increaſe of 500 *l.* for 36 Years at 6 *per Cent.* which you will find to be 4073.5998 *l.* *&c.* from which if you ſubſtract the ſaid Principal 500 *l.* the Remainder is 3573.599 *l.* *&c.*=3573 *l.* 12 *s.* *fere.* And ſo much was due to the Heir at his return.

Queſt. 15. There is an Annuity of 30 *l.* to continue 37 Years; the Queſtion is what it is worth *in ready Money*, Compound Intereſt being com-
uted at 6 per Cent. per Annum.

ted from 500 *l.* leaves 442.1034, *&c.*=
2 *s.* ¾ *d.* and ſo much is the preſent Worth o
aforeſaid Annuity.

Queſt. 16. What Annuity to continue 40 Y
will 500 *l.* purchaſe, Compound Intereſt b
computed at *6 per Cent. per Annum?*

It is evident by the tenth Rule of this Cha
that if you find out the preſent Worth of 1 *l.*
nuity for any number of Years, and at any
of Intereſt, it may eaſily be found what An
to continue the ſame Number of Years, any
Sum will purchaſe, at the ſame rate of Int
by one ſingle Rule of Three Direct: There

Find out the preſent Worth of 1 *l.* Annui
continue 40 Years at *6 per Cent.* by the Me
uſed in ſolving the laſt Queſtion, which w
found to be 15.04632 *l.*=15 *l.* 11 *d.* which
of Money will purchaſe an Annuity of 1 *l.* to
tinue 40 Years at *6 per Cent.* therefore to
what Annuity 500 *l.* will purchaſe for the

COCKER's

Artificial Arithmetick,

SHEWING

The GENESIS or FABRICK of the Logarithms, and their Uſe in the Extraction of Roots, ſolving of Queſtions in Anatociſm or Compound Intereſt, and in the other Rules of Arithmetick, in a Method not uſually practiſed.

Nil tam difficile eſt quod non Solertia vincat.

The Meaning of ſuch Characte
are uſed in the enſuing Treati

+ IS the Sign of Addition, and is as m
to ſay *plus*, ſignifying that the Numb
Quantities between which it is placed, are
added together; as $4+7$ ſignifieth that 4
are to be added together.

— Is the Sign of Subſtraction, and as m
to ſay *minus*, ſignifying that the Number
followeth it, is to be ſubſtracted out of the
ber which precedeth it; as $8-5$ ſignifieth
is to be ſubſtracted from 8.

× Is the Sign of Multiplication, and ſig
that the Numbers between which it is p
are to be multiplied together; as 6×8 ſig
that 6 and 8 are to be multiplied togethe

= Is a Sign of Equality, and ſignifiet
the Numbers or Magnitudes between whi
placed, are equal; as $3+6=7+2$ ſignifieth
and 6 are equal to 7 and 2: Likewiſe $18-6=$
$=12$, and $4\times7=28$, *&c.* If this be not a

THE SECOND BOOK. OF *Artificial Arithmetick.*

CHAP. I.

I. A*Rtificial Arithmetick* is performed by Artificial Numbers, very fitly called Logarithms.

II. Logarithms are borrowed Numbers which differ among themſelves by Arithmetical Proportion, as the Numbers which they ſignify, differ by Geometrical Proportion.

III. Logarithmetical Arithmetick is an artificial Uſe of Numbers, invented for eaſe in Calculation, wherein each natural Number is ſo fitted with an artificial, that what is uſually produced by multiplication of natural Numbers, is here effected by *the addition of their* artificial Numbers; and *what natural* Numbers perform by Diviſion, is

he

here effected by the Subſtraction of their artificial Numbers, and what natural Numbers do perform by long and tedious Operations in the Extraction of Square, Cube, Biquadrate, *&c.* Roots, is here eaſily effected by Bipartition, Tripartition, Quadripartition, *&c.* of their artificial Numbers; and ſo the hardeſt Parts of Calculation are avoided by an eaſy Poſtaphæreſis, as our Trigonometrical Calculators of late have ſufficiently experienced, by avoiding very tedious Multiplications and Diviſions in the Uſe of the Tables of natural Signs, Tangents, Secants, to the everlaſting Credit of the Honourable * Author of this late and incomparable Invention.

IV. The Parts of *Artificial Arithmetick* are the ſame with natural Arithmetick; but we ſhall treat of them in this Order, *viz.* Firſt, of the Nature of Logarithms; Secondly, of their Geneſis, or the Invention of the Table of Logarithms; and Thirdly, of the Uſe of the Logarithms in Multiplication, Diviſion, the Extraction of Roots, *&c.*

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* *The Lord* Nepair *Baron of* Merchiſton *in* Scotland.

CHAP. II.

Of the Nature of Logarithms.

Ogarithms are Numbers ſo fitted to Proportional Numbers, that themſelves retain Differences.

t there be aſſigned a Series or Rank of bers in Geometrical Proportion, as thoſe e Column A, 1, 2, 4, 8, 32, *&c.* And there be as other Numplaced over ſt them in metical Proportion, that is, g equal Diffes, as thoſe in Columns B, C, E, or any other bers whatſoever of the like na- Then, oraſmuch as Numbers in Columns B, C, D, E, are of equal Difference g themſelves, therefore ſhall they be the Lohms *of the Number* in the Column A, each of the

A	B	C	D	E
1	0	2	5	0
2	1	4	8	3
4	2	6	11	6
8	3	8	14	9
16	4	10	17	12
32	5	12	20	15
64	6	14	23	18
128	7	16	26	21
256	8	18	29	24
512	9	20	32	27
1024	10	22	35	30
2048	11	24	38	33
4096	12	26	41	36

the reſpective Number againſt which it is placed. So in the Column B, the Number 4 is the Logarithm of 16 in the Column A; and in the Column C, the Number 10 is the Logarithm of 16 in the Column A; and in the Column D, 29 is the Logarithm of 256 in the Column A, *&c.*

And as the Numbers in the ſaid Columns B, C, D, E, are Logarithms of the reſpective Numbers in the Column A, ſo they may be Logarithms of any other Rank or Series of Numbers in Geometrical Proportion.

II. If four Numbers are Arithmetical Proportionals, either continued, or diſcontinued, the Sum of the Means is equal to the Sum of the Extremes.

Let us chooſe 8, 10, 12, 14, in the Column C; I ſay, that the Sum of the Extremes 8 and 14 is equal to the Sum of the two Means 10 and 12. For, $8+14=10+12=22$. Or if they are diſcontinued, as 10, 12, 22, 24, in the Column C; for $10+24=12+22=34$. The like of any other; this being a peculiar Property of all Numbers that are arithmetically Proportional.

III. If four Numbers are in Geometrical Proportion, either continued, or diſcontinued, the Product ariſing from the Multiplication of the two Extremes, is equal to the Product of the two Means.

So 4, 8, 16, 32, in the Column of A, are Geometrical Proportionals continued; and the Product of the Extremes 4 and 32 is equal to the Product of the Means 8 and 16; for $4\times32=8\times16=128$.

Alſo 4, 8; 64, 128, are Geometrical Proportionals diſcontinued; and the Product of 4 and 128, the Extremes, is equal to the Product of 8 and 64, the two Means; for 4×128=8×64=512.

Hence it follows, that what Geometrical Proportionals perform by Multiplication, the ſame will the Logarithms (being Arithmetical Proportionals) perform by Addition.

Let there be given four Geometrical Proportionals in the Column A, *viz.* 8, 16, 128, and 256; and let their Logarithms be 8, 10, 16, and 18 in the Column C; I ſay, that as 8 × 256, the Product of the Extremes, is equal to 16×128, the Product of the Means, ſo is 8+18, the Sum of the Logarithms of the Extremes, equal to 10+16, the Sum of the Logarithms of the Means. Therefore,

If three Numbers are given to find the fourth Proportional, it may be found by Addition and Subſtraction of their Logarithms; for, as in natural Numbers if you multiply the ſecond and third together, and divide their Product by the firſt, the Quote will be the fourth Proportional Number; ſo if you add the Logarithms of the ſecond and third together, and from their Sum ſubſtract the Logarithm of the firſt, the Remainder will be the Logarithm of the fourth Proportional Number.

Example.

Let there be given 2, 16, 64, and let it be required to find a fourth Proportional Number thereto, *which is 512.*

The

will remain 27, which is the Logarithm the 4th Proportional Number ſought for.

$$12+18-3=30-3=27.$$

And

$$2 : 16 :: 64 : 512$$

IV. By what hath been ſaid, you may p that to natural Numbers there may be fit vers kinds of Logarithms, but we ſhall pit upon that kind which were framed by Mr. at the Requeſt of the Baron of *Merchiſto* hath choſen theſe Geometrical Proport *viz.* 1, 10, 100, 1000, 10000, 100000, which Numbers he hath aſſumed the Loga following, *viz.* for the Number 1, the rithm 0.000000, for 10 the Log. 1.0000 100 the Log. 2.000000, for 1000 the

A	B
1	0.000000
10	1.000000
100	2.000000
1000	3.000000
10000	4.000000
100000	5.000000
1000000	6.000000
10000000	7.000000
100000000	8.000000
1000000000	9.000000
10000000000	10.000000

The Numbers in the Column A are the Series of Geometrical Proportionals, and the Numbers in the Column B, are the reſpective Logarithms of each of thoſe Geometrical Proportionals; them-ſelves being arithmetical Proportionals; where note, that the Figures 1, 2, 3, 4, *&c.* which are ſeparated from the reſt by a Point or Prick, are called the Indices or Characteriſticks of the Logarithm, becauſe they declare how many places the Numbers by them ſignified do conſiſt of; the Characteriſtick of any Logarithm being always an Unit leſs than the number of places which the Number by it ſignified doth conſiſt of; as in the foregoing Table you may perceive that the Logarithm of 1, is 0.000000, and the Logarithm of 10 is 1.000000, and the Logarithm of 100 is 2.000000, *&c.* ſo that the Index or Characteriſtick of 1, and of all Numbers from 1 to 10 is 0. and the Charaſteriſtick of 10, and of all Numbers *from 10 to 100 is 1*; and the Characteriſtick *of 100, and of all Numbers* from 100 to 1000 is 2, and the Characteriſtick of each Number being an Unit leſs than the number of places of which the Number by it ſignified doth conſiſt, as was ſaid before.

The Logarithms of this kind ought all to conſiſt of an equal number of places, that is to ſay, they ought not to be one Log. of 10 places, another of 8, *&c.* but all of them to be of 6, of 7, of 8, *&c.* places.

CHAP. III.

Of the Geneſis or Fabrick of the Logarithms.

I. THE Logarithm of 1 being aſſumed to be 0.000000, and the Logarithm of 10 to be 1.000000, the Logarithm of 100 to be 2.000000, *&c.* In the next place it will be requiſite to ſhew the way and manner of calculating the Logarithms of the intermediate Numbers, *viz.* of the Numbers between 1 and 10, which are 2, 3, 4, 5, *&c.* and between 10 and 100, which are 11, 12, 13, 14, 15, 16, *&c.* and between 100 and 1000, which are 101, 102, 103, 104, *&c.* which to do, obſerve the following Rules.

II. Find ſo many continual Means between 1 and 10, till that continual Mean which cometh neareſt 1, may be a mix'd Number leſs than 2 and ſo near 1, that it may have as many Cyphers placed before the ſignificant Figures of the Numerator, as you intend your Logarithms to conſiſt of Places: But our Directions here ſhall be for the making a Table of Logarithms to conſiſt of 7 places; wherefore find ſo many continual Means between 1 and 10, till the laſt may have 7 Cyphers placed before the ſignificant Figures of its Numerator; in order whereunto, annex to the Number 10 a competent number of Cyphers, (*viz.* 28, becauſe the Work may be the more exact) and extract the ſquare Root of that Number ſo enlarged, which being done, you will find its ſquare Root to be 3.16227766016837; this being done, annex to the ſaid Root 14 Cyphers more, and extract the ſquare Root thereof, which you will find to be 1.77827941003892.

Again, annex to the Root laſt found 14 Cyphers more, and extract the ſquare Root thereof, which you will find to be 1.33352143216332; and thus proceeding ſucceſſively by annexing of Cyphers, and a continual Extraction of the ſquare Root, until you have found a ſquare Root, or continual Mean, having 7 Cyphers placed before the ſignificant Figures of its Numerator, which will be found, after 27 ſeveral Extractions, to be 1.00000001715559.

So the 3 laſt continual Means between 10 and 1, will be found to be
1.00000006862238
1.00000003431119
1.00000001715559

All which 3 continual Means are leſs than 2, and ſo near 1, that there are 7 Cyphers placed before the ſignificant Figures of each of their *Numerators.*

Having found 27 ſeveral Means betwee
and 1, place them ſucceſſively one under the o
as in the Column A of the following Table;
make another Column, B, to contain the re
tive Logarithms of thoſe continual Means.

And becauſe biparting the Logarithm of
Number produceth the Logarithm of the ſ
Root of that Number, therefore take the L
rithm of 10, which is 1.000000, and place
the Column B, over againſt 10; then bipa
(that is, divide it by 2) and you will have 0.50
which is the Logarithm of 3.16227766010
the ſquare Root of 10; then take half of tha
garithm, *viz.* 0500000, which is 0.50000,
place it for the Logarithm of 1.7782794100
the 3d mean Proportional, (or Square of the
of 3.162277660, *&c.*) and ſo by continual B
tition, you will at length find that 0.000
7450580 will be the Logarithm of the laſt
tinual Mean, *viz.* the Logarithm of 1.000
715559, as in the following Table.

A	B
Continual Means.	Their Logarithr
10.00000000000000	1.0000000000000
3.16227766016837	0.5000000000000
1.77827941003892	0.2500000000000
1.33352143216332	0.1250000000000
&c.	&c.
1.00000006862238	0.00000000029802
1.00000003431119	0.00000000014901
1.00000001715559	0.00000000007405

III. Any Number whatſoever being given, how to make the Logarithm thereof.

When it is required to make the Logarithm of any Number, extract ſo many continual Means between the given Number and 1, until the Mean which cometh neareſt 1, may be a mix'd Number leſs than 2, and ſo near 1, that it may have 7 Cyphers placed before the ſignificant Figures of its Numerator; which being done, you may eaſily find out the Logarithm of that continual Mean, by help of the foregoing Table: and then by doubling, and redoubling the Logarithm of the ſaid continual Mean, as many Times as you found continual Means by Extraction, ſo ſhall you at laſt have the Logarithm of the given Number.

You make the Logarithm of any Number whatſoever by this and the laſt Rule.

As for Example.

Let us pitch upon the Number 2, and make its Logarithm.

To do which, annex to the Number 2 a competent Number of Cyphers, *viz.* 28, and extract the Square Root thereof, which you will find to be 1.41421356237309 for the firſt continual Mean, to which ſaid Mean annex 14 Cyphers more, and extract the Square Root thereof, and ſo proceed, by annexing of Cyphers and extracting of Roots, till the neareſt Mean Proportional Number to 1, may have ſeven Cyphers placed before the ſignificant Figures of its Numerator, which *after 23 ſeveral* Extractions you will find *be 1.0000000862658.*

Then to find out the Logarithm of this continual Mean ſay by the Rule of Three Direct;

As the ſignificant Figures of the twenty fifth Mean Proportional in the foregoing Table, *viz.* 6862238,

Is to its reſpective Logarithm 29802322;

So are the ſignificant Figures of the laſt continual Mean found between 1 and 2, *viz.* 8262958,

To its reſpective Logarithm 35885571.

Now if you prefix before the Logarithm laſt found 8 Cyphers, it will be 000000003588557 1, which being doubled and redoubled 23 times, (becauſe there were 23 continual Means found between 1 and 2) there will at laſt be produced 0.301029987 97568, which is the Logarithm of the Number 2, which was required; but becauſe we intend the Table of Logarithms to conſiſt but of 7 Places, and becauſe two Nines follow the ſixth Place, therefore make the Figure 2 to be 3, and ſo ſhall the Logarithm of 2 be 0.301030, cancelling the following Figures as ſuperfluous.

The Logarithm of 2 being found, you may eaſily find the Logarithms of 4, 5, 8, 16, 20, 25, 32, 40, 50, 64, *&c.* by artificial Multiplication and Diviſion, which is by adding and ſubtracting of Logarithms; for if you take the Logarithm of 2 out of the Logarithm of 10, there will remain the Logarithm of 5, and the Logarithm of 2 doubled, gives you the Logarithm of 4; then add the Logarithm of 4 to the Logarithm of 2, and you have the Logarithm of 8, and to the Logarithm of 8 add the Logarithm of 2, and it gives you the Logarithm of 16, and the Logarithm of 5 added to the Logarithm of 4, gives *the* Logarithm of 20, and the Logarithm of 6 *doubled* gives the Logarithm of 25, *&c.*

In the next Place you are to get the Logarithms f 3, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 7, 53, 59, 61, 67, 71, 73, 79, 89, 97, *&c.* by elp of which all the reſt may be calculated.

IV. The firſt Figure of every Logarithm, which s ſeparated from the reſt by a Point or Prick, is ery properly called the Index or Characteriſtick f the Logarithm, which ſheweth the Nature of he Number by it ſignified, *viz.* whether it be oſitive, or negative; and if poſitive, of what Number of Places it doth conſiſt, and if negaive, what Place of the Decimal Fraction the firſt Figure of the Number by it ſignified ſhall poſſeſs, as in the following Table.

So the Logarithm of	will be
46768	4.670134
4676.8	3.670134
467.68	2.670134
46.768	1.670134
4.6768	0.670134
.46768	1.670134
.046768	2.670134
.0046768	3.670134
.00046768	4.670134

Whereby you may perceive that the Logarithms Of abſolute and defective Numbers are the ſame, Only the Characteriſtick of a defective Number is marked with the Note of Defection; for the Log. of the abſolute Number 46768 is 4.670134, the Characteriſtick 4, ſhewing the Number by it ſignified to conſiſt of 5 Places, as is already ſaid in he fourth Rule of the ſecond Chapter, and the Log. of the mixed Number 46.768 is 1.670134, which is the *ſame* with the former, only the *Characteriſtick is 1,* which ſheweth the Integral

Number by it ſignified to conſiſt of two Places, the reſt being a Decimal Fraction. Likewiſe the Logarithm of the Decimal 46768 is — 1.670134, which is ſtill the ſame with the former, only its Characteriſtick being marked with a Note of Defection ſheweth it to be the Logarithm of a Decimal Fraction; and becauſe the Characteriſtick is — 1, it ſheweth that the firſt Figure of the Number by it ſignified, doth poſſeſs the firſt Place of the Decimal, or Place of Primes: Again, the Logarithm 46703 is ſtill the ſame; and if you look for it in the Table of Logarithms, not regarding the Index, you will find it to be the Logarithm of 46768; but becauſe its Index is defective, I conclude it to be the Logarithm of a Decimal, and becauſe the Index, or Characteriſtick is 4, therefore I conclude that the firſt Figure of the Number ſignified by it, muſt poſſeſs the fourth Place of the Decimal; wherefore place 3 Cyphers before it, and you have .0006768 for the Decimal ſignified by the Logarithm 4.670134. This being well underſtood, the reſt will eaſily be attained by the following Directions.

* * *

CHAP. IV.

e Uſe of the *Table* of *Logarithms.*

HE Uſe of the Table of Logarithms is two-fold, *viz.* Firſt, To find therein the ithm of any given Number, or to find the er appropriated to any given Logarithm. ondly, To reſolve divers neceſſary Problems thmetick, Geometry, Trigonometry, Aſ- ny, *&c.*

ncerning the firſt of theſe I ſhall not med- ecauſe our Limits will not afford ſufficient to inſert a Table of Logarithms; and the s already publiſhed by others are ſufficiently ined in that Point, as Mr. *Briggs*, Mr. r, Dr. *Newton*, Mr. *Wingate*, Mr. *Norwood*, *hilips*, *&c.* every one ſhewing how by their s to find the Logarithm of any Number, or umber to any Logarithm; therefore I ſhall ed to ſhew their Uſe in Arithmetick, *viz.* to multiply, divide, and extract Roots, *&c.* by. And Firſt,

To multiply by the Logarithms.

Multiplication by the Logarithms there are ſes, *viz.* the Characteriſticks of the Loga- s of the Factors are either both Affirmative, th *Negative*, or elſe they are one Affirma- *and the other* Negative.

I. When they are both Affirmative.

When the Characteriſticks of the Logarithms of the Factors are both affirmative, then the Sum of thoſe Logarithms is the Logarithm of the Fact or Product.

Examples.

Multiply 34——————Log. 1.531479
by 26——————Log. 1.414973

Product 884——————Log. 2.946452

Multiply 28.86——————Log. 1.460296
by 8.9——————Log. 0.949390

Product 2568.54 ——Log. 2.409686

Note, That if you carry 10 to the Characteriſtick, it is Affirmative, as in the laſt Example.

II. When they are both Negative.

When the Logarithms of the Factors have their Characteriſticks both negative or defective, then the Sum of their Logarithms is the Logarithm of their Product, the Sum of their Characteriſticks being alſo negative, as in the following Examples.

Multiply .004——————Log. 3.602060
by .02——————Log. 2.301030

Product is .00008 ——— Log. 5.903090

Multiply .025——Log. 2.397940
by .42——Log. 1.623249

roduct is .01050——Log. 2.021189

ultiply .093——Log. 2.968483
by .058——Log. 2.763428

Product .005394——Log. 3.731911

And here note, that when you carry 10 to the haracteriſticks it is Affirmative, and muſt be a-ted out of their Sum, as in the two laſt Exam-es.

III. When they are Heterogeneal, *viz*. the one Affirmative, and the other Negative.

When the Characteriſticks of the Factors are e one Negative and the other Affirmative, then d the Logarithms together; and when you come the Characteriſticks, take their difference, and ce it for the Characteriſtick of the Product, king it either Affirmative or Negative, accord-g to the Affection of that wherein lay the Ex-s. And here note, that if you carry any thing the Characteriſticks, it is Affirmative, and muſt added to the Affirmative Characteriſtick: as the following Examples.

Multiply 348——Log. 2.541579
by .64——Log. 1.806180

Product 222.72——Log. 2.347759

Multiply 348——Log. 2.541579
by .0064——Log. 3.806180

Product *2.2272*——Log. 0.347759

Multiply 3.48———Log. 0.541579
by .0064———Log. 3.846180

Product 022273———Log. 2.347759

Multiply 3562———Log. 3.551693
by .008———Log. 3.903089

Product 28.496———Log. 1.454782

CHAP. V.

Diviſion by the Logarithms.

I. TO ſubtract the Logarithm of one N ber out of the Logarithm of anot is the ſame (and produceth the ſame Ef with Diviſion in natural Numbers, the L rithm, remaining being the Logarithm of Quotient.

II. In Diviſion, by the Logarithms, there three Caſes, *viz.* Firſt, when the Characteriſ of the Dividend and of the Diviſor are Affirmative. 2dly, When they are both N tive. And, 3dly, When they are Heteroge *viz.* the one Affirmative, and the other N *tive.* Of which in their order.

I. When they are both Affirmative.

III. When the Characteriſticks of the Dividend, and of the Diviſor, are both Affirmative, then if you ſubtract the Logarithm of the Diviſor out of the Logarithm of the Dividend, the Remainder will be the Logarithm of the Quotient. And if you borrow 10 from the Characteriſticks, it is Affirmative.

Examples.

Divide 468 ———Log. 2.670246
by 12———Log. 1,079181

Quotient 39———Log. 1.591065

Divide 144———Log. 2.158362
by 16———Log. 1.204120

Quotient 9 —— ——Log. 0.954242

Theſe Examples are ſo plain that they need no Explanation.

II. When they are both Negative.

IV. When the Characteriſticks of the Dividend and of the Diviſor are both Negative, ſubtract the Logarithm of the Diviſor from the Logarithm of the Dividend, and the Remainder is the Logarithm of the Quotient; and if you borrow 10, it muſt be paid to the Index of the Diviſor, *Affirmatively.*

Examples.

Divide .48———Log. 1.681241
(1) by .12———Log. 1.079181

Quotient .4———Log. 0.602060

Divide .036———Log. 2.556303
(2) by .18———Log. 1.255272

Quotient .2———Log. 1.301031

Divide .156———Log. 1.193125
(3) by .39———Log. 1.591064

Quotient. .4———Log. 1.602061

Divide .0171———Log. 2.232996
(4) by .9———Log. 1.954242

Quotient. .019———Log. 2.278754

The firſt and ſecond of the foregoing Exa
ples are eaſily underſtood; and as for the th
and fourth, all the difficulty therein is cauſed
borrowing 10 at the next Figure to the Char
teriſticks, as in the third Example, in ſubtraćti
5 out of 1. Now to make good the 10 borrow
I pay 1 to the Characteriſtick of the Diviſ
and becauſe the ſaid 1 is Affirmative, and the ſ
Characteriſtick Negative, therefore ſubtract
from the Characteriſtick of the Diviſor, and th
remains nothing; wherefore I take (0) out of
Characteriſtick of the Dividend, and there
mains 1 for the Characteriſtick of the Quoti

'he ſame is to be underſtood in the fourth Ex-mple and in all others of the ſame nature.

A general Obſervation drawn from the third and fourth Rules foregoing

V. If when the Characteriſticks of the Divi-:nd and Diviſor be Homogeneal, (that is, both ffirmative, or both Negative) the Characteriſ-ck of the Diviſor is greater than the Characte-ſtick, of the Dividend; then in this caſe ſub-act the Characteriſtick of the Dividend out of ıat of the Diviſor, placing the Remainder for ıe Characteriſtick of the Quotient, changing its gn, *viz.* if it be Affirmative make it Negative, ıd if it be Negative make it Affirmative; re-embring the Directions under the laſt Rule, 'hen you borrow from the Characteriſticks.

Obſerve the following Examples.

Divide 6.4——Log. 0.806180
(1) by 80——Log. 1.903090

Quotient .08——Log. 2.903090

Divide 6.4——Log. 0.806180
(2) by 800——Log. 2.903090

Quotient 008——Log. 3.903090

Divide 6.3——Log. 0.799340
(3) by 78.75——Log. 1.896251

Quotient .08——Log. 2.903089

Divide 75 ——--Log. 1.875061
(4) by .0015 ——--Log. 3.176091

Quotient 500 ——--Log. 2.698970

Divide .64 ——--Log 1.806180
(5) by 008 ——--Log. 3.903090

Quotient 80 ——--Log. 1.903090

Divide 16.56 ——--Log. 1.219060
(6) by 460 ——--Log. 2.662758

Quotient .0036 ——--Log. 2.556302

III. When they are Heterogeneal, *viz.* the o Negative, the other Affirmative.

VI. When the Characteriſticks of the Divide and the Diviſor are Heterogeneal, proceed as the two firſt Caſes till you come to the Charact riſticks, and then inſtead of ſubtracting the o Characteriſtick from the other, add them tog ther, ſo ſhall their Sum be the Characteriſtick the Quotient, and it is of the ſame kind with t Characteriſtick of the Dividend.

But here note, that when you borrow 10 at t next Figure to the Characteriſtick, it muſt paid to the Characteriſtick of the Diviſ Affirmatively, *viz.* if the Characteriſtick of t Diviſor be Affirmative, then add 1 to it for th you borrowed; and if it be Negative, ſubtract from it; as in the following Examples.

Divide 144 —— Log. 1.158362
(1) by 12 —— Log. 1.079181

Quote .012 —— Log. 2.069181

J

Divide 64———--Log. 1.806180
(2) by 08———--Log. 2.903090

Quote 800———Log. 2.903090

Divide 64———--Log. 1.806180
(3) by 800———--Log. 2.903090

Quote 0008———Log. 4.903090

In the ſecond of the foregoing Examples, I borow 1, (in the place next the Characteriſtick) by ubtracting 9 out of 8; wherefore to make it good I ſubtract 1 from —2, (the Characteriſtick of the Diviſor) becauſe it is Negative, and the Remainder, which is 1—I add to 1 (the Characteriſtick of the Dividend) and their Sum is 2 for the Characteriſtick of the Quotient, which is Affirmative, becauſe the Characteriſtick of the Dividend is Affirmative.

And in the laſt Example I likewiſe borrow 1 from the Characteriſtick; wherefore to make it good, I add 1 to the Characteriſtick of the Diviſor, (becauſe it is Affirmative) and that makes it , which added to—1 (the Characteriſtick of the Dividend) makes—4 for the Characteriſtick of he Quotient, which here is Negative, becauſe he Characteriſtick of the Dividend is Negative.

Other Examples for Exerciſe may be ſuch as follow.

Divide 648 ——--Log. 2.11575
by 36 ——--Log. 1.556303

Quote 1800 —— --Log. 3.255272

Divide

Divide	64	Log.	0.806180
by	.08	Log.	2.903090
Quote	80	Log.	1.903090
Divide	68	Log.	1.832509
by	.08	Log.	2.903090
Quote	850	Log.	2.929419

CHAP. VI.

To raiſe the Powers of Numbers, viz. To find the Square, Cube, Biquadrate or ſquared Square, &c. of any Number. Alſo to extract the Square, Cube, Biquadrate, &c. Roots of any Number by the Logarithms.

I. BY the third Section of the Second Chapter of this Book, it is evident, that if you add the Logarithms of two Numbers together, the Sum will be the Logarithm of their Product; and by the firſt and fourth Sections of the 9th Chapter of my Decimal Arithmetick, it appeareth, that any Number multiplied by it ſelf, produceth its Square; wherefore if you double (or multiply by 2) the Logarithm of any Number, it will produce the Logarithm of its Square, which if duly conſidered, you will find that to ſquare, cube, *&c.* any Number is nothing elſe but to multiply the Logarithm of the given Number by the Index of the Power you would raiſe it to, *viz.* If you would find the Square of any Number, multiply the Logarithm thereof by 2, ſo ſhall the Product thereof be the Logarithm of its Square; and if you would find the Cube of any Number, multiply its Logarithm by 3, and the Product thereof will be the Logarithm of its Cube; and if you would find the Biquadrate of any Number, multiply its Logarithm by 4, and it will produce the Logarithm of its Biquadrate, *&c.* As in the following Example.

Let it be required to find the Square of 12.

The Logarithm of 12 is 1.079181
2
2.158362

Which being multiplied by 2, produceth 2.158362, which is the Logarithm of 144, *viz.* the Square of 12.

Again, let it be required to find the Square of 94.

The Logarithm of 94 is——1.9731278
2
———
3.9462556

Which being multiplied by 2, produceth 3.9462556, which is the Logarithm of 8836, which is the Square of 94.

II. But if the Characteriſtick of the Logarithm be negative, that is, if the given Number, whoſe Square, Cube, Biquadrate, *&c.* you would find, be a Decimal Fraction, obſerve, that in multiplying the next Figure to the Characteriſtick, the Ten, or Tens to be born in mind are Affirmative, and are to be deducted out of the Product of the negative Characteriſticks.

Obſerve the ſeveral Examples following:

What is the Square of .7? log.—1.845098
.7 2
——— ———
Facit .49 log.—1.690196

What is the Square of .09? log.—2.954242
.09 2
——— ———
Facit .0081 log.—3.908484

What is the Cube of .12? log.—1.079181
3

Fa. .1728, for .12×12×12=.1728 log.—3.237543

What is the Cube of .05? log.—2.698970
3

Fa. .000125, for .05×05×05=00125 log. 4.096910

What is the Biquadrate of .9? log.—0.954242
4

Fac. .6561, for .9×9×9×9=.6561 log.—3.816968

What is the Biquadrate of .08? log.—2.903090
4

Fac. .00004096, for .08×08×08×08 =00004096, whose log. is —5.612360

What is the fifth Power of .6? log.—0.778151
5

Facit .7776=.6×6×6×6×6 log.—3.890755

The like is to be obſerved of all others.

To extract the Square, Cube, Biquadrate, &c. Roots of any given Numbers by the Logarithms.

III. From a due Conſideration of the firſt Section of this Chapter, it may eaſily be perceived, that

To extract the Cube Root of any Number is to *bipart (or divide by 2)* its Logarithm, ſo ſhall that biparted Logarithm be the Logarithm of the Square Root deſired.

Example.

Let it be requir'd to extract the Square Root of 75832.

q.=75832——log. 4.879852
2)
√=275.37——log. biparted 2.439926

Let it be requir'd to find the Square Root of 4489.

q.=4489——log. 3.652149
2)
√=67——log. biparted 1.826074

In the firſt of theſe Examples the Logarithm of 75832 is 4.879852, which being biparted (or divided by 2) gives 2.439926 for the Logarithm of 265.37, the Root required.

And in the ſecond Example 3.652149 (the Logarithm of 4489 being biparted) gives 1.826074 for the Logarithm of 67, its Square Root.

So will the Square Root of 36783 be found to be 191.789 *fere*, and the Square Root of 3866 will be 62.17717 *fere*, and the Square Root of 95 will be found to be 9.7468, *&c.*

To extract the Cube Root of any Number is to tripart (or divide by 3) its Logarithm, ſo ſhall this triparted Logarithm be the Logarithm of the Cube Root required.

Examples.

Let it be required to extract the Cube Root of 157464.

The Cube 157464 ——————— log. 5.197181
3) ———
√c. 54 ——————— log. triparted 1.732393

Let it be required to find the Cube Root of 187237601580329.

The Cube 187237601580329 log. 14.27239308
3) ———
√c. 47209 —— its triparted log. 4.75746436

In the firſt of theſe Examples, where it is required to extract the Cube Root of 157464, its ogarithm is 5.197181, which being divided by hath for its third part 1.732393, which is the ogarithm of 54, the Cube Root of 157464, hich was required. And the ſame is to be obrved in finding the Cube Root of 1872376015 329 by the Logarithms, or of any other poive Number whatſoever; as you may ſee by e following Examples.

he Cube=8 ——————— its log. 0.903090
3) ———
c.=2 ——————— its log. 0.301030

he Cube 125 ——————— its log. 2.096910
3) ———
c.=5 ——————— its log. 0.698970

The Cube 15.625 ———— its log.—1.193820
3)————
√*c.*=2.5 ———— its log.—0.397940

To extract the Biquadrate Root of any Number.

To extract the biquadrate Root of any given Number, do thus, *viz.* Take its Logarithm, and divide it by 4, ſo ſhall the fourth part thereof be the Logarithm of the biquad. Root required: as in the following Example:

Let it be required to extract the Biquadrate Root of 256.

Biquadrate Number given 256—its log. 2.408239
4)————
Its √ biquad.=4———— its log. 0.602059

Here the Logarithm of the given biquadrate Number, *viz.* 256, is 2.408239, which being divided by 4, giveth 0.602059 for the Logarithm of (4) the biquadrate Root required.

In like manner, if you would extract the Root of the fifth Power of any Number given, divide its Logarithm by 5, ſo ſhall the Quotient be the Logarithm of its Root. And if you would find the Root from the ſixth Power of any Number, divide its Logarithm by 6, and the Quote is the Logarithm of the Root deſired, *&c.*

To extract the Square, Cube, &c. Roots of negative Numbers by the Logarithms.

IV. But here you are to obſerve, in extracting the Square, Cube, Biquadrate, or any other Root of a negative Number or decimal by the Logarithms, that if you cannot evenly divide the Index or Characteriſtick of the Logarithm without the Remainder; then add to the ſaid Characteriſtick, ſo many Units, till it may be divided without any Remainder, and place the Quotient for a new Characteriſtick (belonging to the Root;) then look how many Units you lent to the Characteriſtick and eſteem them ſo many Tens to be prefixed to the Logarithmetical Figure immediately following the Characteriſtick; then proceed to finiſh the Work, ſo ſhall this new Logarithm be the Logarithm of the Root required, which will alſo be negative.

Examples follow.

What is the Square Root of .144?

Square given =.144 its log.—1.158362
2) ————
Its $\sqrt{q}$.=.37947 its log. biparted—1.579181

What is the Square Root of .00324?

Square given=.00324 log.—3.510545
2) ————
Its $\sqrt{q}$.=.05621 log. biparted.—2.755272

What is the Cube Root of .000512?

Cube given=.000512 its log. —4.709269
3) ————
Its $\sqrt{c}$.=.05 its log. triparted—2.903089

In the firſt of theſe Examples, where it is required to extract the Square Root of .144, its *Log. is—1.158362,* which (according to the

racteriſtick, therefore I increaſe the next
by adding 10 to it (or prefixing 1 befor
then it is 11, *&c.* So I find the Loga
the Root to be—1,579181, *viz.* .37947.

And in the third Example, where it is
to find the Cube Root of .000512, the
its Logarithm is—4, which cannot be ev
vided by 3, therefore I add 2 to it to m
and the Quotient is—2 for the Index of
garithm of the Root; then becauſe I ad
the Index—4, therefore I increaſe the
gure to it with 2 tens, making it 27, *&c*
the Cube Root required found to be .05.

the like in extracting of any other Roots: *ıerwiſe you may uſe the following Table.*

						A	B
2					1	2	1
					3	4	2
					5	6	3
					7	8	4
					9	10	5
3				1	2	3	1
				4	5	6	2
				7	8	9	3
4			1	2	3	4	1
			5	6	7	8	2
5		1	2	3	4	5	1
		6	7	8	9	10	2
6	1	2	3	4	5	6	1
	7	8	9	10	11	12	2
	50.	40.	30.	20.	10.	0	

The Uſe of the foregoing Table.

the foregoing Table, the Figures 2, 3, 4, 5, 6, l on the left hand, are the Indices of Pow- ·hoſe Roots are required to be extracted, ;y are Diviſors by which they divide the Lo- ım of any given Power, in order to find :s Root; as the Number 2 (which is the ·moſt) is the Diviſor for finding the Loga- ı of the Square Root of any Number; and 3)iviſor *for the* Cube; and 4 for the Biqua- *&c. The* Figures placed between the per- pendicul pendicular Line, and the ſeveral Lines of Connection, and under A are the Characteriſticks of the Logarithms of Negative or Decimal Numbers, whoſe Roots are required to be extracted; and the Figures placed on the right hand of the perpendicular Line under B, are the Characteriſticks of the Logarithms of the ſeveral Roots; and the Numbers at the bottom of the Table, *viz.* 50, 40, 30, *&c.* are the Numbers to be added, or rather prefixed to the firſt Figure of the Logarithm next the Characteriſtick, whoſe Negative Index is found in the ſame Series or Column even with the Diviſor, *&c.*

Example.

Let it be required to extract the Cube Root of .405224.

The Logarithm of the given Number is —1.6076951.

And the Diviſor whereby to extract the Cube Root is 3, which I find in the foregoing Table on the left hand; then on the right hand of its Line of Connection, I find the Characteriſtick of the Logarithm—1.6076951, which is—1; and juſt againſt it on the right hand under B, I find—1 for the Characteriſtick of the Logarithm of the Root; and in the bottom Line under the ſaid—1, in the ſame Series, I find 20, which is to be prefixed to the Figure in the Logarithm next the Characteriſtick, *&c.* and having finiſhed Diviſion, I find the Logarithm of its Root to be—1.8692317, which is .74.

So if the Logarithm—6.418426 were to be *divided by* 4; Firſt, look for the Diviſor 4 on the left hand of the Table, and find the Characteriſtick —6, behind its Line of Connection, juſt againſt which, on the right hand of the perpendicular Line, you have—2 for the Index or Characteriſtick of the Quotient; and at the bottom, juſt underneath—*6* you have 20, which being added to (4) the firſt Figure of the Dividend next the Characteriſtick, makes it 24, in which the Diviſor 4 is contained *6* times, *&c.* See the Work.

Biquad. propoſed .0000026207 *log.*—6.418426
4)————
The √ (4)=.040235 *log.*—2.604606

CHAP. VII.

Of the Uſe of Logarithms in Comparative Arithmetick.

PROP. I.

Having three Numbers given, to find a fourth Proportional.

THIS is nothing elſe but the Work of the Rule of Three; and it may be thus performed, *viz.*

Add the Logarithm of the ſecond and third Numbers together, and from their Sum ſubſtract the Logarithm of the firſt, ſo ſhall the Remainder be the Logarithm of the fourth: as in the following Example.

The 3 given Numbers are 3, 24, and 108, unto which it is required to find a fourth Proportional.

3 : 24 :: 108 : 864

The Operation by the Logarithms.

As 3—its *log.* ———————— 0.477121

Is to 24 its *log.* ———————— 1.380211 } add
So 108 its *log.* ———————— 2.033424 }

The Sum of the 2 laſt *log.* is 3.413635

From which if you ſubſtract }
the Log. of the firſt, the Re- } 2.936514
mainder is ———————— }
which is the Logarithm of 864, the fourth Proportional required.

The former Work may be ſomewhat ſhortned, if inſtead of the Logarithm of the firſt you take its Complement Arithmetical (which is nothing elſe but to ſet down what every Figure wants of 9, till you come to that next the right hand, and then ſet down what it wants of 10) and then add them all 3 together, and cancel the firſt Figure of the Sum on the left hand; for then will the Sum be the Logarithm of the Anſwer, as will appear by working the foregoing Examples.

As 3 Compl. Arith. of its *log.* — 9.522879 }
Is to 24 — its *log.* — 1.380211 } add
So is 108 — its *log.* — 2.033424 }

To 864 — its *log.* — 12.936514

PROP. II.

Between two Numbers given to find a Mean Proportional.

WHen the Logarithms of the Numbers propounded are Homogeneal, *viz.* both Affirmative, or both Negative, add them together, then bipart that Logarithmetical Sum, ſo have you the Logarithm of the Mean Proportional required; which Logarithm ſo found, is of the ſame kind with the Logarithms of the Number given.

Let it be required to find a Mean Proportional between 18 and 6.

18 — *log.* 1.25527
6 — *log.* 0.77815

2)2.03342

10.392 *log.* 1.01671

In this Example the Logarithm of 18, and 6 being added together, make 2.02340, which is the Logarithm of 108; and that Logarithm being *divided by* 2 (which is the ſame with extracting *the Square* Root of its ſignificant Number, as in the third Rule of the ſixth Chapter) the Quotient is 1.01671, the Logarithm of 10.392, which is the Mean Proportional required.

Example.

Let it be required to find a Mean Proportional between .018 and .006.

.018 its *log.*——————2.25527
.006 its *log.*——————3.77815

2)—4.03342
.010392 *log.*——————2.01671

II. But when the Characteriſticks of the Logarithms of the given Numbers be Heterogeneal, *viz.* the one Affirmative, and the other Negative; add the Logarithms together as before, till you come to the Characteriſtick, then ſubſtract the leſſer Characteriſtick out of the greater, (according to the third Rule of the fourth Chapter) which being done, bipart the Logarithmetical Production, ſo ſhall the Quote be the Logarithm of the Mean Proportional required, which will always be of the ſame kind with that Logarithm of the given Numbers whoſe Index is greateſt: as in the following Examples.

Example

Example 1.

What is the mean Proportional between 36 l .5 ? Facit 4.2427.

Here the *Log.* of $\left\{\begin{matrix}36\\ .5\end{matrix}\right\}$ is $\left\{\begin{matrix}1.556302\\ -1.698970\end{matrix}\right.$

Their Sum is ——————— 2) 1.255272

hich being divided by 2, gives
e *Log.* of the mean Prop. which } 1.627636
the *Log.* of 4.2427.

Example 2.

What is the mean Proportional between 12 d .75 ? Facit 3.

The *Log.* of $\left\{\begin{matrix}12\\ .75\end{matrix}\right\}$ is $\left\{\begin{matrix}1.079181\\ -1.875061\end{matrix}\right.$

Their Sum is ——— 2) 0.954242

hich being divided by 2,
ives the *Log.* of 3, the mean } 0.477121
roportional required.

for .75 : 3 :: 3 12 : 3

PROP.

PROP. III.

Between two Numbers given to fin *Mean Proportionals.*

WHether the Numbers given be Hon neal, or Heterogeneal, ſubſtract the garithm of the leſſer Extreme from the I rithm of the greater Extreme, then take ⅓ (difference of the ſaid Logarithms, and add it t Logarithm of the leſſer Extreme, ſo will the be the Logarithm of the leſſer Mean: then the ſame difference to the ſaid Logarithm o leſſer Mean, and the Sum will be the Logai of the greater Mean; ſtill obſerving the l deliver'd in the fourth and fifth Chapters o Book, in adding and ſubſtracting of Logarith

Examples.

Example 1. Let it be required to find Mean Proportionals between 144 and 12.

The Log. of	144 is	2.158362
	12	1.079181
The Difference ——	3)	1.079181
⅓ of the Differ. ——		0.359727
Leſſer Mean	27.473 *log.*	1.438908
Greater Mean	61.899 *log.*	1.798635

E

Example 2.

Let it be required to find two mean Propor-
nals between .75 and .05.

The Log. of {.75, .05} is {—2.875061, —2.698970}

The Difference 3) 1.176091

⅓ part of the Diff. is 0.392030

Leſſer Mean is .12331 Log. 1.091000

Greater Mean is .30411. Log. 1.483030

Example 3.

.et it be required to find two mean Propor-
ials between .125 and .05.

The Log. of {.125, .05} is {—2.096910, —2.698970}

Their Difference. 3) 3.397940

⅓ of their Difference. 1.132646

leſſer Mean is .67860 Log. 1.831616

greater Mean is 9.2101 Log. 1.964262

P R O P. IV.

ee Numbers given to find a Fourth in a Duplicate Proportion.

ake the Logarithms of the two Numbers
have one and the ſame Denomination, and

ſubſtract the leſſer Logarithm from the greater, and double the Remainder, (that is, multiply it by 2;) then if the firſt Number be leſs than the ſecond, add the ſaid double Difference to the Logarithm of the other Number; ſo will the Sum be the Logarithm of the fourth Number, or Number required; as for Example.

The ſuperficial Content of a Circle, whoſe Diameter is 14 Inches, is 154 ſquare Inches, I demand the Content of another Circle, whoſe Diameter is 25 Inches. *Facit* 491.07 ſquare Inches. See the Operation.

Diameter 14 Inches.	Log.	1.146128
Diameter 25 Inches.	Log.	1.397940
The Difference of the Log.		0.251812
		2
Their Difference doubled.		0.503624
The given Content.	Log.	2.187521
The Content required 491.07.	Log.	2.691143

But if the firſt Number be greater than the ſecond; then inſtead of adding the double Difference to the other Number, ſubſtract it therefrom, ſo ſhall the Remainder be the Logarithm of the Number required; as in the following Example.

There is a Circle whoſe Diameter is 28 Inches, and its ſuperficial Content is 616 ſquare Inches; I demand what is the ſuperficial Content of another Circle, whoſe Diameter is 25 Inches. *Facit* 491.07 ſquare Inches, as in the former Example.

Diameter 28 Inches.	Log.	1.447158
Diameter 25 Inches.	Log.	1.397940
The Difference of the Log.		0.049218
		2
The Difference doubled.		0.098436
The given Content 616.	Log.	2.789580
Content required 491.07.	Log.	2.691143

PROP. V.

aving Three Numbers given, to find a fourth in a Triplicate Proportion.

TRiple the Difference of the Logarithms of two given Terms, which have the ſame enomination; then if the firſt Term be leſs an the ſecond, add the ſaid triple Difference the Logarithm of the other Term, ſo ſhall the m be the Logarithm of the fourth Term re-ired; as in the following Example.

There is a Bullet whoſe Diameter is 4 Inches, d its Weight is 9 *l.* I demand what Weight a llet of the ſame Metal will be, whoſe Diame-r is 8 Inches. *Facit* 72 *l.* View the following ork.

Diameter 4 Inches. Log.—0.60
Diameter 8 Inches. Log.—0.90

The Difference of the Logarith. 0.30

The Difference tripled is, 0.90
The given Weight 9 *l.* Log.—0.95

The Weight requir'd is 72 *l.* Log. 1.85

But if the firſt Term be greater than th
cond, then ſubſtract the ſaid tripled Differ
from the Logarithm of the other Term, ſo
the Remainder be the Logarithm of the fo
Number required: as for Example.

There is a Bullet whoſe Diameter is 8 In
and its Weight is 72 Pounds, I demand the We
of another Bullet of the ſame Metal, whoſe
ameter is 4 Inches. *Facit* 9 *l.* See the O
tion, it being the Converſe of the former.

Diameter 8 Inches. Log. 0.90
Diameter 4 Inches. Log. 0.60

The Difference of the Log. is, 0.30

The Difference tripled is, 0.90
The given Weight 72. Log. 1.85

The Weight required 9 *l.* Log. 0.95

* * *

CHAP. VIII.

f *Anatociſm, or Compound Intereſt; wherein is ſhewed how by the Logarithms to anſwer all Queſtions concerning the Increaſe or preſent Worth of any Sum of Money or Annuity, for any Term of Years, or at any Rate of Intereſt: According to the ſix fundamental Theorems invented and laid down by Mr.* Oughtred, *in his Treatiſe,* De Anatociſmo ſive Uſura Compoſita, *annexed to his* Clavis Mathematica.

WHEN any Queſtion in Compound Intereſt is propoſed, it will fall under one f the ſix Caſes following, *viz.*

1. To find the Increaſe or Amount of any Sum f Money, put out at Compound Intereſt, for ay number of Years, and at any Rate of Intereſt ropounded.

2. To find the preſent Worth of any Sum of Money due at the end of any Number of Years o come, Rebate being allowed at any Rate of ompound Intereſt.

3. To *find the Increaſe* or Amount of an An*uity, being forborn* for any number of Years, *any Rate of* Compound Intereſt.

4. To find what Annuity any Sum o
due at the end of any Term of Years
will purchaſe at any rate *per Cent.*

5. To find the preſent Worth of any
to continue any number of Years, allov
bate at any rate *per Cent.*

6. To find what Annuity any Sum o
will purchaſe for any number of Year
any rate of Intereſt propoſed.

II. When any Queſtion in Compound
is propounded, find out the Intereſt o
let 1 *l.* with its Intereſt, be the rate o
implied in the Queſtion: As if any Queſ
propoſed at 8 *per Cent.* the Intereſt of
Year is .08, and the Rate of Intereſt is
at 6 *per Cent.* the Rate is 1.06, *&c.* of w
out the Logarithm.

III. When in any Annuity or Debt, the
be half-yearly, quarterly, or monthly,
are to divide the Logarithm of the ſaid R
4, or 12, *&c.* ſo ſhall that Quotient b
garithm of the Rate; as ſuppoſe any
were propounded at 8 *per Cent.* the Ra
tereſt here implied is 1.08, for

100 : 108 : : 1 : 1.08

Which ſaid Rate is for yearly Paym
Logarithm whereof is 0.0334237; but i
ments are to be half-yearly, then if yo
the ſaid Log. by 2, it will give you o
for the Log. of the Rate; and if the
be quarterly, divide the ſaid Logarith
by 4, and it will give you 0.00835

Logarithm of the Rate; and if the Payments be monthly, then if you divide the ſaid Logarithm of 1.08 by 12, it will give you 0.0027853 for the Logarithm of the Rate, &c. and this is generally the firſt thing to be obſerved in every Queſtion, as you will find by the following Examples.

CASE I.

To find the Increaſe or Amount of any Sum of Money put out at Compound Intereſt for any Term of Years, and at any Rate of Intereſt propounded.

Queſt. 1. If 50 *l.* 16 *s.* be put out at 8 *per Cent.* Compound Intereſt, for 7 Years, I demand how much will then be due to the Creditor. Facit 87 *l.* 1 *s.* 2 ¾ *d.*

IV. Multiply the Logarithm of the Rate by the Number of Years, and the Product will give you the Logarithm of the Amount of 1 *l.* for the propoſed Time; to which if you add the Logarithm of the Sum propounded, the Sum will be the Logarithm of the Anſwer.

The Operation by the Logarithms.

The Log. of 1.08 the Rate prop.	0.0334237	
The Numb. of Years propounded	7	
The Log. of the Increaſe of 1 *l.* for 7 Years	0.2339659	add
The Log. of (50.8 *l.*) the Sum propoſed	1.7058637	
The Log. of (87.061) the Anſwer	1.9398296	

Which is 87 l. 1 s. 2 ¾ d. fere.

Queſt. 2. What is the Amount of 76 *l.* 4 *s.* for 3 Years at 8 *per Cent.* Facit 97 *l.* 17 *s.* 1 *d.*

The Operation by the Logarithms.

The Log. of (1.08) the given Rate *per Annum.*	0.0334237
	4)
Which divided by 4, gives the Log. of the quarterly Rate.	0.0083559
	13
	0.0250677
	083559
Which multiplied by 13, the Quarters in 3 Years, gives the Log. of the Increaſe of 1 *l.*	1.086267
The Log. of (76.2) the given Sum.	1.8819547
The Log. of (97.854) the Anſwer.	1.9905814

Queſt. 3. If 50 *l.* be put forth at Intereſt for 20 Years at 6 ¼ *per Cent.* I demand how much it will be increaſed to at the end of the ſaid Time? Facit 168 *l.* 1 *s.* 10 *d.*

The Rate of Intereſt here propoſed is 6¼=6.25 *per Cent.* therefore to find out the Rate of 1 *l.* for a Year, ſay, by the Rule of Proportion,

l.		*l.*		*l.*		*l.*
100	:	106.25	::	1	:	1.0625

So that the Rate of Intereſt implied in the Queſtion fit for Calculation by the Log. is 1.0625, according to the ſecond Rule of this Chapter.

Behold

The

The Operation.

he Log. of (1.0625) the given Rate ———	0.0263289	
he number of Years propound.	20	
he Log. of the Amount 1 *l.* in 20 Years ———	0.5265780	add
he Log. of (50) the Sum propoſed ———	1.6989700	
he Log. of 168.090 ——	2.2255480	

Vhich is 168 *l.* 1 *s.* 10 *d.* very near; and ſo nuch will 50 *l.* be increaſed to in 20 Years, at 5 *l.* 5 *s.* *per Cent.*

CASE II.

o find the preſent Worth of any Sum of Money due at the end of any number of Years to come, Rebate being allowed at any Rate of Compound Intereſt.

V. When it is required to find the preſent vorth of any Sum of Money, firſt find the Anount of 1 *l.* for the propoſed Time, and at the ate of Intereſt propounded; then find the Loarithm of the Sum propoſed to be rebated, and rom it ſubſtract the Logarithm of the Amount of *l.* (found as before) and the Remainder will be e Logarithm of the preſent Worth of the Sum ropoſed: as in the following Example.

Queſt. 4. What is 30 *l.* that is due 7 Years ence, worth, to be paid preſently, allowing Reate at 8 *per Cent.* Facit 17 *l.* 10 s. 1 ½ d. as you ay *perceive by*

The

The Operation by the Logarithms.

The Log. of (1.08) the propoſed Rate——0.033424
The Time propoſed —— —— 7

The Log. of the amount of 1 *l.* for 7 years 0.233968
The Log. of 30 —— —— —— 1.477121

The Log. of (17.506) the Anſwer —— 1.243153
which is 17 *l.* 10 *s.* 1 ½ *d.* and ſo much is the preſent Worth of 30 *l.* due 7 Years hence.

Queſt. 5. What is the preſent Worth of 120 *l.* due 2 Years hence, allowing Rebate at *6 per Cent.* Facit 106 *l.* 15 *s.* 11 *d.*

The Operation of the Logarithms.

The Log. of (1.06) the propoſed Rate 0.025306
The Time propoſed —— —— —— 2

The Log of the amount of 1 *l.* for 2 years 0.050612
The Log. of 120 —— —— —— —— 2.079181

The Log. of (106.79) the Anſwer —— 2.028569
which is 106 *l.* 15 *s.* 11 *d.* and ſo much is the preſent Worth of 120 *l.* due 2 Years hence.

CASE III.

To find the Increaſe or Amount of any Annuity, being forborn any number of Years, at any Rate of Compound Intereſt.

VI. For reſolving Queſtions concerning the forbearance of Annuities, you are (by the fourth

Rule) firſt, to find out the Amount of 1 *l.* for the Time, and at the Rate of Intereſt propounded.

Secondly, find out the Logarithm of the ſaid Amount made leſs by 1, and alſo the Log. of the Rate made leſs by 1, and ſubſtract the latter from the former; ſo ſhall the Remainder be the Log. of the Amount of 1 *l.* Annuity for the Term of Years propounded, to which if you add the Logarithm of the propoſed Annuity, the Sum will be the Logarithm of the Amount or Increaſe of the ſaid Annuity: as in the following Example.

Queſt. 6. What will be the Amount or Increaſe of 48 *l.* 16 *s.* *per Annum*, for 7 Years, Compound Intereſt being computed at 8 *per Cent.* Facit 435 *l.* 8 *s.* 5 ½ *d.* *fere.* See

The Operation by the Logarithms.

The log. of (1.08) the given rate is	0033424	mult.
The Time propounded ——	7	
The Log of (1.7138) the Amount of 1 *l.* for 7 Years——	0.233968	
1.7138—1=.7138 its Log.	—1.853577	ſubſt.
1.08—1=.08 its Log.	—2.903090	
The difference of the Log. which is the Increaſe of 1 *l.* Annuity is —— —— ——	0.950487	add
The Log. of (48.8) the Annuity propoſed —— ——	1.688419	
The Log. of (435.422) the Amount of the propoſed Annu.	2.638906	

which is 435 *l.* 8 *s.* 5 ½ *d.* very near; and ſo *much will be* the Increaſe of an Annuity of 48 *l.* 16 *s.* in 7 Years, at 8 *per Cent.* Compound Intereſ

Qu

Queſt. 7. There is an Annuity of 50 *l.* forborn to the end of 10 Years, I demand how much is then due, Compound Intereſt being computed at *6 ¼ per Cent.* Facit *666 l. 16 s.* as you will find by

The Operation by the Logarithms.

The Log. of (1.0625) the Rate	0.0263289	mult-
The Term of Years ———	10	
The Log. of (1.8335) the Amount of 1 *l.* for 10 Years.	0.2632890	
1.8335—1=.8335 its Log.	—1.9209056	ſubt —
1.0625—1=.0625 its Log.	—2.7958800	
The Log. of the Amount of 1 *l.* Annuity for 10 Years ———	1.1250256	add
The Log. of (50) the Annuity propoſed ———	1.6989700	
The Log. of (*666.80*) the Amount of the Annuity propoſed ———	2.8239956	

which is *666 l.* 16 *s.* and ſo much will be due at the end of the ſaid Time.

CASE IV.

To find what Annuity any Sum due at any time to come, will purchaſe, to continue for any time, and at any Rate of Intereſt propoſed.

VII. The Operation in this Caſe is the ſame in every reſpect with that in the former Caſe, only whereas in the laſt Caſe you ſubſtracted the Log. of the Rate leſs 1, from the Log. of the Increaſe *of 1 l. leſs* 1, ſo in this you muſt ſubſtract the Log. *of the* Increaſe of 1 *l.* leſs 1 from the Log. of the *leſs 1*, as in the following Example.

Queſt. 8. There is 705 *l.* due at the end of 7 ears to come, I demand what Annuity to coninue 7 Years, the ſame will purchaſe, Compound ntereſt being allowed at 8 *per Cent.* Facit 9.015 *l.*=79 *l.* 0 *s.* 4 *d.* *fere*, as you may find if ou obſerve

The Operation by the Logarithms.

he Log. of (1.08) the propoſed Rate —— —— ——	0.033424	mult.
he propoſed Time————	7	
he Log. of the Increaſe of 1 *l.* for 7 Years 1.738——— ——	0.233968	
.08—1=.08 its Log.	—2.903090	ſubt.
.7138—1=.7138 its Log.	—1.853577	
he Log. of the Value of 1 *l.*	—1.050513	add
he Log. of (705) the Purchaſe-Money. —— —— ——	2.848189	
he Log. of the Purch. (79.015)	1.898702	

hich is 79 *l.* 0 *s.* 4 *d.* *fere.*

Queſtions of this Nature may be ſolved at two perations, by the ſecond and ſixth Caſes: Firſt, y the Rule in the ſecond Caſe, find the preſent orth of the Sum propounded; then by the ſixth, nd what Annuity ſuch a Sum will purchaſe.

CASE V.

o find the preſent Worth of an Annuity, to continue any Term of Years, howſoever payable, viz. *either Yearly, Half-yearly, or Quarterly, Rebate being allowed at any Rate* per Cent.

VIII. *Find out the* Logarithm of the Rate, *nd multiply it by* the number of Years or Quar-

ter

ters, according as the Annuity is payable, and that will produce the Logarithm of the Increaſe of 1 *l.* for the propoſed Time; to which add the Log. of the Rate made leſs by 1, and ſubſtract that Sum from the Log. of the Increaſe of 1 *l.* made leſs by 1, ſo ſhall the Remainder be the Log. of the preſent worth of 1 *l.* Annuity, for the Time propoſed; to which add the Logarithm of the propoſed Annuity, and the Sum will be the Logarithm of the preſent worth of the given Annuity: as for Example.

Queſt. 9. What is the preſent worth of an Annuity of 30 *l.* payable by yearly Payments, and to continue 30 Years, allowing Rebate after the Rate of 8 *per Cent. per Annum?* Facit 337 *l.* 14 *s.* 9 ½ *d.* as appears in

The Operation by the Logarithms.

The Log. of (1.08) the prop. Rate 0.033424 } mult.
The Term of Years —— —— 30 }

The Log. of (10.063 *l.*) the Increaſe of 1 *l.* for 30 Years. } 1.002720 } add
The Log. of (80) the Rate leſs 1 — 2.903090 }

—1.905810 ſubſt

The Log. of 10.063—1=9.063 0.957272

The Log. of the preſent worth of 1 *l.* Annuity ——— ——— } 1.051462 } add
The Log. of (30) the propoſed Annuity ——— ——— } 1.477121 }
The Log. of (337.74) the preſent worth of the propoſed Annuity ——— ——— ——— } 2.528583

vhich is 337 *l.* 14 *s.* 9 ½ *d.*

CASE

CASE VI.

l out what Annuity to continue any Term of
s, any given Sum of Money will purchaſe at
Rate of Compound Intereſt.

When you would know what Annuity any
Sum will purchaſe; firſt (as in the forego-
ıles) find out the Logarithm of the Rate,
multiply by the propoſed Time, ſo will
'roduct be the Logarithm of the Increaſe
to which add the Logarithm of the Rate
eſs by 1, and from that Sum ſubſtract the
thm of the ſaid Increaſe of 1 *l.* made leſs
ſo will the Remainder be the Logarithm
ıt 1 *l.* will purchaſe for the propoſed time;
ch if you add the Logarithm of the given
ıſe-Money, the Sum will be the Logarithm
Annuity that the given Sum will pur-
as for Example.

ſt. 10. What Annuity to continue 7 Years,
ıyable by quarterly Payments, will 246 *l.*
ſe, allowing Rebate at 8 *per Cent?* Facit
ı *l.*

The

The Operation by the Logarithms.

The Log. of 1.08 the given Rate *per Annum* —— —— } 0.033424
4)——

which divided by 4, gives the log. of (1.0194) the Rate *per* Quar. } 0.008356 } mult.
The Quarters in 7 Years —— 28

66848
16712

The Log. of the Increaſe of 1 *l.* for 28 Quarters, *viz.* 1.7138 } 0.233968 } add
1.0194—1=.0194 its Log. —2.287801

Sum —2.521769 } ſubt.
1.7138—1=.7138 its Log. —1.853577

The Log. of the purchaſe of 1 *l.*—2.668192 } add
The Log. of the prop. Sum 264 2.421604

The Log. of (12.297) the Annuity which the ſaid Sum will purchaſe —— —— } 1.089796
which is 12 *l.* 5 *s.* 11 ½ *d.*

More variety of Queſtions might be ſtated, but theſe to the Ingenious are ſufficient.

FINIS.

COCKER's
Algebraical Arithmetick:

CONTAINING

e Doctrine of Compoſing and Reſolving
n EQUATION; with all other Rules
requiſite for the underſtanding of that myſte-
rious Art, according to the Method uſed by
Mr. JOHN KERSEY, in his incomparable
Treatiſe of *ALGEBRA.*

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to foribus Academiæ inſcribi juſſit; *Nemo Arithmetices ignarus hic Ingrediatur.*

[illegible]

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[illegible]
[illegible]
[illegible]

[illegible]

Algebraical Definitions.

⁂

CHAP. I.

Concerning the Conſtruction of Coſſick Powers, and the Way of expreſſing them by Letters, together with the Signification of all ſuch Characters or Marks as are uſed in the enſuing Treatiſe.

THE Analytical Art, generally called Algebra, is that by which, when a Problem or hard Queſtion is propounded, we aſſume the Quantity or Number ſought, as if it were really nown; and with this aſſumed Quantity, and he *Quantity* or Quantities given, we proceed by undeniable Conſequences, until the Quantity firſt aſſumed is found to be equal to ſome Quantity or Quantities really known, and is therefore itſelf alſo known.

II. Algebra is either Numeral or Literal.

III. Numeral Algebra is ſo called, becauſe all the given Quantities in any Queſtion are expreſſed by natural Numbers, and the Number or Quantity ſought is ſolely repreſented by ſome Letter or Character, taken at the pleaſure of the Artiſt.

IV. Literal Algebra is ſo called, becauſe when a Queſtion is reſolved after this Method, the known or given Quantities as well as the unknown, are all expreſſed by Letters of the Alphabet, or ſome other convenient Marks or Characters; and this is alſo called Specious Algebra. And when a Queſtion is reſolved after this manner, at the end of the Operation, there is diſcovered a Canon, directing how the Queſtion propoſed, or any other of the like nature, may be ſolved: and therefore is Literal Algebra accounted more excellent than Numeral Algebra, for that produceth not a Canon without extraordinary difficulty; becauſe the Numbers firſt given are by Arithmetical Operations ſo interwoven and confounded, that it may ſeem a Task too tedious for the moſt ingenious Artiſt to trace out their Footſteps.

V. The Doctrine of Algebra conſiſts in the *knowledge* of certain Quantities, called Coſſick *Powers*, which we ſhall immediately explain.

VI. In a Series, or Rank of Geometrical Proportionals continued, proceeding from Unity or One, whether they be aſcending or deſcending, all the Numbers or Terms except the firſt (which is ſuppoſed to be Unity) are called Coſſick Numbers or Powers; as for Example, in this Rank of continual Proportionals, *viz.* 1, 2, 4, 8, 16, 32, 64, 128, 256, *&c.* the ſecond Term (2) is called the Root or firſt Power; the third Term (4) is called the Square or ſecond Power; the fourth Term (8) is called the Cube, or third Power; the fifth (16) is called the Biquadrate, or fourth Power; (32) is the fifth Power, (64) is the ſixth Power, *&c.*

In like manner, if you take a Rank of Geometrical Proportionals continued, and deſcending from Unity, *viz.* 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, *&c.* or 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, *&c.* or 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, *&c.* The ſecond Term is called the Root or firſt Power, the third Term is called the ſecond Power, the fourth Term is called the third Power, *&c.*

VII. Whence it is evident that the Square or Second Power is generated by the Multiplication of the Root or firſt Power into itſelf; and the Cube or third Power is generated by multiplying the ſecond Power by the Root, or by multiplying the Root three times into it ſelf; and the Biquadrate or fourth Power is produced by multiplying of the third Power by the Root, or by multiplying the Root four times into itſelf; and *the fifth Power is* produced by multiplying the fourth Power by the Root, *&c.* As for Example:

If you take 2 for a Root, multiply it by itſelf, it produceth 4 for the Square or ſecond Power of the Root 2: Again, multiply 4 by the Root 2, and it produceth 8 for the third Power, or Cube of the Root 2: Again, if you multiply the Cube 8 by the Root 2, it produceth the fourth Power, or Biquadrate of the Root 2, *&c.*

In like manner, if 3 were propoſed for a Root, it being multiplied by itſelf, produceth 9 for the Square, or ſecond Power of 3; and 9 being multiplied by the Root 3, produceth 27 for the Cube, or third Power of the Root 3, *&c.*

And alſo if $\frac{1}{2}$ be propoſed for a Root, and it be multiplied by itſelf, it produceth $\frac{1}{4}$ for the Square or ſecond Power of (the Root) $\frac{1}{2}$; and $\frac{1}{4}$ (the Square) being multiplied by (the Root) $\frac{1}{2}$, it produceth $\frac{1}{8}$ for the Cube, or third Power of (the Root) $\frac{1}{2}$, *&c.*

Whence it is evident, that the 4th, 6th, or 7th Powers of any Roots may be found out, without any reſpect at all had to the intermediate Powers, between the Root and the Power required: as, ſuppoſe there were given the Root 3, and it were required to find the fifth Power of it; I take 3, and ſet it down 5 times in order, thus, 3, 3, 3, 3, 3, and multiply them all into each other, according to the Rule of continual Multiplication, and the laſt Product (which is 243) is the fifth Power of the Root 3, which was required.

Again, Let it be required to find the fourth *Power* of 5; I take 5, and ſet it down 4 times, *thus*, 5, 5, 5, 5; then do I multiply them con-

lly, and find the laſt Product to be *625*,
h is the 4th Power (of the given Root) as
equired. The like may be obſerv'd in the
ıg of any other Power of any other given

III. If there be a Series of Geometrical Pro-
onals continued, and againſt each Power
be placed Numbers orderly, repreſenting
Number or Degree of Diſtance of each Power
the Root; ſuch Numbers are called the In-
or Exponents of the Pow-
becauſe they ſhew how of-
ne Root is involved into it
or the production of ſuch a
er; as in the Rank or Scale
gebraical Powers placed in
Margin, proceeding from
Root 2, to the 10th Power
of, which is 1024, under
h is written the word Pow-
and then againſt each par-
r Power, on the Left-hand
of, is expreſſed the Index, or
nent of that Power, ſhew-
ow often the Root is in-
d or multiplied into it ſelf,
oduce that Power: As for
nple; againſt the Number
placed the Number *6*,
ı ſheweth that *64* is the *6*th Power of its
, or that its Root is multiplied *6* times into
, to produce the Number *64*. The like is
underſtood of any other.

Indices.	Powers.
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

ewiſe, if any two or more Indices or Exponents be added together, their Sum will be an Exponent, ſhewing what Power will be produced by the Multiplication of thoſe Powers belonging to thoſe Exponents or Indices which you add together; as in the foregoing Table, let it be required to find out what Power of the Root 2 will be produced by multiplying 128 (its 7th Power) by 8 (its 3d Power;) in order to which I take 3 and 7, the reſpective Indices of the given Powers, and add them together, and their Sum is 10; which ſheweth that the 3d Power, and the 7th Power of any Number or Root being multiplied together, will produce the 10th Power of that Root; ſo in our Example, 128 being multiplied by 8, produceth 1024, which is the 10th Power of the Root 2.

In like manner, the Indices 3 and 5 being added together, make 8 for a new Exponent; which ſheweth that 32 and 8 (the Powers belonging to thoſe Exponents) being multiplied together, will produce the 8th Power, *viz.* 256; as appears by the ſaid Table. The like of any other.

So that you ſee that the Addition of Indices anſwers to the Multiplication of their correſpondent Powers.

And in like manner will the Subtraction of Indices or Exponents anſwer to the Diviſion of their correſpondent Powers, obſerving always to make the Power correſponding to the Subtrahend (or Index to be ſubtracted) to be the Diviſor.

IX. When a Queſtion is propounded, and its Solution is to be ſearched out by the Algebraick Art, the Number or Magnitude ſought is generally called a Root, and it muſt be repreſented or ſignified by ſome Character or Symbol, as muſt be alſo all the Powers proceeding from the ſaid Root, according to the Tenour of the Queſtion; in order to which, there may be taken ſome Letter of the Alphabet, at the pleaſure and diſcretion of the Artiſt, as *a*, *b*, *c*, or *d*, *&c.* to expreſs the ſaid Root: but to avoid confuſion in Operation, by the commixture of known and unknown Quantities, our modern Analyſts have been accuſtomed to aſſume Vowels to repreſent unknown Quantities, and to put Conſonants to ſignify known or given Quantities.

X. If for the Number or Quantity ſought there be put or aſſumed the Vowel *a*, then its Square will be *aa*, that is, *a* being multiplied by it ſelf, produceth *aa*, that is *a* ſquared, or the Square of *a*, for *a* times *a* is *aa*; and the Cube or third Power raiſed from the Root *a*, is *aaa*, that is, *a* times *aa*, is *aaa*; and the fourth Power accordingly is *aaaa*: and after the ſame manner may any higher Power of *a* be ſignified.

In like manner, if for the Quantity or Number ſought there be aſſumed the Letter *e*, then ſhall the Square raiſed therefrom be *ee*, and the third Power *eee*, and the fourth Power *eeee*, and the fifth Power, *eeeee*, &c.

Alſo if *b*, or any other Conſonant be put for a given or known Quantity, then its Square will *be bb, its Cube bbb*, and its Biquadrate bbbb, &c.

But by ſome Analyſts the Powers of *a*, or any other Letter, either Vowel or Conſonant, are expreſſed by placing the Index or Exponent of the Power in a ſmall Character, juſt after the Symbol, even with the Head thereof, *viz.* a, a^2, a^3, a^4, *&c.* ſignify the Root *a*, its Square, its Cube, and its Biquadrate, *&c.* which may be further exemplified by the following Table.

A Table ſhewing the Powers of Numbers, and how to expreſs the Simple Powers by Alphabetical Characters.

The Root, or firſt Power.	1	2	3	4	*a*	a^1
The Square, or ſecond Power.	1	4	9	16	*aa*	a^2
The Cube, or third Power.	1	8	27	64	*aaa*	a^3
The Biquadrate, or fourth Power.	1	16	81	256	*aaaa*	a^4
The fifth Power.	1	32	243	1024	*aaaaa*	a^5
The ſixth Power.	1	64	729	4096	*aaaaaa*	a^6
The ſeventh Power.	1	128	2187	16384	*aaaaaaa*	a^7
The eighth Power.	1	256	6561	65536	*aaaaaaaa*	a^8
The ninth Power.	1	512	19683	262144	*aaaaaaaaa*	a^9
The tenth Power.	1	1024	59049	1048576	*aaaaaaaaaa*	a^{10}

XI. The Numbers made uſe of in ſolving of Algebraical Queſtions, are either abſolute Numbers, or Numbers prefix'd.

Abſolute Numbers are thoſe which are disjunct from any kind of Magnitude or Quantity, either known or (unknown) required, but ſtand ſimply of themſelves, without having relation to any thing elſe, as 5, 10, 20, 100, $\frac{3}{4}$, and $\frac{1}{3}$, are called abſolute Numbers.

Numbers prefix'd are ſuch as are immediately prefix'd to ſome Letter or Letters, ſignifying an Algebraical Quantity, either known or required, ſuch as are $2a$, $4a$, $10a$, $100a$, $\frac{1}{2}a$, $\frac{3}{4}a$, $3aa$, $5bbb$, $3a$, $5b^3$; which Numbers ſo prefix'd, ſhew how often the Quantity to which they are prefixed, is to be taken, as $4a$ ſignifieth that a is to be taken 4 times, and $5bbb$, or $5b^3$ ſignifieth that the Cube or third Power of b is to be taken 5 times, $\frac{1}{2}a$ is half of a, and $\frac{2}{3}b$ is two thirds of b. The like is to be underſtood of any other. And,

Note, That when you have any Algebraical Quantity or Letter or Character, not having any Number prefix'd to it, then 1, or Unity, muſt be imagined to be prefix'd, as a, or $1a$, b, or $1b$, *&c.*

XII. As in Vulgar and Decimal Arithmetick, ſo in Algebraical Arithmetick, the Operations are performed either by abſolute Numbers, or by Alphabetical Characters, in all the fundamental Rules, *viz.* Addition, Subtraction, Multiplication, Diviſion, and the Extraction of Roots. And note, that

Where it is required to perform the Work by abſolute Numbers, the Operation is in every reſpect the ſame as in Common Arithmetick

tick. But where it is performed by Alphabetical Letters, there is an abſolute neceſſity of uſing ſome Characters, to ſignify the Operation: an Explanation of which Characters take as followeth.

XIII. The Character (+) is a Sign of Affirmation or Addition, which is placed between two Quantities, ſignifying that the 2 Numbers or Quantities between which it is placed, are to be added together, and is as much as to ſay *plus*; as $3+6$ ſignifieth the Sum of 3 and 6, and is as much as to ſay, 3 *plus* 6, or 3 more 6, which is 9; and $4+7+9$ ſignifieth the Sum of 4, 7, and 9, which is 20; ſo $a+b+c$ ſignifieth the Sum of *a*, *b*, and *c*.

And here note, that when there is no Mark or Character before any Letter or Quantity, then it is Affirmative, and the Mark (+) is ſuppoſed to ſtand before it; as *a* is $+a$, or $+1a$, and *b* is $+b$, or $+1b$, and *bcd* is $+bcd$: the like of others.

XIV. This Character (—) is a negative Sign, and always belongeth to the Quantity or Number which followeth, it denying it to be, and ſignifieth a fictitious Number, or Quantity, leſs than nothing.

So -7 is a feigned Number leſs than nothing by 7, *viz.* as the Height of the Sun above the Horizon may be affirmed to be 7 *deg.* or $+7$ *deg.* ſo when it is depreſſed 7 Degrees below the Horizon, its Height may be ſaid to be -7 *deg.* that is, 7 *deg.* leſs than nothing.

But when the ſaid Sign or Character is placed between two Numbers or Quantities, it ſignifieth that the Number or Quantity which followeth

eth it, is to be ſubtracted out of ſome Number or Quantity going before it, as $12-8$ ſignifieth that 8 is to be ſubtracted out of 12; or it ſignifieth the exceſs of 12 above 8, or the difference between 12 and 8, which is 4: ſo $a-b$ ſignifieth the exceſs of a above b, and it is as much as to ſay (a leſs b) ſo $a+b-c$ ſignifieth that c is to be ſubtracted from the Sum of a and b.

XV. This Character ($\times$) is the Sign of Multiplication, and ſignifieth that the Numbers or Quantities between which it is placed, are to be multiplied together; as 4×5 ſignifieth the Product of 4 and 5, which is 20; ſo $3\times5\times8$ ſignifieth the Product of the continual Multiplication of 3, 5, and 8, *viz.* 120.

Likewiſe $b\times c$ ſignifieth the Product of the Multiplication of b by c, and $b\times c\times d$ ſignifieth the Product made by the continual Multiplication of b, c, and d, into each other.

But for the moſt part, Analyſts ſignify the Multiplication of literal Quantities, by ſetting the Letters together like Letters in a Word; as ab is the ſame with $a\times b$, and abc is the ſame with $a\times b\times c$; and this is indeed to be preferred before the other, as moſt convenient and fitteſt for Operation.

XVI. This Character ($\backsim$) ſignifieth the Difference between the two Quantities between which it is placed, when it is not known in which of them the Exceſs lieth: So $b\backsim c$ ſignifieth the Difference between b and c, when it is not known whether b be greater or leſſer than c.

XVII. The ſaid 4 Characters defin'd in the 13th, 14th, 15th, and 16th Sections foregoing, *viz.* $+$, $-$, $\times$ and $\backsim$, may oftentimes have relation to ſuch a Compound Quantity following the Character, as hath a Line drawn over each part of it: as for Example, $c+\overline{b\backsim d}$, by which you are to underſtand, that the Quantity (c) is to be added to the Difference between the Quantities (b and d) in which of them ſoever the Exceſs lieth.

Likewiſe $a-\overline{b\backsim c}$, which ſignifieth that the Difference between b and c is to be ſubtracted from the Quantity expreſſed by a.

Alſo $a\times\overline{b+c}$ ſignifieth that the Sum of b and c is to be multiplied by the Quantity a: where take notice, that in regard there is a Line drawn over the two Quantities b and c, the Sign × hath reference to the Multiplication of a into the Quantity c, as well as the Quantity b, which immediately followeth it; but if the ſaid Line were omitted, and the Quantities were thus expreſſed, $a\times b+c$, it would ſignify the Quantity c to be added to the Product of the Multiplication of a and b.

Furthermore, $b-\overline{c+d}$ ſignifieth that the Quantities c and d are or muſt be ſubtracted from the Quantity b; whereas if there were not a Line over the Quantities c and d, it would ſignify that the Quantity d is to be added to $b-c$.

And $c\backsim\overline{d+e}$ ſignifieth the Difference between the Quantity c, and the Sum of d and e; whereas if the Line were not over d and e, it would ſignify the Quantity e to be added to the Difference between c and d.

XVIII. This Character ($\surd$) is a radical Sign, and ſignifieth that the Square Root of the Quantity or Quantities following it, is to be extracted; as $\surd\ 36$ ſignifieth the Square Root of 36, *viz.* 6.

So $\surd\ ab$ ſignifieth the Square Root of the Product of the Quantities a and b, and $\surd\ abc$ is the Square Root of the Product of the continual Multiplication of the Quantities of a, b, and c.

But when you would repreſent the Root of a Power that is higher than a Square, then immediately after the ſaid radical Sign, expreſs the Index or Exponent of its Power in a Parentheſis, as followeth, *viz.* $\surd\ (3)\ 64$ ſignifieth the Cube Root of 64, which is 4; $\surd\ (4)\ 81$ ſignifieth the Biquadrate Root of 81, *viz.* 3.

Alſo $\surd\ (3)\ ab$, ſignifieth the Cube Root of the Product of the Multiplication of the Quantities, a and b; and $\surd\ (4)\ cd$ ſignifieth the Biquadrate Root of the Product of the Multiplication of the Quantities c and d.

And the ſaid radical Sign doth oftentimes belong to ſuch a Compound Quantity following it, as hath a Line over every part of it: as for Example, $\surd\overline{b+c}$ ſignifieth the Square Root of the Sum of the Quantities b and c; ſo $\surd\ (3)\ \overline{a+b-c}$ ſignifieth the Cube Root of the Remainder, when the Quantity c is ſubſtracted from the Sum of the Quantities a and b; and $\surd\ (4)\ \overline{aa+b-c}$ ſignifieth the Biquadrate Root of the Remainder, when the Quantity c is ſubtracted from the Sum of the Square of a added to b.

Likewiſe $a+\surd\overline{bb+c-d}$ ſignifieth that to the Quantity a is to be added the Square Root of the Remainder, when the Quantity d is ſubtracted from the Sum of the Square of the Quantity bb, nd the Quantity *c*: And theſe and ſuch like are y Analyſts generally called univerſal Roots.

After the ſame manner may be expreſſed the univerſal Square Root of $b+\overline{aa+c}$ thus, *viz.* $\sqrt{(2)}\overline{b+\sqrt{aa+c}}$, which ſignifieth the Square Root of the Sum when *b* is added to the Square Root of $\overline{aa+c}$.

XIX. This Character (=) ſignifieth an Equation, or Equality of the Magnitudes, or Quantities between which it is placed, and imports as much as theſe Words, *viz.* $3+4=7$, which is as much as to ſay, the Sum of 3 and 4, or $3+4$ is equal to 7; ſo $7+9=12+4=16$ imports that he Sum of 7 and 9 is equal to the Sum of 12 nd 4, which is equal to 16; and $9=12-3$, ſignifieth that 9 is equal to the Exceſs of 12 above 3.

Alſo $4\times5=2\times10=16+4=20$, ſignifieth that he Rectangle or Product of 4×5 is equal to the Rectangle or Product of 10×2, which is equal to he Sum of 16 and 4, equal to 20.

Likewiſe $\frac{24}{6}=\frac{8}{2}$ ſignifieth that the Quotient of 24 divided by 6, is equal to the Quotient of 8 divided by 2.

Again, $a+b=c-d$ ſignifieth that the Sum of a and *b* is equal to the Exceſs of *c* above *d*; and $+d=\frac{f}{g}$ ſignifieth that the Sum *c* and *d* is equal to *the Quotient* of *f* divided by g; and $c=r-s$ *ſignifieth* that the Rectangle of b and

c is equal to the Exceſs of r above s; and $a=\sqrt{cc+\frac{1}{4}cc}-\frac{1}{2}c$ ſignifieth that a is equal to the Remainder, when $\frac{1}{2}c$ or $\frac{c}{2}$ is ſubtracted from the univerſal Square Root $cc+\frac{1}{4}cc$. This will be made plain and eaſy to the ingenious Practitioner by the enſuing Example of this Treatiſe.

XXI. This Character ($\sqsubset$) ſtands for the Word (greater) ſignifying the Number, or Quantity ſtanding on the left-hand of the ſaid Character to be greater than that on the right-hand thereof; as $8 \sqsubset 3$ ſignifieth that 8 is greater than 3: Alſo $a+b \sqsubset c$ ſignifieth that the Sum of a and b is greater than c, &c.

XXII. This Character ($\sqsupset$) ſtands for the Word (leſs) and it ſignifieth that the Number or Quantity ſtanding on the left-hand thereof, is leſſer than that on the right-hand; as $4+3 \sqsupset 20-8$ ſignifieth that the Sum of 4 and 3 is leſs than the Exceſs of 20 above 8. Likewiſe $c-d \sqsupset d+e$ is thus read, *viz.* the Remainder of d being ſubtracted from c is leſſer than the Sum of d and e.

XXIII. This Character (: :) is always placed in the middle between 4 Geometrical Proportionals, as in the following Examples, *viz.* $2 : 4 :: 9 : 18$; is thus to be read, *viz.* as 2 is to 4, ſo 9 is to 18: or after the manner of the Rule of 3, if 2 require 4, 9 will require 18. Alſo $b : c :: d : e$ is thus read, as b is to c, ſo is d to e. And $a+e : b :: c+b : \frac{bc+bb}{a+e}$ is as much as to ſay, as the Compound Quantity $a+e$ is to the Quantity b, ſo is the Compound antity $c+b$ to the Quotient of the Compound antity $bc+bb$ being divided by $a+e$.

XXIV. This Character (∺) placed after any mber of Quantities, exceeding Two, declareth ſaid Numbers or Quantities after which it is ced, to be continual Geometrical Proportio- ls: ſo 2, 4, 8, 16, 32, 64 ∺, ſignifieth the d Numbers to be continual Proportionals Geo- trical; for, as 2 is to 4; ſo is 4 to 8, and ſo is to 16, and ſo is 16 to 32, and ſo is 32 to 64, :. Alſo theſe Quantities, *viz.* a. b. c. d. e. ∺ continual Proportionals Geometrical; for as s to b; ſo is c to d, and ſo is d to e.

* * *

CHAP. II.

Addition of Algebraical Integers.

AS in Common Arithmetick, ſo in Algebraical, Addition finds out the Aggre- te, or Sum of two or more given Quan- ies, however expreſſed, Numerally or Lite- ly.

II. When the Quantities given to be added are ke, and have like Signs, collect the Numbers *fixed to each* Quantity into one Sum, and thereto annex the Letter, or Letters of any one of the given Quantities, and then prefix the Sign of Affirmation or Negation, *viz.* $+$ or $-$; ſo ſhall the Quantity thus found be the Sum deſired.

And here note, that every Quantity which hath no Number prefix'd to it, is ſuppoſed to have the Number 1 prefixed; ſo is $a=1a$. and $b=1b$.

Example.

What is the Sum of $3b+b+2b$? Facit $6b$. for the Sum of the Numbers prefixed to each Quantity, *viz.* 3, 1, and 2, is 6; to which if I annex the Character b, it will be $6b$, which muſt have the Sign $+$ prefixed to it, or elſe it muſt be imagined ſo to be, then will $+6b$ be the Sum of the given Quantities. So is $5ab$ the Sum of $3ab+2ab$; and $-4cd$ the Sum of $-3cd-cd$.

More Examples of this Rule.

Quantities to be added.	$3aa$	$-4aa$	$+15abc$
	$2aa$	$-aa$	$+11abc$
	aa	$-2aa$	$+25abc$
Sum	$6aa$	$-7aa$	$+51abc$

III. When the Quantities given to be added together are like, but have unlike Signs; then ſubtract the leſſer Number prefixed from the greater, and to the Remainder annex the Letter or Letters by which any one of the given Quantities is expreſſed; and thereto prefix the Sign of $+$ or $-$, according to the Sign of that prefixed ıber wherein lay the Exceſs, ſo ſhall this new ntity be the Sum of the Quantities propoun-

Example.

:t it be required to add $+5cd$ to $-2cd$, the will be found to be $+3cd$; for firſt I ſub- -2 from $+5$, and there remains 3, to which nex cd, ſo will there be $3cd$, to which I prefix Sign $+$, becauſe it belongs to the Number 5, rein lay the Exceſs, ſo have I $+3cd$ for the required. See the Work.

$$\text{add}\begin{cases}+5cd\\-2cd\end{cases}$$

$$\text{Sum } +3cd$$

gain, if it were required to $4aa$ to $-7aa$, the Sum ld be found to be $-3aa$, be- the Sign $-$ belongs to the ıber 7 wherein lay the Ex- See the Work in the gin.

$$\text{add}\begin{cases}-7aa\\4aa\end{cases}$$

$$\text{Sum} -3aa$$

More Examples of the laſt Rule.

$$\begin{array}{l|r|r|r|r}
\text{be ad-}\ \text{d} \begin{cases}\\ \\ \end{cases} & \begin{array}{r}5abcd\\-abcd\end{array} & \begin{array}{r}6aae\\-9aae\end{array} & \begin{array}{r}-15ggg\\11ggg\end{array} & \begin{array}{r}-30bcd\\14bcd\end{array}\\
\hline
\text{Sum} & 4abcd & -3aae & -4ggg & -16bcd
\end{array}$$

And here note, that if the Numbers preſi to the given Quantity be equal, and they h different Signs, their Sum will be o; ſo if it w required to add $+8bcd$ to $-8bcd$, their Sum be o, the negative Sign deſtroying the affir tive.

LVI. When the Quantities given to be added more than 2, and have different Signs, then cording to the ſecond Rule of this Chapter, br the Quantities, having like Signs, into one S that is, the Affirmative Quantities into one Si and the Negative into another; then by the fo going third Rule, add thoſe two Quantities to ther, ſo ſhall their Sum be the Number ſought

Example.

Let it be required to add the Sum of $3aa+$ $-2aa-5aa$. Firſt, by the ſaid ſecond Rule, I f the Sum of $3aa+7aa$ to be $10aa$, and the Sum $-2aa-5aa$ to be $-7aa$; then by the ſaid th Rule, I find the Sum of $10aa-7aa$ to be $3aa$, $+3aa$; ſo that I conclude the Sum of $3aa+$ $-2aa-5aa$ to be $+3aa$.

More Examples of this Rule follow.

To be added	$+3cc$	$-2cd$	$-13bcd$
	$-7cc$	$-7cd$	$+17bcd$
	$+8cc$	$+5cd$	$+7bcd$
	$-4cc$	$+9cd$	$-8bcd$
Sum	0	$+5cd$	$+3bcd$

V. When the ſimple Quantities given to be added together, be unlike, then (how many ſoever there be) ſet them one after another in the ſame Line, without altering their Signs.

Example.

What is the Sum of $4b$ added to $3cd$? Facit $4b+3cd$ for the Sum.

More Examples of this Rule follow.

To be added.	$2b$ aa	$3bc$ $2cd$	$8ab$ $3be$
Sum	$2b+aa$	$+3bc+2cd$	$+8ab+3be$

To be added	$3e$ $2b$ $-ae$	$+2fg$ $-3gh$ $-4rs$
Sum	$3e+2b-ae$	$+2fg-3gh-4rs$

Algebraical Addition of Compound Integers.

VI. The Addition of Compound Algebraical Integers is eaſily performed by the help of the foregoing Rules of this Chapter, whether the Compound Quantities to be added are alike, or unlike; as you may eaſily perceive by the Work of the following Examples.

Let it be required to find the Sum of $3a+b$ *and* $5a+3b$; *their* Sum will be $8a+4b$; for

$3a+5a=8a$, and $3b+b=4b$, whoſe Sum is $8a+4b$, by the ſecond Rule of this Chapter.

Alſo the Sum of $6cd+3bb$ and $2cd-5bb$ will be found to be $8cd-2bb$, for (by the ſecond Rule of this Chapter) $6cd+2cd=8cd$, and by the third Rule, the Sum of $3bb-5bb=-2bb$; which two Sums added together, by the fifth Rule of this Chapter, will be $8cd-2bb$.

Moreover, if it were required to find the Sum of theſe Compound Quantities, *viz.* $15gg+8a-20$ and $3gg-3a+12$, it will be $18gg+5a-8$; for $15gg+3gg=18gg$ by the ſecond Rule, and the Sum of $8a-3a=5a$ by the third Rule, and by the ſame $12-20=-8$, the Sum of which 3 Sums is $18gg+5a-8$ by the fifth Rule of this Chapter.

And the Sum of $8b-16+2cd$ and $24-5b+3cd$ is $3b+8+5cd$. And here note, that in ſetting down Compound Quantities to be added together, it matters not which of them you ſet firſt, ſo that to every Quantity there is prefixed its proper Sign; as $3a+b-cc$ is the ſame with $b+3a-cc$, and with $-cc+b+3a$, &c.

More Examples of the Addition of Compound Algebraical Integers.

$$\begin{array}{rr|r}
\text{To be added} & \left\{\begin{array}{r} 4ccc-18+3ab \\ 8ccc+16-8ab \\ 3ccc+12+6ab \end{array}\right. & \begin{array}{r} 10ab+12aa-6d \\ -8ab-8aa+9d \\ -2ab-2aa-2d \end{array} \\
\hline
\text{Sum} & 15ccc+10+ab & 0\ +2aa+d
\end{array}$$

To

$$\begin{array}{ll|l}
\text{To be added} & \left\{\begin{array}{l} 16cde+4db+5p \\ -8cde-2db \\ -3cde-db \end{array}\right. & \begin{array}{l} 9c^4+4ef-dg \\ 4c^4+2dg-ab \\ 2c^4+ef+dg \end{array} \\
\hline
\text{Sum} & 5cde+db+5p & 15c^4+5ef-ab+2dg \\
\hline
\end{array}$$

CHAP. III.

Subtraction of Algebraick Integers.

I Shall not here need to give you a Definition of the Nature of Subtraction, but ſhall only give 'ou a general Rule for finding out the Remainer, Exceſs, or Difference of any two Quantiies, and that in all Caſes whatſoever.

I. When a Quantity, ſingle or compound, is iven to be ſubtracted from another, then change he Sign or Signs of the Quantity to be ſubtracd, into the contrary Signs, that is + into —, nd — into +; which being done, add the two iven Quantities together by the Rule of the regoing ſecond Chapter; ſo ſhall their Sum be e Difference or Remainder ſought.

Example 1.

Let it be required to ſubtract $3a$ from $8a$.

The Quantity here given to be ſubtracted is $3a$, which, according to the ſecond Rule of the ſecond Chapter, is $+3a$, therefore muſt its Sign $+$ be changed into $-$, ſo will it be $-3a$, which being added to $8a$ (by the third Rule of the ſecond Chapter) their Sum will be $5a$, for $8a-3a=5a$, and is the difference between the Quantities $8a$, and $3a$.

Example 2.

Let it be required to ſubtract $-3bc$ from $4bc$.

Here becauſe $-3bc$ is the Quantity to be ſubtracted, therefore muſt its Sign $-$ be changed into $+$, ſo will it be $+3bc$, which being added to $4bc$, by the ſecond Rule of the ſecond Chapter, their Sum is $7bc$, for $3bc+4bc=7bc$; and ſo much is the Remainder, when $-3bc$ is ſubtracted from $+4bc$.

Example 3.

Let it be required to ſubtract $-3bde$ from $-9bde$.

Here, becauſe $-3bde$ is the Quantity to be ſubtracted, therefore muſt its Sign $-$ be changed into $+$, ſo will it be $+3bde$, which being added to $-9bde$, according to the third Rule of the ſecond Chapter, their Sum will be $-6bde$; for $+3bde-9bde=-6bde$; and ſo much is the Remainder when $-3bde$ is ſubtracted from $-9bde$.

Example 4.

Subtract $3cd$ from $8de$. The Sign of $3cd$ being changed, it will be $-3cd$, which being added to $8de$, by the fifth Rule of the ſecond Chapter, their Sum will be $8de-3cd$, which is the Remainder, when $3cd$ is ſubtracted from $8de$.

Example 5.

What is the Remainder when $-3bc$ is ſubtracted from $2cd$? Facit $3bc+2cd$, or $2cd+3bc$.

In all which Examples you ſee that the Sign of the Quantity given to be ſubtracted is changed into the contrary Sign.

More Examples of Subtraction of Simple Algebraick Integers.

	Example 6.	*Example* 7.
From	$3cd$	$-5bc$
Subtract	cd	$-8bc$
Remainder	$3cd-cd$	$-5bc+8bc$
Remainder contracted.	$2cd$	$3bc$

Example

	Example 8.	*Example* 9.
From	$+3da$	$-cde$
Subtract	$-da$	$+2cde$
Remainder	$3da+da$	$-cde-2cde$
Remainder contracted	$+4da$	$-3cde$

And when it is required to ſubtract a Compound Integer from a Compound Integer, the Operation will not in any wiſe differ from the former, obſerving always to change + into —, and — into +, as will appear by the following Examples.

Example 10.

From $3a+4b$ let it be required to ſubtract $2a-b$. Here $2a-b$ being the Quantity to be ſubtracted from the other, its Signs muſt be changed into the contrary Signs; and then inſtead of $2a-b$ you will have $-2a+b$, which being added to $3a+4b$, the Sum will be $a+5b=3a+4b-2a+b$, and ſo much is the Remainder, when Subtraction is performed according to the Tenour of the Queſtion. See the Work laid down as followeth.

From $3a+4b$
Subtract $2a-b$

Remainder $3a+4b-2a+b$

Remainder contracted } $a+5b$

Example 11.

'rom $3aa-2dc+ab$ let it be required to ſub-
t $aa+3ab-3dc$. The Quantity here given
e ſubtracted is $aa+3ab-3dc$, whoſe Signs
g changed, it will then be $-aa-3ab+3dc$,
ch being added to $3aa-2dc+ab$, the Sum
be $3aa-2dc+ab-aa-3ab+3dc$, which ac-
ling to the ſixth Rule of the ſecond Chapter,
ual to $2aa+dc-2ab$. See the following O-
tion.

rom $3aa-2dc+ab$
btract $aa+3ab-3dc$

emainder $3aa-2dc+ab-aa-3ab+3dc$

emainder
ntracted } $2aa+dc-2ab$

ut when the given Quantities are unlike, then
e the Quantity to be ſubtracted immediately
the Quantity out of which it is to be ſubtrac-
in the ſame Line, changing its Signs; which
Quantity, when the ſaid Quantity is ſo annex-
is the Remainder required; which will admit
o *Contraction*, becauſe the Quantities are
e.

Example

Example 12.

Let it be required to ſubtract $3ab+2$ $7bc+6cd$. The Quantity given to be ſub is $3ab+2aa$, which annexed to the othe Quantity, changing its Signs, will give 7 $-3ab-2aa$, which is the Remainder re See the following Work.

From $7bc+6cd$
Subtract $3ab+2aa$

Remainder $7bc+6cd-3ab-2aa$

More Examples of Subtraction in Compoun braick Integers.

From	$3aa+2bc$	$8rd-3dc$
Subtract	$2aa+4bc$	$-2rd-9cd$
Remain.	$3aa+2bc-2aa-8bc$	$8rd-3dc+2$
Rem. cont.	$aa-2bc$	$10rd+6dc$

From $6ace+3cd-bc$
Subtract $4ace-cd-5bc$

Remain. $6ace+3cd-bc-4ace+cd+5bc$

Rem. cont. $2ace+4cd+4bc$

om $3aa-2bc+5ab$
ibtract $2aa-2bc-ab$

emain. $3aa-2bc+5ab-2aa+2bc+ab$

em.
ont. } $aa+6ab$

rom $8a^4+3bc-3d$ | $3cd+5a$
ubtract $5a^4$ | $2ab-3c$

Remain. $8a^4+3bc-3d-5a^4$ | $3cd+5a-2ab+3c$

Rem.
ont. } $3a^4+3bc-3d$

As in Natural Arithmetick, the Remainder and
he Sum ſubtracted being added together will be
qual to the Number from which the Subtraction
made, ſo it is likewiſe in Algebraical Arith-
etick; for if you add the remaining Quan-
ty to the Quantity ſubtracted, the Sum will be
qual to the Quantity out of which the Subtrac-
on is made. As in the firſt Example of this
hapter; where it is required to ſubtract $3a$
om $8a$, and the Remainder is $5a$; now if to $5a$
ou add $3a$, the Sum will be $5a+3a=8a$. And
the twelfth Example, where it is required to
btract $3ab+2aa$ from $7bc+6cd$, the Remainder
found to be $7bc+6cd-3ab-2aa$; to which
you add the Number ſubtracted, *viz.* $3ab+$
aa, the Sum will be $7bc+6cd$, equal to the gi-
n Quantity out of which Subtraction is made;
r $3ab+2aa$ being added to $-3ab-2aa$, they de-
ſtroy

ſtroy each other, becauſe their Signs are unlike. And this may ſerve for a ſufficient (and indeed the only) Proof of the Work.

CHAP. IV.

Multiplication of Algebraick Integers.

I. IN Multiplication of Algebraical Quantities, there are always two Quantities given, to find out a third.

II. The two Quantities given are called the Factors, and the third Quantity invented or found by the ſaid Factors, is called the Product, Fact, or Rectangle.

III. When the given Factors are ſingle Quantities, alike or unlike, if they have not natural Numbers prefix'd to them, the Fact is diſcovered at firſt ſight, and is performed by joining both the Quantities together in one, without any Character between them, like Letters in a Word.

But ſpecial regard muſt be had to the Signs of *the* given Quantities, in all kinds of Multiplication, whether by Simple or Compound Quan-ties, and whether with or without Numbers efixed to them, the Nature of the Product holly depending thereupon, *viz.* If the Signs the Quantities to be multiplied together be ike, that is, both +, or both −, then the Sign the Product or Fact will be +; but if they be different kinds, *viz.* the one +, and the oer−, then the Sign of the Product will be −, you will find by the ſeveral Examples followig.

Example 1. What is the Product of *a* multiied by *b*? Facit *ab*.

Here becauſe both the Factors are ſigned with -, therefore the Sign of the Product is +.

In like manner, if the given Factors had been -*a* and −*b*, the Product would have been (*ab* *ba*) the ſame as before, becauſe the Signs of e Factors are both alike, *viz.* both −.

But if the given Factors had been + *a* and *b*, or −*a* and +*b*, then the Product or Fact ould have been −*ba* or −*ab*, becauſe the Signs the Factors are unlike, *viz.* the one + and e other −: Obſerve the like in all Caſes whatever.

Example 2. What is the Product of *abc* multied by *cd*? Facit *abccd* or + *abccd*.

And if you had been to multiply −*abc* by −*cd*, Product would have been the ſame, *viz.* *abccd*, +*abccd*.

But if the Factors had been −*abc* by + *cd*, or *abc* by −*cd*, then the Fact would have been *abccd*, becauſe the Signs of the Factors are like.

More Examples of the like nature.

Multiplicand	$+ab$	$-ac$	$-aef$
Multiplier	$+ade$	$-ac$	$+aaf$
The Product	$+aabde$	$+aacc$	$-aaaeff$

IV. When the Quantities given to be multiplied are ſingle, or ſimple Quantities, (whether alike, or unlike) having natural Numbers prefixed to them, then in ſuch Caſes let the natural Numbers be multiplied together, and to their Product annex the Product of the given Algebraical Quantities, they being multiplied together, as in the laſt Rule, ſo ſhall this new Quantity found, be the Product required; as in the following Examples.

Example 1. Let it be required to multiply $3a$ by $9a$.

Firſt, I multiply the Numbers prefixed to both Quantities, the one by the other, *viz.* 9 by 3, and their Product is 27, to which I annex the Letters contained in both Quantities, *viz.* aa, and they make $27aa$, which is the Product, or Fact required.

Example 2. Let it be required to multiply $3aa$ by $4b$. Here, firſt, I multiply the Numbers prefixed together, *viz.* 3 and 4, and they make 12; to which Product I annex the Letters of both Quantities given, *viz.* aa and b, and they make $12aab$ for the Product required.

Example 3. What is the Product of $-3abc$ by $-5cd$?

Here, firſt, I multiply the given Numbers prefixed, *viz.* 3 and 5, and they produce 15, to which I annex the Letters in both the Quantities, *viz.* *abc* and *cd*, and they make $15abccd$, to which I prefix the Sign $+$, becauſe the Signs of the given Quantities were both alike, *viz.* $-$, and then will the Product or Fact be $+15abccd$.

Example 4. What is the Product of $+6ab$ multiplied by $-3cd$?

Firſt, multiply the Numbers prefixed, *viz.* 6 and 3, and the Product is 18, to which I annex the Letters in both the given Quantities, *ab* and *cd*, and it makes $18abcd$, to which I prefix the Sign $-$ (becauſe the Signs of the Factors were unlike, *viz.* the one $+$, and the other $-$) and then the Product will be $-18abcd$.

More Examples of the like Nature.

Multiplicand	$8fg$	$48be$	$20dff$	$14ghk$
Multiplier	$6rs$	$6f$	$3ff$	6
Product	$48fgrs$	$288bef$	$60df^4$	$84ghk$

V. When Compound Quantities are to be multiplied, the Operation (in effect) is the ſame with Multiplication of ſimple Quantities delivered in the foregoing Rules, for you are to multiply every particular Quantity in the Multiplicand, by each particular Quantity in the Multiplier, (not regarding whether you begin the Work at the right-hand or the left) and then let the ſeveral Products be joined together according to the Rules of the Algebraical Addition, and that Sum will be the Product required. The following Examples will make the Rule plain.

Example 1.

In the firſt place let it be required to multiply a Compound Quantity by a Simple, *viz.* $ab+d$ by a. And in order thereto, firſt I multiply a into ab, and the Product is aab, and then into d, and it produceth ad, ſo is $aab+ad$ the Product required; each Member of the Product being Affirmative, becauſe all Parts of the Factors were Affirmative.

Example 2.

Let it be required to multiply $aa+ab-c$ by b. The Product of aa by b is aab, and the Product of ab by b is abb, and the Product of $-c$ by b is $-cb$; all which particular Products being joined, and one Compound Quantity compoſed thereof, it will give $aab+abb-bc$ for the Product required. See the Work in the Margin.

Multiply $aa+ab-c$
by b

Product $aab+abb-bc$

Example 3. Let it be required to multiply the ompound Quantity $ac+dg$ by the Compound uantity $c+d$.

$$\begin{array}{ll} \text{Multiply} & ac+dg \\ \text{by} & c+d \\ \hline & acc+cdg \\ & acd+ddg \\ \hline \text{Product} & acc+cdg+acd+ddg \\ \hline \end{array}$$

Firſt, multiply each Member of the Multipliand by c, and the Product is $acc+cdg$; then ultiply each Member of the Multiplicand by d, nd the Product is $acd+ddg$; which two Products eing joined together, by the Rule of Algebraical ddition, the Sum is $acc+cdg+acd+ddg$, which the Product required, as appears by the Opeation.

Example 4. What is the Product of $da+bc$ muliplied by $da-ab$?

$$\begin{array}{lrr} \text{Multiply} & da+bc & \\ \text{by} & da-ab & \\ \hline & ddaa+dabc & \\ & & -aadb-abbc \\ \hline \text{Product} & ddaa+dabc & -aadb-abbc \\ \hline \end{array}$$

Firſt, Multiply the Multiplicand $da+bc$ by da he *firſt Member* of the Multiplier) and it proceth $ddaa+dabc$; then multiply the ſaid Multiplican plicand by $-ab$ (the ſecond Member of the Multiplier) and it produceth $-aadb-abbc$; which two Quantities being joined together, give $ddaa+dabc-aadb-abbc$ for the Product requir'd: as you may ſee in the Operation.

Example 5. What is the Product of $a+b-c$ multiplied by $a+b-c$?

$$\begin{array}{lrrrrrr}
\text{Multiply} & a & +b & -c & & & \\
\text{by} & a & +b & -c & & & \\
\hline
 & aa & +ab & -ac & & & \\
 & & +ab & & +bb & -bc & \\
 & & & -ac & & -bc & +cc \\
\hline
\text{Product} & aa & +2ab & -2ac & +bb & -2bc & +cc \\
\hline
\end{array}$$

Firſt, Multiply each Member of the Multiplicand by a (the firſt Member of the Multiplier) and it produceth $aa+ab-ac$; then multiply each ſaid Member in the Multiplicand by b (the 2d Member of the Multiplier) and it produceth $ab+bb-bc$; then multiply each Member of the ſaid Multiplicand by $-c$ (the third and laſt Member of the Multiplier) and the Product is $-ac-bc+cc$; which ſaid three Products being joined together, according to the Rules of Algebraical Addition, will give $aa+2ab-2ac+bb-2bc+cc$, which is the Square of $a+b-c$, or Product required; as appears by the whole Operation.

And if there are natural Numbers prefixed to any of the Compound Quantities, the Operation will not be different from the foregoing Examples of this Rule, regard being had to the fourth

Rul

ule of this Chapter; as will appear by the fol-
owing Example.

Example 6. What is the Product of $3b+2c$ ultiplied by $4b-3c$?

Multiplicand $3b+2c$
Multiplier $4b-3c$

$12bb+8bc$
$-9bc-6cc$

Product $12bb-bc-6cc$

First, by the fourth Rule, multiply each Mem-
er of $(3b+2c)$ the Multiplicand by $4b$, and the
oduct is $12\,bb+8bc$; then multiply the ſaid Mul-
plicand by $-3c$, and the Product is $-9bc-6cc$,
hich being added to $12bb+8bc$, the Sum will be
$bb-bc-6cc$, which is the Product required.

Example 7. What is the Product of $2a+2e-8$ ltiplied by $2a-5$?

Multiply $2a+2e-8$
by $2a-5$

$4aa+4ae-16a$
$-10a-10e+40$

Product $4aa+4ae-26a-10e+40$

First, $2a+2e-8$ being multiplied by 2a, pro-
eth $4aa+4ae-16a$; for $2a\times2a=4aa$, and
$2a\times2$

$2a\times2e=4ae$, and $2a\times-8=-16a$, which is marked with $-$, becauſe the Signs of the Factors are unlike, *viz.* -8 and $+2a$. Secondly, $2a+2e-8$ being multiplied by -5, produceth $-10a-10e+40$; for $-5\times+2a=-10a$, and $-5\times+2e=-10e$, and $-5\times-8=+40$, all which Quantities being joined together by the Rules of Algebraical Addition will give $+4aa+4ae-26a-10e+40$, which is the Product required. See the Work.

More Examples in Multiplication of Compound Algebraical Integers.

$$\begin{array}{l|l|l}
\text{Multiplicand} & 3b+2c & 3cd+3b-6 \\
\text{Multiplier} & 3b & 8 \\
\hline
\text{Product} & 9bb+6bc & 24cd+24b-48 \\
\hline
\end{array}$$

$$\begin{array}{ll}
\text{Multiplicand} & 2ab+bcd \\
\text{Multiplier} & 3ab-bcd \\
\hline
 & 6aabb+3bbacd \\
 & \quad -2bbacd-bbccdd \\
\hline
\text{Product} & 6aabb+bbacd-bbccdd \\
\hline
\end{array}$$

$$\begin{array}{ll}
\text{Mult.} & 3a+bb-16 \\
\text{Mult.} & 4a-6c-4 \\
\hline
 & 12aa+4abb-64a \\
 & \quad -18ac-6bbc+96c \\
 & \qquad -12a-4bb+64 \\
\hline
\text{Product} & 12aa+4abb-76a-18ac-6bbc+96c-4bb+64 \\
\hline
\end{array}$$

$$\begin{array}{ll}
\text{ltiplicand} & 3ab+4cc-2a \\
\text{ltiplier} & 2ab-3cc-a \\
\hline
 & 6aabb+8ccab-4aab \\
 & \qquad -9abcc-12c^4+6acc \\
 & \qquad\qquad -3aab-4acc+2aa \\
\hline
\text{luct} & 6aabb-abcc-7aab-12c^4+2acc+2aa \\
\hline
\end{array}$$

CHAP. V.

Diviſion of Algebraick Integers.

IN Diviſion in Algebraical (as in Diviſion
in Common Arithmetick) there are two
antities given to find out a third; which
antity ſought is called the Quote, or Quo-
ıt; and of the Quantities given, that which
ɔ be divided is called the Dividend, and the
ıntity by which it is to be divided is called
Diviſor.

I. When it is required to divide one Number
Quantity by another, if you place the Divi-
d for the Numerator, and the Diviſor for the
ıominator of a Fraction, that Fraction ſo
ıpoſed *is equal* to the Quotient that would
ariſe

Quantity is to be divided by another, Quantity that is the Dividend, for the Nu of a Fraction, and the Quantity that is t ſor, for the Denominator.

So if it were required to divide the Q by the Quantity a, I would place them t $\frac{b}{a}$, which ſignifieth the Quotient of b by a.

In like manner, if it were required t abe by cd, the Quotient would be $\frac{abe}{cd}$; an were to be divided by $3cd$, the Quotient w $\frac{3ac}{3cd}$; and, if bcd were to be divided by Quotient would be $\frac{bcd}{7}$.

The ſame is to be obſerved in Diviſion pound Algebraical Integers; for if it we red to divide $a+b$ by c, the Quotient w $\frac{a+b}{}$; and if $5b$ were to be divided by

Examples of Diviſion according to the foregoing Rule.

idend	cd	$3dce$	$3aa+e-f$
iſor	fg	$15b$	$a+b+c$
otient	$\frac{cd}{fg}$	$\frac{3dce}{15b}$	$\frac{3aa+e-f}{a+b+c}$
idend	a^4b	$15bb$	$6aa$
iſor	$3b^3$	$7cc$	3
otient	$\frac{a^4b}{3b^3}$	$\frac{15bb}{7cc}$	$\frac{6aa}{3}$

I. When in any Quotient that is expreſſed ac-
ding to the foregoing Rule, there are the
e Letter or Letters repeated in every Part or
mber of the Numerator and Denominator, you
cancel ſuch Letter or Letters; but be ſure
what you cancel in one Part, you cancel the
ſame in all the reſt: ſo ſhall this new Quan-
be a true Quotient, equivalent to what it was
ore the ſaid Letters were cancelled.

xample. What is the Quotient of bd di-
d by b? According to the foregoing
le, the Quotient is $\frac{bd}{b}$; but becauſe the Letter
found both in the Numerator and in the
ominator, therefore cancel b in both of
them,

them, and then you will find the Quoti d, for $\frac{bd}{b} = d$.

Again, let it be required to divide by acd, the Quotient is $\frac{ab + ad}{acd}$; and bec Letter a is found in every Member of merator and Denominator, caſt it out one, and then you will have $\frac{b + d}{cd}$ for th tient.

Likewiſe if you were to divide $ab +$ by $abd + abf$, the Quotient you will fin $\frac{ab + abc + abe}{abd + abf}$ which being contracted by ca ab in each Member of the Numerator an minator, there will be found $\frac{c + e}{d + f}$ for t tient.

And $bb + b$ being to be divided by b, t tient will be $\frac{bb + b}{b}$, and by cancelling b i Part, there will be $b + 1$ for the Quotient viated for $\frac{bb + b}{b} = \frac{1bb + 1b}{1b}$; and by cancelli every Part, there will be $\frac{1b + 1}{1}$ and $\frac{1b + 1}{1} =$

The ſame is to be obſerved whether t be + or —: So if it be required to di + cbe by $bde - bce$, the Quotient will be And becauſe be is found in each Quantity, cel it, and the Quotient contracted or abl ted will be found to be $\frac{a + c}{d + c}$.

Examples of Contractions or Abbreviations in viſion of Algebraick Integers, according to the going Rule.

vidend	aaa	$ad-ak+a$	$bcd+crd$
viſor	aa	a	$cgd-cmd$
otient	$\frac{aaa}{aa}$	$\frac{ad-ak+a}{a}$	$\frac{bcd+crd}{cgd-cmd}$
otient tracted	a	$d-k+1$	$\frac{b+r}{g-m}$

If when it is required to divide a ſimple mpound Algebraick Quantity by a ſimple tity, there be prefixed to every Member a er, or Numbers, that may be divided by ther Number without any Remainder, then d of the given prefixed Numbers, prefix the ent of each of the ſaid Numbers divided ſaid common Meaſurer, not neglecting to any Letter that may be found in each part Numerator and Denominator, according foregoing third Rule. As for Example.

ide $16bc$ by $4b$. Here, according to regoing Rule, the Quotient is $\frac{16bc}{4b}$; but e the prefixed Numbers 16 and 4 will of 4 for a common Meaſure, therefore de them both by 4, and the Quotients . and 1, which I prefix to the given *ities inſtead of* 16 and 4, and then the

Quotient will be $\frac{4bc}{1b}$, or $\frac{4bc}{b}$; and becauſe b contained both in the Numerator and Den minator, cancel it; ſo have you $\frac{4c}{1}=4c$ for . Quotient required.

Moreover, $15abc +- 12bd$ being given to be vided by $3bg$, the Quotient will be found to $\frac{5ac +- 4d}{g}$.

For, I firſt diſcover that the prefixed Numb 15, 12 and 3, have 3 for their common Meaſu by which they being ſeverally divided, give 5, and 1, which being prefixed to the ſaid Qu tities (after b, which is found in every Quanti is cancelled) there will be $\frac{5ac +- 4d}{g}$ for the tr Quotient required.

More Examples of Contraction in Diviſion, accor ing to the two laſt Rules.

Divide by	$8bc$ $4b$	$28cd$ $16d$	$32abc$ $12acd$
Quotient	$\frac{8bc}{4b}$	$\frac{28cd}{16d}$	$\frac{32abc}{12acd}$
Quotient contracted	$2c$	$\frac{7c}{4}$	$\frac{8b}{3d}$

V. When, in compound Quantities, one or mo Letter or Letters is repeated in every Memb then will the remaining Letters in each Quanti

even

venly divide the ſaid compound Quantity without any Remainder, and the Quotient will be he Letter or Letters repeated in each Number s aforeſaid. As for Example

1. What is the Quotient of $ba+ca$ divided by $+c$? Here it is evident that the Quotient will e a; for Proof whereof, take the Diviſor $b+c$, nd multiply by the Quotient a, according to he fifth Rule of the 4th Chapter, and the Product will be $ba+ca$ equal to the Dividend.

2. Likewiſe, if it were required to divide $bad+cad$ by $b+c$, the Quotient will be found to be ad.

3. Alſo by the ſame reaſon, if you divide $2ab-2acd-2adb$ by $b-cd-db$, the Quotient will be $2a$; and if you divide the ſame Dividend by $2b-2cd-2db$, the Quotient will be a.

More Examples of the like Nature.

Dividend	$6ab+2adc$	$4bc+3cd-c$
Diviſor	$6b+2dc$	$4b+3d-1$
Quotient	a	c

The reaſon why -1 is the laſt Number of the aſt Example is, becauſe $-c$, or $-1c$, is the laſt Number of the Dividend; for according to the 2th Rule of the firſt Chapter, when a Quantity ath no Number prefixed to it, it is ſuppoſed to ave the Number 1 before it.

And here note, that as in the Multiplication of lgebraick Integers + by +, and − by −, produceth +, and + by − produceth −, ſo in Divi- on, if you divide + by +, or − by −, the

Sign of the Quote will be +; but if you divide + by −, or − by +, the Sign of the Quote will be −; ſo if $3ab$ be divided by $3a$, the Quote will be b, or $+b$; and if $-3ab$ be divided by $-3a$, the Quote will be $+b$; for if you multiply $-3a$ by $+b$, the Product will be $-3ab$, by the third Rule of the fourth Chapter foregoing. Alſo if you divide $+3ab$ by $-3a$, or $-3ab$ by $+3a$, the Quotient will be $-b$; for $+3a$ being multiplied by $-b$, produceth $-3ab$; and $-3a$ being multiplied by $-b$, produceth $+3ab$.

VI. From a due Conſideration of the manner of operating the Examples of the laſt Rule, a way may be diſcovered to divide a Compound Quantity by a Simple or Compound Quantity, and to find out the true Quotient, when it likewiſe will be a Compound Quantity; the Practice of which will be made plain by the following Examples.

Example 1. Let it be required to divide $ba+ca$ by a. Having placed the Dividend and Diviſor as is uſual in Vulgar Arithmetick, and as you ſee in the following Operation;

$$\begin{array}{l} a)\,ba+ca\,(b+c \\ \quad\;\; -ba \\ \hline \quad\;\; +ca \\ \quad\;\; -ca \\ \hline \quad\;\; (0) \end{array}$$

then do I ſeek how often a is contained in ba (the firſt Member of the Dividend) and the Anſwer is b times, therefore I put b in the Quotient, and thereby I multiply (a) the Diviſor, and the Product is $+ba$, which muſt be ſubtracted from ba in the Dividend; and therefore I change its Sign into $-ba$, by the firſt Rule of the 3d Chapter, and there remaineth 0, then do I bring down $+ca$ the next Member of the Dividend, and divide it by a, and the Quotient is $+c$, by which I again multiply (a) the Diviſor, and the Product is ca, which ſubtracted from ca, there remaineth 0: and ſo the Work of Diviſion is ended, and I find the Quotient of $ba+ca$, divided by a, to be $b+c$; for Proof whereof, if you multiply $b+c$ by a (the Diviſor) the Product will be $ba+ca$ equal to the given Dividend.

Example 2. Let it be required to divide $ba+ca+be+ce$ by $b+c$.

Having diſpoſed of the Dividend and Diviſor in order to the Work, with a crooked Line, behind which to place the Quotient, as in Common Arithmetick, then, firſt, I ſeek how often b (the firſt Member of the Diviſor) is contained in ba, (the firſt Member of the Dividend) and there ariſeth a, which I put in the Quotient, and thereby multiply each Member of the Diviſor, *viz.* $b+c$, and the Product is $ba+ca$, which place under the two firſt Quantities of the Dividend towards the left-hand, *viz.* under $ba+ca$, and by the firſt Rule of the ſecond Chapter, ſubtract it therefrom, ſo will the Remainder be 0; to which I bring down the remaining Part of the Dividend, *viz.* $be+ce$, and divide be by b, and there ariſeth in the Quotient e, by which I multiply the whole Diviſor $b+c$, and the Product is $be+ce$, which ſubtracted from the Dividend

$be+ce$, the Remainder is 0. See the
Work, as followeth.

Quotient.

$$\begin{array}{l} b+c)\ ba+ca+be+ce\ (a+e \\ \quad\quad\ \ ba+ca \\ \quad\quad\ \ \overline{0\quad 0+be+ce} \\ \quad\quad\quad\quad\quad\ be+ce \\ \quad\quad\quad\quad\quad\ \overline{0\quad\ 0} \end{array}$$

So that the Quotient is $a+e$. Now to
the Work, multiply the Diviſor, $b+c$, b
Quotient ($a+e$) according to the fifth Ru
the fourth Chapter, and the Product you wil
to be $ba+ca+be+ce$, which is equal to the
Dividend; and therefore I conclude the O
tion to be truly performed.

Example 3. In like manner, if you
$ba+bd+ca+cd-ae-de$ by $a+d$, the Qu
will be found to be $b+c-e$, according to th
lowing Work:

Quo

$$\begin{array}{l} a+d)\ ba+bd+ca+cd-ae-de\ (b+c- \\ \quad\quad\ \ ba+bd \\ \quad\quad\ \ \overline{0\quad 0+ca+cd} \\ \quad\quad\quad\quad\ \ +ca+cd \\ \quad\quad\ \ \overline{\quad\quad\ 0\quad 0-ae-de} \\ \quad\quad\quad\quad\quad\quad\ -ae-de \\ \quad\quad\quad\quad\quad\quad\ \overline{\ 0\quad\ 0\ } \end{array}$$

The Work of the laſt Example explain'd.

In the foregoing Example, firſt I divide *ba* (the
rſt Member (of the Dividend) by *a* (the firſt
uantity or Member of the Diviſor) and there a-
ſeth *b* in the Quotient, which is $+b$ (becauſe
he Signs of the Dividend and Diviſor are +) and
hereby I multiply the Diviſor $a+d$, and the Pro-
uct is $ba+bd$, which I place under the two firſt
embers of the Dividend, as you ſee in the Work,
nd ſubtract it therefrom, and the Remainder is
, to which I bring down the two next Quanti-
ies, *viz.* $+ca+cd$.

Then do I divide $+ca$ by a, and there ariſeth
n the Quotient $+c$, (becauſe the Dividend and
iviſor are both ſigned +) by which I multiply
he ſaid Diviſor, and the Product is $+ca+cd$,
hich I place under the Dividend, and ſubtract
t therefrom, and there remaineth 0; to which I
nnex the two next and laſt Members of the Divi-
end, *viz.* $-ae-de$, and divide $-ae$ by $+a$, and
he Quotient is $-e$, (becauſe the Signs of the Di-
idend and Diviſor are different, *viz.* the one +,
nd the other —) and thereby I multiply the
hole Diviſor, and the Product is $-ae-de$, which
btracted from ($-ae-de$) the Dividend, the Re-
ainder is 0; and ſo the Work is finiſhed, and
he Quotient ariſing by this Diviſion is $b+c-e$;
s you may prove at your leiſure.

If the Quantities or Members of the Dividend
f the foregoing Example are not placed in the
me Order that is there expreſſed, the Effect of
he Operation will be the ſame; as you may ſee
y the *following* Work:

Diviſor Dividend Quotient

$a+d)\ ba+ca-ae+bd+cd-de\ (b+c-e$

$ba+bd$

$ca-ae$

$ca+cd$

$-ae-de$

$-ae-de$

$0\quad 0$

Firſt, I divide ba by a, and the Quote is b, by which I multiply the Diviſor $(a+d)$ and the Product is $ba+bd$, which I ſubtract from $ba+ca$, and the Remainder is (by the Rule of the third Chapter) $ba+ca-ba-bd$, which being contracted by the Rules of Addition, is $ca-bd$, (for $+ba$ and $-ba$ expunge each other) then to this Remainder do I bring down the two next Quantities of the Dividend, *viz.* $-ae+bd$, which being annex'd to the ſaid Remainder $ca-bd$, it then makes (for a new Dividual) $ca-bd-ae+bd$; but $-bd$ and $+bd$ deſtroy each other, and therefore the Dividual contracted is $ca-ae$, which I divide by $a+d$, as before, and the Quotient is $+c$, by which I multiply the Diviſor, and the Product is $ca+cd$, which being ſubtracted from the ſaid Dividual $ca-ae$, the Remainder is $ca-ae-ca-cd$, which being contracted, is $-ae-cd$, to which I join the two next Quantities in the Dividend, *viz.* $+cd-de$, and it makes $-ae-cd+cd-de=-ae-de$ (for $-cd$ and $+cd$ deſtroy each other) for a new Dividual, which I divide by the ſaid Diviſor $a+d$, and the Quote is $-e$, by which I multiply the Di-ſor, and the Product is $-ae-de$, which ſubracted from $-ae-de$ (the Dividual,) the Reainder is $-ae-de+ae+de=o$, and ſo the Vork is finiſhed, and I find the Quotient to be $+c-e$, as before.

Note.

But here note, by the way, that it doth not lways fall out that you are to divide he firſt Member of the Dividual by he firſt of the Diviſor, but by ſome ther Member which you can diſcover will do the Vork without making a Fraction: as in the following Example.

Example 4. Let it be required to divide $aa-ee$ y $a+e$.

Firſt, I divide aa by a, and there ariſeth in the uotient a, by which I multiply the Diviſor $a+e$, nd the Product is $aa+ae$, which ſubtracted from e Dividend ($aa-ee$,) the Remainder is $-ee$ $-ae$ for a Dividual; and then do I not ſeek how ften a is contained in ee, for then the Anſwer ould be a Fraction, but I divide ee by its corſpondent Diviſor $+e$, and there ariſeth $-e$, be written in the Quotient next after a, but ot $+e$, becauſe — divided by + quotes —; then multiply the whole Diviſor $a+e$ by $-e$, and e Product is $-ae-ee$, which ſubtracted from e ſaid Dividual $-ee-ae$, the Remainder is o, is the Work ended, and I find the Quotient to e $a-e$. See the Operation.

$$\begin{array}{l} a+e)\ aa-ee\ (a-e \\ \qquad\quad aa+ae \\ \hline \qquad\quad -ae-ee \\ \qquad\quad -ae-ee \\ \hline \qquad\qquad 0\quad 0 \end{array}$$

Example 5. If it were required to divide $abd+baa+bbd$ by $aa+bd$, the Quotient be found to be $a+b$, as appears by the W

$$\begin{array}{l} aa+bd)\ aaa+abd+baa+bbd\ (a+b \\ \qquad\qquad aaa+abd \\ \hline \qquad\qquad 0 \qquad 0+baa+bbd \\ \qquad\qquad\qquad\quad baa+bbd \\ \hline \qquad\qquad\qquad\qquad 0 \quad 0 \end{array}$$

Example 6. If $aaa-abb+abd+baa-bb$ be divided by $a+b$, the Quote will be $aa-b$ as by the Operation:

$$\begin{array}{l} a+b)\ aaa-abb+abd+baa-bbb+bbd\ (aa- \\ \qquad\ aaa+baa \\ \hline \qquad\qquad -abb+abd \\ \qquad\qquad -abb-bbb \\ \hline \qquad\qquad\quad +abd+bbd \\ \qquad\qquad\quad +abd+bbd \\ \hline \qquad\qquad\qquad 0 \quad 0 \end{array}$$

Example 7. Let it be required to divide $abbc+9bbcc+24bcc+24abc+16cc$ by $3\,bc+4c$, e Quotient will be found to be $6ab+3bc+4c$. e the following Operation:

iviſor . Dividend Quotient

$3bc+4c)\ 18abbc+9bbcc+24bcc+24abc+16cc\ (6ab+3bc+4c$

$18abbc+24abc$

$+9bbcc+24bcc$

$+9bbcc+12bcc$

$+12bcc+16cc$

$+12bcc+16cc$

$0 \quad 0$

When Algebraical Diviſion, according to the ules before delivered, will not exactly perform e Work without any Remainder, then you may ce the Dividend and Diviſor Fraction-wiſe, ich is indeed the moſt general Practice amongſt gebraiſts; or elſe proceed in Diviſion as far as u can, by the preceding Method, and then ce the Remainder for a Numerator over the viſor; as in the following Example, where $-bb+ac$ is divided by $a+b$, and the Quotient $a-b+\frac{ac}{a+b}$, the Remainder being $+ac$.

$a+b)\ aa-bb+ac\ (a-b\frac{ac}{a+b}$

$aa+ab$

$-ab-bb+ac$

$-ba-bb$

$0 \quad 0+ac$

CHAP. VI.

The Doctrine of Algebraical Fractions.

And FIRST,

Of Reduction.

I. HE that intends a conſiderable Proficiency in this myſterious Art, muſt be very well acquainted with the Doctrine of Vulgar Fractions, a mean Knowledge therein not being ſufficient; for all Operations whatſoever in Algebraick Fractions have their dependance thereupon. Being wrought in every reſpect as Vulgar Fractions, they are to be known by the Rules contained in the ſeveral Chapters foregoing; and there are very few Queſtions ſolved algebraically, but what have one or more Fractions concern'd in their Operation.

To reduce Fractions, having unequal Denominators, to Fractions of the ſame Value, having a common Denominator.

II. When you would reduce Algebraical Fractions to a common Denominator, multiply the Numerator of the firſt Fraction into the Denominator or Denominators of the reſt, ſo ſhall the Product be a Numerator equal to the Numerator of the firſt Fraction; likewiſe multiply the Numerators of the ſecond, third, *&c.* Fractions into all the Denominators except its own, and the ſeveral Products ſhall be ſo many new Numerators; then multiply all the Denominators continually, ſo ſhall the Product be a common Denominator to all the Numerators found out, as before.

Example 1. Reduce $\frac{a}{b}$ and $\frac{d}{c}$ to a common Denominator. Multiply the Numerator a (of the firſt Fraction) into the Denominator (c) of the ſecond Fraction, and the Prodnct is ac, for a Numerator $=a$; then multiply (d) the Numerator of the ſecond Fraction into (b) the Denominator of the firſt, and the Product is db, for a Numerator $=d$; then multiply the Denominators together, *viz.* b into c, and the Product bc, is the Denominator common to both the Numerators, ſo will the two new Fractions be $\frac{ac}{bc}$ and $\frac{bd}{bc}$, for $\frac{ac}{bc}=\frac{a}{b}$ and $\frac{db}{bc}=\frac{d}{c}$.

Example 2. What Fractions are $=\frac{a}{c}$, $\frac{b}{a}$ and $\frac{c}{d}$ having an equal or common Denominator? Facit $\frac{aad}{cad}$ $\frac{bcd}{cad}$ and $\frac{cca}{cad}$, for $a\times a\times d=aad$, $=$ the Numerator a, and $b\times c\times d=bcd=$ the Numerator , and $c\times c\times a=cca=$ the Numerator c, and $\times a\times d=cad$, which is the common Denominator.

To reduce an Algebraical Fraction to its loweſt Terms Equivalent.

III. When in the Numerator and Denominator of an Algebraical Fraction, the ſame Letter or Letters are contained, then cancel the ſame in both, and if there be any Numbers prefix'd, if you can diſcover any Number that will divide them both, without any Remainder, then prefix thoſe Quotients inſtead of the Numbers prefix'd before; ſo ſhall this new Fraction be of the ſame Value with the Fraction propoſed.

So will $\frac{cbd}{cbb}$ be reduced to $\frac{d}{b}$ by cancelling *cb* in the Numerator and Denominator.

Alſo $\frac{24bde+32bda}{8bdd}$ by being reduced to its loweſt Terms, will be $\frac{3e+4a}{d}$ by cancelling *bd* in every part, and dividing the prefixed Numbers by 8. More Examples follow.

$$\frac{cbd}{cde}=\frac{b}{e} \quad \Big| \quad \frac{16rbsd}{20rsgq}=\frac{4bd}{5gq}$$

$$1+\frac{ghm}{gh}=1+m \quad \Big| \quad \frac{24adc}{12ade+16adb}=\frac{6c}{3e+4b}$$

Or if you can (in a Compound Algebraical Fraction) diſcover a Quantity that will divide the Numerator and Denominator without any Remainder, (according to the ſixth Rule of the fifth Chapter) then ſhall the Quotients be a

ne

w Numerator and a new Denominator, equal the Fraction in its given Terms: as in the folwing Examples.

$$\frac{ba+da}{b+d}=a \quad\Big|\quad \frac{aa+2ae+ee}{a+e}=a+e$$

$$\frac{+2ae+ee}{a-e}=ae \quad\Big|\quad \frac{aaaa-bbbb}{aa-bb}=aa-bb$$

To reduce an Integral Quantity to an Algebraical Fraction.

IV. Multiply the given Quantity by the intended Denominator, ſo ſhall the Product be the Nuierator required: as in the following Examples.

Let it be required to reduce the Quantity b to Fraction, having ad for its Denominator. To o which, I multiply the given Quantity b by ad, ńd the Product is the Numerator, *viz.* bad, ſo iall $\frac{bad}{ad}$ be the Fraction required; for $\frac{bad}{ad}=b$.

Alſo, if it were required to reduce the Quantity to a Fraction, whoſe Denominator ſhould be $+c$, it would be $\frac{be+ce}{b+c}$.

V. If it be required to reduce a mixt Quantity) a Fraction, multiply the integral Quantity by ie Denominator of the fractional Part, and join ie Product to the Numerator of the fractional art, ſo ſhall the Sum be the Numerator: as for *xample.*

Reduc

Reduce $a+b+\frac{c}{d}$ to an improper Fract. Multiply the integral Part $a+b$ by the Denominator d, and the Product is $da+db$, which being added to the Numerator c, makes $da+db+c$ for the Numerator; to which placing d for a Denominator, it gives $\frac{da+db+c}{d}=a+b+\frac{c}{d}$ for the Anſwer.

VI. When you are to expreſs an Algebraick Integer Fraction-wiſe, without an aſſigned Denominator, then make the given Quantity the Numerator, and 1 the Denominator.

So will ab be $\frac{ab}{1}$ and cd will be $\frac{cd}{1}$ and $a+b$ will be $\frac{a+b}{1}$, &c.

Theſe things are ſo plain, that they need no farther Explanation by Examples.

CHAP. VII.

Of Addition and Subtraction of Algebraical Fractions.

I. WHEN the Fractions given to be added together, have an equal or common Denominator, add the Numerators together, and place their Sum for a Numerator over the common Denominator, which new Fraction ſhall be the Sum of the given Fractions; but if they hav e not a common Denominator, reduce them the ſecond Rule of the ſixth Chapter, and then ceed as before.

Example 1. What is the Sum of $\frac{a}{b}$ and $\frac{ac}{b}$? Fa- $\frac{a+ac}{b}$. For the Sum of the Numerators, *viz.* a d ac, is $a+ac$, which placed over the Denomi- or b, gives $\frac{a+ac}{b}$ for the Sum required.

Example 2. So alſo the Sum of $\frac{abc}{f}$ and $\frac{dg}{f}$ will $\frac{abc+dg}{f}$.

Example 3. And the Sum of $\frac{a+b-c}{d}$ — $\frac{a+b+c}{d}$ — d $\frac{2a-2b}{d}$ will be found to be $\frac{4a}{d}$; for the Sum of e Numerators is $a+b-c+a+b+c+2a-2b$ $4a$.

Example 4. What is the Sum of $\frac{a}{b}$ and $\frac{c}{d}$? cit $\frac{ad+cb}{bd}$. The given Fractions being reduced to ommon Denominator, by the ſecond Rule of the th Chapter, are $\frac{ad}{bd}$ and $\frac{cb}{bd}$ whoſe Sum is $\frac{ad+cb}{bd}$.

Example 5. What is the Sum of $\frac{a}{b}$ $\frac{b}{c}$ and $\frac{3f}{d}$? cit $\frac{acd+bbd+3fbc}{bcd}$.

II. When it is required to gather mixt Quantities into one Sum, then add the fractional Parts together by the foregoing Rule, and likewiſe bring the Integers into one Sum, and the Sum of theſe two Sums will be the Sum required.

Example.

What is the Sum of $bb+\frac{ab}{c}$ and $cd+\frac{a+b}{d}$?

The Sum of the Fractions being added, by the foregoing Rule, is $\Big\}\ \frac{abd+ca+cb}{cd}$

To which Sum if you add the integral Parts of the propounded mixed Quantities, the Sum required will be $\Big\}\ bb+cd+\frac{abd+ca+cb}{cd}$

Subtraction of Algebraical Fractions.

III. If the two given Fractions have not a common Denominator, then reduce them to ſuch (by the ſecond Rule of the 6th Chapter) then (by the Rule of the third Chapter) ſubtract the Numerator of the Fraction to be ſubtracted from the Numerator of the other Fraction, and place the Remainder for a Numerator over the common Denominator; which new Fraction ſhall be the Remainder ſought: as in the following Examples.

If you would ſubtract $\frac{ab}{e}$ from $\frac{bc}{e}$, take the Numerator *ab* from the Numerator *bc*, and the Remainder is *bc*—*ab*, which being placed over the De-ıominator *e*, it will give $\frac{bc-ab}{e}$ for the Re-ıder, or Difference ſought.

lſo, let it be required to ſubtract $\frac{ab+c-18}{b+c}$ ı $\frac{bc+5c-24}{b+c}$.

'he Difference of the nerators of the gi- Fractions, is $\Big\}$ $bc+5c-24-ab+c-18$ $=bc+4c-6-ab$

hich Remainder or Difference ıg made a Numerator to the mon Denominator, will give Difference ſought, which is $\Big\}$ $\frac{bc+4c-6-ab}{b+c}$

nd if it were required to ſubtract $a+\frac{cd-bc}{c}$ ı $aa+\frac{ab}{c}$, the given mixt Quantities will (by fifth Rule of the ſixth Chapter) be reduced $\frac{+ab}{c}$ and $\frac{ae+cd-be}{c}$.

'hich will be redu- to theſe Fractions of ;ame Value, having ımmon Denomina- *viz.* $\Big\}$ $\frac{caae+abe}{ce}$ and $\frac{cae+ccd-ccb}{ce}$

nd if from the Numera- *caae+abe* you ſubtract Numerator *cae+ccd—ccb* e will be given for the *ainder ſought, viz.* $\Big\}$ $\frac{caae+abe-cae-ccd+ccb}{ce}$.

In

In like manner, if from d it be required to ſubtract $\frac{a+c}{b-c}$; firſt by the fourth Rule of the ſixth Chapter, reduce the Quantity a to the improper fractional Quantity $\frac{ab-ac}{b-c}$, and therefrom ſubtract the given Fraction $\frac{a+c}{b-c}$, ſo will you have the Remainder ſought, which is $\frac{ab-ac-a-c}{b-c}$.

CHAP. VIII.

Multiplication and Diviſion of Algebraical Fractions.

I. WHEN it is required to multiply two Algebraical Fractions the one by the other, the Work is the ſame as in Vulgar Fractions; for if you multiply the Numerators of the given Fractions together, and likewiſe their Denominators together, and place their reſpective Products for a new Numerator, and a new Denominator, that new Fraction ſhall be the Product required.

Example 1. What is the Product of $\frac{9a}{b}$ multiplied by $\frac{ab}{c}$? Facit $\frac{9aab}{cb}$; for $9a\times ab=9aab$, which is the Numerator, and $c\times b=cb$ the Denominator.

Example 2. What is the Product of $\frac{3a+2c}{2cb}$ multiplied by $\frac{4cd}{b+c}$? Facit $\frac{12cda+8ccd}{2cbb+2ccb}$? for $4cd\times 3a+2c=12acd+8ccd$, which is the Numerator.

Example 3. What is the Product of $a\frac{bc}{d}$ multiplied by $8c+\frac{ad}{b}$? Facit $\frac{8dacb+8ccbb+aadd+abdc}{db}$; for $a+\frac{bc}{d}=\frac{da+bc}{d}$ and $8c+\frac{ad}{b}=\frac{8cb+ad}{b}$ by the Rule of the 6th Chapter, and $\frac{da+bc}{d}\times\frac{8cb+ad}{b}=$ $\frac{dacb+8ccbb+aadd+abdc}{db}$ which is the Prod. requir'd.

Example 4. What is the Product of ab multiplied by $\frac{a+b}{c}$? Facit $\frac{aab+abb}{c}$; for $ab=\frac{ab}{1}$, and $\frac{ab}{1}$; $\frac{a+b}{c}=\frac{aab+abb}{c}$, which is the Product requir'd.

II. If it ſo chance that you have a Fraction to be multiplied by an Integer, that is equal to the Denominator *of the Fraction,* then take the Numerator *for the Product.*

Example. What is the Product of $\frac{aa-2ae+ee}{a-c}$ being multiplied by $a-c$? Facit $aa-2ae+ee$. The reaſon of which is plain; for the Numerator being multiplied by the Integer, and the ſame Integer being put as a Denominator to the Product, the Quotient ariſing by the Diviſion of the Numerator by the ſaid Denominator, will be equal to the Numerator of the given Fraction; ſo $\frac{a}{b}\times b=\frac{ab}{b}=a$.

III. When it is requir'd to divide one Algebraical Fraction by another, if they have a common Denominator, cancel the Denominator, and divide the Numerator of the Dividend by that of the Diviſor, ſo ſhall that Quote be the Quotient ſought.

So if it were required to divide $\frac{abc}{d}$ by $\frac{bcc}{d}$, the Quotient will be found to be $\frac{abc}{bcc}=\frac{a}{c}$; for having caſt away the common Denominator d, and divided abc, the Numerator of the Dividend, by bcc, the Numerator of the Diviſor, the Quotient will be $\frac{abc}{bcc}$, which is $\frac{a}{c}$, by cancelling bc in the Numerator and Denominator.

IV. When the given Algebraical Fractions have not a common Denominator, then multiply the Denominator of the Diviſor into the Numerator of the Dividend, and the Product is a new Numerator; alſo multiply the Numerator of the Diviſor into the Denominator of the Dividend, and the Product is a new Denominator; which re

Fract

:ion is the Quotient ſought: And this is a ge- Rule in all Caſes whatſoever, and is the with Diviſion in Vulgar Fractions, only ing to the Algebraick Rules.

cample. What is the Quotient of $\frac{a}{b}$ being di- l by $\frac{a}{c}$? Facit $\frac{ca}{ba}=\frac{c}{b}$, for $c\times a=ca$, (the Numerator) and $a\times b=ba$, the new Deno- tor.

kewiſe, if it were required to divide $\frac{3a+b}{e}$, $\frac{b}{b}$, the Quotient would be $\frac{3aa+4ab+bb}{2bbe}$; for the ıerator of the Dividend is $3a+b$, and the De- nator of the Diviſor is $a+b$, and $\overline{a+b}\times b=3aa+4ab+bb$, which is the Numerator; :he Numerator of the Diviſor is $2bb$, and the ɔminator of the Dividend is e, and $2bb\times e=$ which is the Denominator. The like is to ɔſerv'd in all Caſes, both in Multiplication, Diviſion of Algebraical Fractions.

CHAP. IX.

he Rule of Three in Algebraical Quantities.

THE Rule of Three in Algebraical Quantities repreſented by Letters, (whether it irect or Inverſe) differs not from the Rule of Three in Vulgar Arithmetick, reſpect l had to the Rules of Algebraical Multiplic and Diviſion, before delivered in this Book (in a direct Proportion) if you multiply tl cond Term by the third, and divide the Pr thereof by the firſt, the Quotient will be th Quantity ſought in Proportion.

Example 1. If b gives c, what will d g Facit $\frac{cd}{b}$.

In this Example, the ſecond and third Qu ties are c and d, which being multiplied t ther, produce cd by the third Rule of the fo Chapter, which being divided by (b) the Quantity, the Quotient is $\frac{cd}{b}$, which is the fo Proportion ſought: for,

$$b : c :: d : \frac{cd}{b}$$

Which may be proved according to the l of the Rule of Three Direct, laid down i Tenth Chapter of my Vulgar Arithmet For,

The Product of the ſecond and third Ter cd. And

The Product of the firſt and fourth Ter $\frac{bcd}{b}$.

And by the 3d Rule of the 6th Chapter, $\frac{bcd}{b}$ (the firſt Term) which was to be proved.

T

ample 2. If $b+c$ require d, what will $d+e$
ire? Facit $\frac{dd+de}{b+c}$: For,

$$b+c : d :: d+e : \frac{dd+de}{b+c}$$

ample 3. If 12 require 36, what will $4ab$
ire? Facit $\frac{144ab}{12}=12ab$: For,

$$12 : 36 :: 4ab : \frac{144ab}{12}=12ab$$

Nor will the Operation be different from
ormer, if any of the three given Quantities
Fraction, or if they be all Fractions, obſer-
the Rules of Multiplication and Diviſion in
braick Quantities; or when any of the given
ns is a mix'd Quantity, let it be reduced to
Form of a Fraction, by multiplying the Inte-
Part by the Denominator, and joining the
uct to the Numerator of the fractional Part,
hen multiply and divide as before.

ample 4. If $b+\frac{c}{d}$ require d, what will $\frac{f}{g}$ re-
? Facit $\frac{ddf}{dbg+gc}$: For if you firſt reduce
to the Form of a Fraction, it will be $\frac{db+c}{d}$
the ſecond Term d being ſet fraction-wiſe,
be $\frac{d}{1}$: *then if* you multiply $\frac{f}{g}$ the third

Term

Term by $\frac{d}{1}$ the ſecond Term, the Product will be $\frac{df}{g}$, which being divided by $\frac{db+-c}{d}$, the firſt Term, the Quotient will be $\frac{ddf}{dbg+-gc}$, which is the fourth Proportional ſought: For,

$$\frac{db+-c}{d} : \frac{d}{1} :: \frac{f}{g} : \frac{ddf}{dbg+-gc}.$$

I ſhall not need here to give any Examples in the inverſe Rule of Proportion in the Algebraick Quantities, the manner of the Operation being the ſame with the former; only the Proportion flows backward, as in the Rule of Three Inverſe in Vulgar Arithmetick.

CHAP.

CHAP. X.

A Collection of ſome eaſy Queſtions wherein the Rules hitherto delivered are exerciſed, taken out of Mr. Oughtred's Clavis Mathematica, *Chap.* 11. *Sir* Jonas More's Arithmetick in Species, *Chap.* 10. *and Mr.* Kerſey's Elements of Algebra, *Chap.* 10. *of the Firſt Book.*

I. THERE are two Quantities or Numbers, whereof the greater is a $(=4)$ and the leſſer is e $(=2)$ What is their Sum? What their Difference? What the Product of their Multiplication? What the Quotient of the greater divided by the leſſer? What the Quotient of the leſſer divided by the greater? What the Sum of their Squares? What the Difference of their Squares? What is the Sum of their Sum, and Difference? What is the Difference of their Sum and Difference? What is the Product of their Sum *and Difference?* What the Square of their Sum?

What the Square of their Difference? What the Square of their Product?

1. The Sum of the Quantities propoſed is $a+e$
2. Their Difference is $a-e$
3. Their Product by Multiplication ae
4. The Quote of the Greater divided by the Leſſer $\frac{a}{e}$
5. The Quote of the Leſſer by the Greater $\frac{e}{a}$
6. The Sum of their Squares $aa+ee$
7. The Difference of their Squares $aa-ee$
8. The Sum of their Sum and Difference $2a$
9. The Difference of their Sum and Difference $2e$
10. The Product of their Sum and Difference $aa-ee$
11. The Square of their Sum $aa+2ae+ee$
12. The Square of the Differ. $aa-2ae+ee$
13. The Square of their Prod. $aaee$

II. There are two Quantities whoſe Sum is b ($=12$) and the greater of them is a ($=8$) I demand what is the Leſſer? What their Difference? What is the Product of their Multiplication? What is the Sum of their Squares? What the Difference of their Squares?

1. The Leſſer is $b-a$
2. Their Difference is $2a-b$
3. The Product is $ab-aa$

4. The

. The Sum of their Squares is	$2aa-2ba+bb$
. The Difference of their Squares is	$2ba-bb$

III. There are two Quantities or Numbers hoſe Difference is d ($=4$) and the greater of hem is a ($=8$) I demand what is the Leſſer? Vhat is their Sum? What their Rectangle or roduct? What the Sum of their Squares? What he Difference of their Squares?

. The Difference or Exceſs being ſubtracted from the greater, gives the leſſer	$a-d$
. Their Sum is	$2a-d$
. Their Product or Rectangle is	$aa-da$
. The Sum of their Squares is	$2aa-2ad+dd$
. The Difference of their Squares is	$2ad-ad$

IV. There are two Numbers, Magnitudes, or Quantities, whereof the Ratio of the greater to he leſſer is as r to s, (or as 3 to 2) and the greater of them is a ($=12$.) I demand what is the eſſer? What is their Sum? What their Difference? What their Rectangle, or Product? hat the Sum of their Squares? and what the ifference of their Squares?

The Leſſer is by the Rule of Three	$\frac{sa}{r}$
. Their Sum is	$a+\frac{sa}{r}$
Their Difference is	$a-\frac{sa}{r}$

4. Thei

4. Their Rectangle or Product is $\frac{saa}{r}$

5. The Sum of their Squares is $aa+\frac{ssaa}{rr}$

6. The Difference of their Squares is $aa-\frac{ssaa}{rr}$

But if the Ratio between the leſſer an greater had been given as *s* to *r*, (or as 2 and the leſſer had been given *e* (=8) then,

1. The greater by the Rule of Three would be $\frac{re}{s}$

2. Their Sum $\frac{re}{s}+e$

3. Their Difference $\frac{re}{s}-e$

4. Their Rectangle, or Product $\frac{ree}{s}$

5. The Sum of their Squares $\frac{rree}{ss}+ee$

6. The Difference of their Squares $\frac{rree}{}-ee$

V. There are two Numbers or Quan whereof the Rectangle or Product is *b* (=36 the greater Quantity is *a* (=12) what is the ſer? what their Sum? what their Differe what the Sum of their Squares? and wha Difference of their Squares?

1. The Product given being divided by (*a*), the leſſer Quantity is $\frac{b}{a}$

Their Sum	$a+\frac{b}{a}$
Their Difference	$-\frac{b}{a}$
The Sum of their Squares	$aa+\frac{bb}{aa}$
The Difference of their Squares	$aa-\frac{bb}{aa}$

But if the Rectangle had been given *b*, as before, and the leſſer Quantity had been given *e* =8) then,

The greater would have been found by Diviſion to be	$\frac{b}{e}$
Their Sum	$\frac{b}{e}+e$
Their Difference	$\frac{b}{e}-e$
The Sum of their Squares	$\frac{bb}{ee}+ee$
The Difference of their Squares	$\frac{bb}{ee}-ee$

* * *

CHAP. XI.

Reduction of Equations.

AN Equation is an Equality between two Quantities of different Names, whether compariſon of Equality be between Simple or Compound Quantities, or both; between which two Quantities there is always this Character, *viz.* $=$.

So in this following Equation, *viz.* $a=3c$, a is ſaid to be the firſt Part, and $3c$ the ſecond Part of the Equation; and ſignifieth that ſome Number or Quantity repreſented by a is equal to three times another Number or Quantity repreſented by c.

So $a=b+c$ ſignifieth that ſome Quantity repreſented by a, is equal to the Sum of two other Numbers or Quantities repreſented by b and c.

The manner of compoſing an Equation will be underſtood by ſolving of the ſeveral Queſtions contained in this and other following Chapters. But when known are mingled with unknown Quantities in an Equation, they muſt be ſo ſeparated or reduced, that the unknown Quantity or Quantities may remain entire on the one ſide, or part, and the known or given Quantities on the other ſide or part of the Equation; which to perform is the Work of Reduction, and which is contained in the ſeveral following Rules of this Chapter.

Note here, that the Quantity unknown or ſought in every Equation is repreſented by the Letter a, or ſome other Vowel, and the Quantity or Quantities known or given are repreſented by Conſonants, as b, c, d, f, &c.

Reduction by Addition.

II. If equal Numbers or Quantities be added to *equal* Numbers or Quantities, the Sums or Totals *will* be equal: and therefore, If it be granted that $a-8=20$

hen by adding $+8$ to each part of the Equation, there ariſeth $a-8+8=20+8$

'hen becauſe in the firſt part of the Equation there is $+8$ and -8, they deſtroy each other by the third Rule of the ſecond Chapter, and it followeth that $a=28$

gain, let this Equation be propoſed to be reduced, *viz.* $a-b=d+b$

Then by adding b to each part of the Equation, there ariſeth $a-b+b=d+b+b$

And, becauſe $-b$ and $+b$ are in the firſt part of the Equation, they deſtroy each other; and the Equation is $a=d+2b$

ikewiſe, if $aa-b-c=ff$

Then by adding $b+c$ to each part of the Equation, there ariſeth $aa=ff+b+c$.

Now from a due Conſideration of the Pre-
ıiſes, it followeth, that if in an Equation there
e any Number or Quantity propoſed with
ıe Sign — before it, then if it be transferred
ɔ the other ſide of the Equation, and plac'd
ıere with the Sign + and cancelled on the
de or part where it now ſtandeth, the effect
ill be the ſame as the adding of that Quan-
y to each part of the Equation; and this, by
Artiſt

Artiſts, is called Tranſpoſition: as in the firſt of the foregoing Examples, where it is granted,

That $a-8=20$

And by tranſpoſing -8 on the other ſide of the Equation, making it there $+8$, it giveth $a=28$

And in the 2d Example, where $a-b=d+b$

By tranſpoſing $-b$, cancelling it on the firſt ſide of the Equation, and making it $+b$ on the other, it is $a=d+2b$

And let it be granted that $a-bb-d=cc$

Then by tranſpoſing of $-bb$ and $-d$, there ariſeth $a=cc+bb+d$.

Reduction by Subtraction.

III. If in any Equation there be any Number or Quantity ſigned with + (on which ſide of the Equation ſoever) if it be cancelled on that ſide, and placed on the other ſide with the Sign − prefixed to it, the Work of Reduction is truly performed; and this is alſo called Tranſpoſition, and is only the Converſe of the foregoing Rule.

Examples.

Let it be granted that $a+8=36$

Then if $+8$ be cancelled, and placed on the other part of the Equation with the Sign −, it will give $a=36-8$

Which Equation being contracted, is $a=28$

Again, let be given $2aa+b=aa+cc$

y the Tranſpoſition of $+b$ on the firſt ſide of the Equation, it is $2aa=aa+cc-b$

ınd by Tranſpoſition of aa on the ſecond ſide of the Equation, it is $aa=cc-b$

ılſo, if $aa+b+c=ba+dd$

ly Tranſpoſition of $b+c$ to the ſecond ſide of the Equation, it is $aa=ba+dd-b-c$

ınd by the Tranſpoſition of ba to the firſt ſide of the Equation, it is $aa-ba=dd-b-c$

Which Method (in reducing of the premiſed Equation) is deduced from this general Axiom, *viz.*

If from equal Numbers or Quantities, equal Numbers or Quantities are ſubtracted, the Re- nainders ſhall be equal.

So in the ſecond Example there is given this Equation, *viz.* $2aa+b=aa+cc$

Firſt, by ſubtracting b from each part of the Equation, there is $2aa=aa+cc-b$

Then I ſubtract aa from each part, and there remaineth $aa=cc-b$

Reduction by Multiplication.

IV. When in an Equation, one or both Parts *are Fractions,* then let them be reduced to a com- *non Denominator* by the 2d, 4th, and 5th Rules of the ſixth Chapter, and then caſting away the Denominator, uſe only the Numerators, ſo ſhall Equations expreſs'd by Algebraical Fractions be reduced to other Equations, conſiſting altogether of Integers. As in the following Examples.

If

$$\frac{a}{8}=9$$

Then by reducing 9 in the ſecond part of the Equation to a Fraction, having 8 for its Denominator, it is

$$\frac{a}{8}=\frac{72}{8}$$

And by caſting away the Denominator which is common to both, it is

$$a=72$$

Again, if

$$a=\frac{bcd}{a+b}$$

Then by reducing *a* on the firſt ſide of the Equation to a Fraction, having $a+b$ for its Denominator, it is

$$\frac{aa+ba}{a+b}=\frac{bcd}{a+b}$$

And by caſting away the common Denominator $a+b$, the Equation is

$$aa+ba=bcd$$

Likewiſe, if

$$\frac{ab}{c}=\frac{dc}{b}$$

The Quantities being reduced to a common Denominator, are

$$\frac{abb}{cb}=\frac{dcc}{cb}$$

And the common Denominator *cb* being caſt away, the Equation is

$$abb=dcc.$$

/. When either part of an Equation is com-
ed of a mixed Quantity or Quantities, let the
egral Part or Parts be reduced to a Fraction
Fractions, and then proceed as in the laſt Ex-
ple.

t is granted, that $b+c+\frac{a}{b}=cd+\frac{bc}{a}$

Firſt, it is reduced to $\frac{bb+bc+a}{b}=\frac{cda+bc}{a}$

ich Fractional Equation
eing reduced according
o the foregoing Rule, is $\Big\}$ $bba+bca+aa=bcad+bbc$

VI. When ſome Power or Degree of the Num-
or Quantity ſought is multiplied into each
t, and each Member of an Equation, then let
t Degree or Power be cancelled in each Part
l Member, ſo will it quite vaniſh, and the Equa-
n will be reduced to more ſimple Terms: As
Example.

Let it be granted that $aa+ba=ac$

raſmuch as a is a Factor in
each Part and Member of
he Equation, therefore it
eing expunged in each,
here ariſeth this Equation $\Big\}$ $a+b=c.$

VII. When (according to the ſecond, third,
rth, and fifth Rules) an Equation is reduced,
l that ſome known Number or Quantity is
ltiplied into the Quantity ſought, then divide
h part of the Equation by that known Quan-
, to the end that the Quantity ſought may

have no Quantity multiplied into it but 1 (or Unity.) As for Example,

If it be granted that $ba=cd$

Then becauſe the Quantity ſought is a multiplied by b, divide each part of the Equation by b, and there ariſeth $a=\frac{cd}{b}$

VIII. When any one Part of an Equation is compoſed of a ſurd Quantity (*viz.* ſuch as hath the Radical Sign prefixed to it) and the other Part is a rational Quantity: then let that rational Quantity be raiſed to the Power ſignified by the Radical Sign, and then caſt away the ſaid Radical Sign, ſo ſhall both Parts of the Equation be a rational Quantity. As,

If it be propoſed that $\sqrt{a}=8$

Square 8, and place its Square in the room of itſelf, caſting away the radical Sign from the firſt Part of the Equation, and then it will be $a=64$

Likewiſe if $\sqrt{a}=cd$

Then by raiſing the ſecond Part of the Equation to its Square, and caſting away the radical Sign from the firſt Part, there ariſeth this Equation, *viz.* $a=ccdd$

Again, if $\sqrt{a}=b+c$.

The ſecond Part of the Equation being ſquared, and the Radical Sign cancelled in the firſt, there ariſeth $a=bb+2bc+cc$

Reduction by Diviſion.

IX. If equal Quantities be divided by equal Quantities, the Quotients thence ariſing will be equal. For,

If $aa=10a$

Then by dividing each part of the Equation by *a*, there ariſeth this Equation $a=10$

And if $aaa=bba+daa$

Then by dividing each Part of the Equation by *a*, there ariſeth $aa=bb+da$

And *da* in the ſecond Part of the Equation being tranſpoſed by the third Rule of this Chapter, there ariſeth this Equation, *viz.* $aa-da=bb$

And if $ba-ca=dd$

Then by dividing each part of the Equation by $b-c$, it is $a=\frac{dd}{b-c}$

CHAP. XII.

To convert Analogies into Equations, and Equations into Analogies.

I. THIS is deduced from this univerſal Theorem, *viz.* That if four Quantities are Proportionals, the Product of the two Means is equal to the Product of the two Extremes; and if three Numbers are Proportionals, the Product of the two Extremes is equal to the Square of the Means.

1. Let there be propos'd theſe four Proportionals; } $a : b :: c : d$

Then by the ſaid Theorem this Equation will follow, *viz.* } $ad = bc$

2. Let there be propos'd theſe three continual Proportionals, *viz.* } $a, b, c \div$

That is to ſay, $a : b :: b : c$

Whence there followeth this Equation, *viz.* } $ac = bb$

II. From a due Conſideration of the Premiſes it is evident that Equations may oftentimes be reſolved into Proportionals, *viz.* when the Product of two Quantities is found equal to the Product

duct of two other Quantities; for then as any one of the Factors, in the firſt ſide of the Equation, is to any one of the Factors in the ſecond part of the Equation, ſo is the remaining Factor of the ſecond part to the remaining Factor in the firſt part: And the Converſe.

Suppoſe that $bc=ad$

From thence may be drawn this Analogy $b : a :: d : c$

The Truth of which may be proved by the firſt Rule of this Chapter; for thereby the ſaid Analogy may be reduced to the given Equation, *viz.* $bc=ad$.

Again, if $3ba=3dc$

Then from thence may be deduced this Analogy, *viz.* $3b : c :: 3d : a$

Or $3b : 3c :: d : a$

Or $b : 3c :: d : 3a$

Likewiſe if $cb=ca+da$

Then may that Equation be reſolved into theſe Proportionals $c+d : b :: c : a$

And if $dd=6ba$

Then it will be found that $6b : d :: d : a$

III. When it happens that there is an Equation between an Algebraical Fraction and an Integer, if the Numerator of the ſaid Fraction can be reſolved into two ſuch Quantities, as being multiplied the one by the other, will produce the ſaid Numerator, then will the ſaid Equation produce the Proportion, *viz.*

As the Denominator of the Fraction is to one of the Factors of which the Numerator is produced, ſo is the other Factor to the Integer, unto which the ſaid Fraction is equal.

Examples.

If it be granted, that $\frac{bc}{d}=a$

Then may that Equation be reſolved into this Analogy $\Big\}\ d : c :: b : a$

For, $d : c :: b : \frac{bc}{d}=a$

Again, if $\frac{cd}{b+d}=a$

Then may that Equation be reſolved into this Analogy $\Big\}\ b+d : c :: d : a$

And alſo if $dd=\frac{ab+ad}{cc}$

Then may the ſaid Equation be reſolved into this Analogy, *viz.* $\Big\}\ cc : b+d :: a : dd$

The Practice of the two laſt Rules will be plainly diſcovered in the next Chapter (in the reſolution of Queſtions producing ſimple Equations) to be of moſt excellent uſe in diſcovering or laying down of Theorems for the ready Reſolution of the Queſtion propoſed, or any other of the ſame nature: which Theorems are to be kept reſerved in ſtore for the finding out of new, and the Confirmation of old Truths.

CHAP

CHAP. XIII.

The Reſolution of Arithmetical Queſtions (Algebraically) which produce Simple Equations.

I. AN Equation is two-fold, *viz.* Firſt, Simple; and, Secondly, Adfected or Compounded.

II. A Simple Equation is, when the Quantity ſought (ſolely poſſeſſing one Part of the Equation) is either expreſſed by a ſingle or ſimple Root, as a, or by a ſingle or ſimple Power, as aa, or aaa, *&c.* as in theſe Equations, *viz.* $a=8$, and $aa=64$, or $4aa=256$, and ſuch like.

III. When a Queſtion is propounded, and to be reſolved Algebraically, then for the Anſwer put a, and for each of the given Numbers put Conſonants: then proceed according to the Tenure of the Queſtion, by Addition, Subtraction, Multiplication, or Diviſion, until an Equation is compoſed; and when the Equation is compoſed, then proceed to reduce it (according to the Rules contained in the 11th Chapter) *until the* Quantity unknown being a, (or *ome Power of a)* do ſolely poſſeſs one part of

the

the Equation, and the known or given Qua the other Part, and then will the Quantity 1 be alſo known.

IV. I ſhall in the Reſolution of every Qu proceed (*gradatim*) Step by Step, accordi the Method uſed by Mr. *Kerſey*, each Step numbred orderly in the Margin, from tl ginning to the end, by 1, 2, 3, 4, *&c.* ſhall only proceed in the Operation literall cauſe otherwiſe this Treatiſe would ſwell bigger Volume than is at preſent intended I ſhall give the Learner a taſte of Numeral bra, in the Solution of two or three of th Queſtions thereby.

Queſt. 1. There are two Numbers whoſ is 48 (or b) and the Exceſs of the greater the leſſer is 14 (or c) I demand what a Numbers.

The Solution literally.

1. For the greater Number put a
2. From which if you ſubtract the Difference c, you will have the leſſer, which is $a-c$
3. The greater and leſſer being added together, will be equal to b, the Sum, whence this Equation $2a-c=b$
4. And by the Transpoſition of $-c$, the Equation is $2a=b+c$
5. Then dividing each part of the Equation by 2, it is $a=\frac{b+c}{2}$

6. And if from $\frac{b+c}{2}$ you ſubtract c, the Exceſs of the greater above the leſſer, the leſſer will be } $\frac{b+c}{2}-c$

So that the Numbers ſought are 31 and 17, for by the fifth Step, a the greater is found to be $=\frac{b+c}{2}$, and b is given 48, and c is given 14; the Sum of which is 62; which divided by 2, gives 31 for the greater: and by the ſixth Step, if from the greater you ſubtract the Difference c, the Remainder will give the leſſer, which is 17; or $\frac{b+c}{2}-c=17$.

Now if the fifth and ſixth Steps are duly conſidered, they will preſent you with this

Theorem.

The Sum of the Sum and Difference of any two Numbers being divided by 2, will give the greater Number; and the Difference of any two Numbers being ſubtracted from half the Sum of the Sum and Difference, the Remainder will give the leſſer Number.

The Solution Numerally.

1. For the greater Number put a
2. From which if you ſubtract the Difference 14, the leſſer is } $a-14$

3. Which added together, will be the Sum, whence this Equation $2a-14=48$

4. And by Tranſpoſition of -14, it will be $2a=62$

5. And both parts of the Equation being divided by 2, will give the Value of *a*, the greater $a=31$

6. From which if you ſubtract 14, the Difference, the Remainder will give the leſſer, by the ſecond Step. 17

So that the Numbers ſought are 31 and 17, which will ſatisfy the Conditions of the Queſtion.

Queſt. 2.

There are two Numbers whoſe Sum is 56 (or *b*) and the leſſer hath ſuch proportion to the greater, as 2 to 5, (or *c* to *d*) I demand what are the Numbers.

1. For the leſſer Number put a

2. Then by the Rule of Three find the greater, *viz.* $\frac{da}{c}$

$$c : d :: a : \frac{da}{c}$$

3. Wherefore the Sum of the two Numbers ſought is $a+\frac{da}{c}$

4. Which Sum being equal to the given Sum, hence this Equation $a+\frac{da}{c}=b$

5. Which Equation being reduced by the 4th and 5th Rules of the 11th Chapter, the Value of *a* will be found to be $\left.\right\} a = \dfrac{cb}{c+d}$

6. And by the 1ſt, 2d, and 5th Steps, the greater Number will be diſcover'd to be $\left.\right\} \dfrac{db}{c+d}$

So that the Numbers ſought are 40 and 16: for *a*, the leſſer, is found to be, by the 5th Step, $\dfrac{cb}{c+d}$ *viz.* the Product of *cb* 56 by 2 divided by $c+d$, the Sum of 2 and 5, *viz.* 7, which is 16, &c.

And if (according to the 3d Rule of the 12th Chapter) the two laſt Steps be turned into Proportionals, it will give this

Theorem.

As the Sum of the Terms which repreſent the Ratio of two Numbers is to the Sum of the Numbers themſelves, ſo is the leſſer Term to the leſſer Number; and ſo is the greater Term to the greater Number.

Therefore if the Sum of two Numbers is given, and alſo their Ratio, the Numbers themſelves are alſo given by this Theorem.

The ſame Queſtion ſolved Numerically.

1. For the leſſer Number put a
2. Then by the Rule of Three the greater Number is found viz. $2 : 5 :: a : \frac{5a}{2}$ $\Big\}$ $\frac{5a}{2}$
3. Then will their Sum be $a + \frac{5a}{2}$
4. And according to the Tenour of the Queſtion, their Sum muſt be equal to the given Sum, whence this Equation $\Big\}$ $a + \frac{5a}{2} = 56$
5. And that Equation being reduced by the 5th and 6th Rules of the 11th Chapter, the Value of a will be found to be $\Big\}$ $\frac{112}{7} = 16$
6. Which being ſubtracted from the given Sum, the greater Number is $\Big\}$ 40

Queſt. 3.

A Gentleman ask'd his Friend (that had four Purſes in his Hand) what Money he had in each Purſe: To whom he anſwer'd, that he knew not; but (quoth he) this I know, that in the ſecond Purſe there are 8 or (b) Crowns more than in the firſt or leaſt Purſe, and in the third 8 Crowns more than in the ſecond, and in the fourth or biggeſt Purſe there are 8 Crowns more than in the third, and twice as many as in the firſt or leaſt: I demand what number of Crowns he had in each ſe.

1. F

1. For the number of Crowns in the firſt Purſe put a
2. Then in the ſecond there is $a+b$
3. And in the third there is $a+2b$
4. And in the fourth $a+3b$
5. Which, according to the Tenour of the Queſtion, is double to that in the firſt, whence this Equation $a+3b=2a$
6. Then by the Tranſpoſition of a from the firſt ſide of the Equation, it is $3b=a$

which diſcovereth the Value of a to be $3b$, or 3 times 8, which is 24, &c. which is the number of Crowns in the firſt Purſe, and conſequently the number of Crowns in each Purſe is 24, 32, 40, and 48, which will ſatisfy the Conditions of the Queſtion.

The ſame Queſtion ſolved Numerically.

1. For the Crowns in the firſt Purſe put a
2. Then in the ſecond there is $a+8$
3. And in the third $a+16$
4. And in the fourth $a+24$
5. Which is double to the number of Crowns in the firſt; whence this Equation $a+24=2a$
6. Which Equation being reduced by the Tranſpoſition of a, diſcovers the Value of a, *viz.* $a=24$

Queſt. 4.

Three Men build a Ship which coſt them 2700 (or b) Pounds, of which B muſt pay double to what A muſt pay, and C muſt pay three times as much as B; I demand the Share that each muſt pay

1. For the Sum to be paid by A, put a
2. Then B muſt pay $2a$
3. And C muſt pay $6a$
4. The Sum of theſe three Quantities are equal to the total Charge, whence this Equation $9a=b$
5. Which being reduced, diſcovers the Value of a, *viz.* $a=\frac{b}{9}$

which is the Sum that A muſt pay, *viz.* 300 *l.* Therefore B muſt pay $\frac{2b}{9}=600\,l.$ which is twice as much as A; and C muſt pay $\frac{6b}{9}=1800\,l.$ which is three times as much as B.

Queſt. 5.

There is a Fiſh whoſe Head is ſuppoſed to be 9 (or b) Inches, and his Tail is as long as his Head and half his Body, and his Body is as long as his Head and his Tail; I demand the length of ſuch a Fiſh.

1. For the length of the Body put a
2. Then will the Tail be $\frac{a}{2}+b$

3. The

3. Then if to the Tail you add the length of the Head, *viz.* b, the Sum is $\frac{a}{2}+2b$

4. Which according to the Ténour of the Queſtion is equal to the length of the Body, whence this Equation $a=\frac{a}{2}+2b$

5. And the ſecond Part of the Equation being clear'd of the unknown Quantity a by Reduction, gives the Value of a, the length of the Body, *viz.* $a=4b$

6. Then according to the Tenour of the Queſtion, if therefrom you ſubtract b, the length of the Head, the Remainder will be the length of the Tail, which is $3b$

By the fifth Step the length of the Body is found to be $4b=36$, and by the ſixth Step the length of the Tail is diſcover'd to be $3b=3\times9=27$. So that the length of the Head is (given) 9 Inches, the length of the Tail 27 Inches, and the length of the Body 36 Inches; which Numbers will ſatisfy the Conditions of the Queſtion.

For, $36=27+9$ the Body,
And $\frac{36}{2}+9=27$ the Tail.

So that the whole length of the Fiſh is $9+27+36=72$ Inches.

Queſt.

Queſt. 6.

A Father lying at the point of Death, left to his three Sons, *A*, *B*, and *C*, all his Eſtate in Money, and divided it thus, *viz.* to *A* he gave $\frac{1}{2}$, wanting 44 (or b) Pounds; and to *B* he gave $\frac{1}{3}$ and 14 (or c) Pounds over; and to *C* he gave the reſt, which was 82 (or d) Pounds leſs than the Share of *B*: now I demand what was the Father's Eſtate.

1. For the Father's Eſtate put a
2. Then will the Share left to *A* be $\frac{a}{2}-b$
3. And the Share of *B* $\frac{a}{3}+c$
4. And by the third Step the Share of *C* is $\frac{a}{3}+c-d$
5. The Quantities in the three laſt Steps being added together, give $\frac{7a}{6}+2c-b-d$
6. Which muſt be equal to the Father's Eſtate, whence this Equation $\frac{7a}{6}+2c-b-d=a$

Which Equation after due Reduction and Tranſpoſition of Quantities, the Value of a is diſcover'd to be $a=6b+6d-12c$

And $6b=6\times44=264$, and $6d=6\times82=492$, and $12c=12\times14=168$; now $264+492-168=588$: ſo that the Father's Eſtate was 588 Pounds, of which *A* had 250 *l.* *B* 210 *l.* and *C.* 128; which Numbers do anſwer the Conditions of the Queſtion.

Queſt. 7.

'o Perſons thus diſcourſed together con-ıg their Money, quoth *A* to *B*, give me 3 of your Crowns, and I ſhall have as ma-you: Nay, quoth *B* to *A*, but if you will ne 3 of your Crowns, I ſhall have 5 times ıny as you. Now I demand how many ns had each Perſon?

r the Number of Crowns ich *A* had, put	$a.$
ıen foraſmuch as adding or *b*) Crowns to *A* will equal to the Crowns re-ining to *B* after he had en 3 Crowns to *A*, there-e *B* will then have left	$a+b$
ıd conſequently if you l thereto the 3 (or *b*) ɔwns which he gave to :he Sum will be the Num-' of Crowns which *B* had irſt, which is	$a+2b$
ɜn if from the Number of ɔwns *A* had at firſt (*a*) ı ſubtract 3 (or *b*) Crowns, re will remain to *A* $a-b$ ɔwns, and giving the ıe to *B*, he will then have	$a+3b$
ıich according to the ınour of the Queſtion is ɜ times as much as what *had left, whence there* *'eth this Equation*	$5a-5b=a+3b$

6. Which Equation being reduced by the 2d and 7th Rules of the 11th Chapter, the Value of *a* is diſcover'd to be } $a=2b$

7. And by the 6th and 3d ſteps, the number of Crowns which *B* had at firſt, are found to be } $a+2b=4b$

So that it is found that *A* had 6 Crown
B had 12 Crowns, which Numbers will
the Conditions of the Queſtion. For,

$$6+3=12-3=9$$

And,

$$12+3=5\times\overline{6-3}=15$$

Queſt. 8.

A Labourer had 576 (or *b*) Pence for tl
ing 60 (or *c*) Quarters of Corn, *viz.* Whe
Barley; for the Wheat he had 12 (or *d*)
per Quarter, and for the Barley he had 6 (
Pence *per* Quarter; I demand how many (
ters of each he threſhed?

1. For the Quarters of Wheat which he threſhed, put } a
2. Then the Quarters of Barley will be } $c-a$
3. The Quantity of Wheat in the firſt Step being multiplied by its Price, produces } da

The Quantity of Barley in he ſecond Step being mul- iplied by its Price, pro- luceth

$$fc-fa$$

The Sum of the Quantities n the two laſt Steps muſt e equal to the given Price f the 60 Quarters, whence his Equation

$$da-fa+fc=b$$

Which being reduced by he ſecond, third, and fifth Rules of the 11th Chapter, he Quantity of Wheat will e diſcover'd to be

$$a=\frac{b-fc}{d-f}$$

And by the ſecond and fth Steps the Quantity of Barley is diſcover'd to be

$$c=\frac{b-fc}{d-f}$$

o that the Quarters of Wheat which he threſh- were 36, and the Quarters of Barley 24.

The Proof.

$2\times36=432$

And

$\times24=144$

And

$32+144=576$, which was to be proved.

Queſt. 91.

. Gentleman bought a Cloke of a Saleſman, ch coſt him 3*l.* 10*s.* or 70 (or *b*) Shillings; deſiring the Saleſman to tell him what he

gained thereby, he ſaid he gained $\frac{1}{4}$ (or c) o
it coſt him; the Queſtion is what the Clo
the firſt Penny?

1. Suppoſe the Clock coſt — a
2. Then he gained — ca
3. The firſt and ſecond Steps being added together, their Sum will be equal to the Sum which the Gentleman gave for it, whence this Equation. — $ca+a=b$
4. Which Equation being reduced by the 9th Rule of the 11th Chapter, the value of a will be diſcover'd to be — $a=\frac{b}{c+1}$

So that it coſt 56 Shillings, $\frac{1}{4}$ of which
Shillings, and $56+14=70$.

And if the Quantity in the fourth Step b
conſider'd, you will find that if the Gai
been any other Part or Parts of the firſt C
the Price it was ſold for had been divided
Fraction repreſenting part of the Gain inc
by 1, the Quote would have been the Anſ

Queſt. 10.

A Gentleman hired a Labourer to w
him for 40 (or b) Days, and made this
ment with him, that for every Day he wrou
ſhould have 20 (or c) Pence, and for ever
that he played he ſhould forfeit 8 (or d)
and at the end of the ſaid 40 Days he r

4 (or *f*) Pence, which was his full due: now emand how many Days he wrought, and how ny Days he played.

For the Number of Days he wrought, put	a
Then the number of Days he played will be	$b-a$
And if the time he wrought (in the firſt Step) be multiplied by 20 (*c*) it will produce the Total he gained by Work, *viz.*	ca
And if the time he played (in the 2d Step) be drawn into 8 (*d*) the Product will be what he loſt by play	$db-da$
And if the total Loſs (in the 4th Step) be ſubtracted from the Gain (in the 3d Step) the Remainder will be what he receiv'd, whence this Equation	$ca+da-db=f$
Which being reduced by the ſecond and ninth Rules of the 11th Chapter, it will diſcover the Value of *a* to be 18, which is the Days that he wrought	$a=\dfrac{f+db}{c+d}$
And from the ſixth and ſecond Steps the number of Days he played are diſcover'd to be 22 Days, *viz.*	$b-\dfrac{f+db}{c+d}$

So that by the ſixth Step it appears he wrought *Days, and by the* ſeventh Step it appears that *played 22 Days.*

Th

The Proof.

$18\times20=360$ and
$22\times8=176$ and
$360-176=184$

Queſt. 11.

A Perſon (in the Afternoon) being aſk'd w a-clock it was, anſwer'd, that $\frac{3}{5}$ (or b) part the time from Noon was equal to $\frac{5}{8}$ (or c) p of the time remaining to Midnight; now (poſing the time from Noon to Midnight to divided in 12 (or d) equal Parts or Hours) I mand what was the preſent Hour of the Day?

1. For the Hour ſought put a
2. Then the time to midnight will be $d-a$
3. Then will $\frac{3}{5}$ (or b) parts of the Hour from Noon be ba
4. And $\frac{5}{8}$ (or c) parts of the time remaining till Midnight, will be $cd-ca$
5. Therefore from the third and fourth Steps there ariſeth this Equation $ba=cd-ca$
6. Which Equation being reduced according to the ſecond and ninth Rule of the eleventh Chapter, gives the Value of a (to be 6 $3\frac{48}{49}$ the Hour ſought) *viz.* $a=\frac{cd}{b+c}$

So that the Hour ſought was $6\frac{4}{5}\frac{8}{25}$ and conſequently the time remaining till Midnight was $5\frac{2}{3}\frac{8}{9}\frac{1}{2}$ Hours, which two Numbers will anſwer the Condition of the Queſtion; for $\frac{3}{5}$ parts of $6\frac{1}{3}\frac{8}{9}\frac{1}{2}$, which is $3\frac{3}{1}\frac{3}{9}$, is equal to $\frac{3}{4}$ parts of $5\frac{2}{3}\frac{8}{9}\frac{1}{2}$, as you may prove at your leiſure.

Moreover, if the laſt Step be converted into Proportionals by the third Rule of the twelfth Chapter, it will give this

Theorem.

As the Sum of the parts of any two Numbers (wherein there is an Equality) is to the Sum of thoſe Numbers, ſo is the given parts of any one of thoſe Numbers to the other Number.

As, ſuppoſe it were required to find out two Numbers whoſe Sum is 27, and ſuch, that $\frac{3}{4}$ of the one may be equal to $\frac{3}{5}$ of the other, the ſame may be found out by the ſaid Theorem. For,

$$\frac{3}{4}+\frac{3}{5} : 27 :: \frac{3}{4} : 15$$

which Number ſo found is the Number ſought, whereof $\frac{3}{5}$ is to be taken; and the other is $27-15=12$, or it may be found by the following Proportion, *viz.*

$$\frac{3}{4}+\frac{3}{5} : 27 :: \frac{3}{5} : 12$$

Queſt. 12.

One asked a Shepherd what was the Price of *his hundred Sheep?* Quoth he, I have not an hun-*dred; but if I had* as many more and half as many more, and 7½ (or b) Sheep, then I ſhould have juſt 100 (or c) I demand how many Sheep he had?

1. For the Number of Sheep he had, put	a
2. Which being doubled is	$2a$
3. And if to the ſecond Step you add half the firſt, it is	$2a+\frac{a}{2}$
4. And if to the third Step there be added 7½ (or b) the Sum is	$2a+\frac{a}{2}+b$
5. Which Quantity in the 4th ſtep is equal to 100 (or c) whence this Equation	$2a+\frac{a}{2}+b=c$
6. Which Equation being reduced by the 5th and 7th Rules of the 11th Chapter, the Value of a will be diſcover'd to be 37, *viz.*	$a=\frac{2c-2b}{5}$

So that the number of Sheep he had were 37; for $37+37+\frac{37}{2}+7\frac{1}{2}=100$.

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* * *

CHAP. XIV.

How to extract the Root of a Square formed from a Binomial; and how by having any two of the Members of ſuch a Square given, to find out the third.

A Binomial is a Quantity conſiſting of two Names or Parts, as $a+b$, or $a-b$, $a+ee$, $b+d$, &c. And when a Square is formed rom ſuch a Root, it will conſiſt of three Memers or Parts, *viz.* two affirmative Squares of he Parts of which the Binomial is compoſed, nd the double Rectangle of thoſe Parts; which louble Rectangle is ſometimes affirmative, and ometimes negative, *viz.* affirmative, when the 'arts of the Binomial are both affirmative, or oth negative; that is, when they are both ſignd with +, or both with —; and negative, hen one of the Parts of the Binomial Root is gned with +, and the other with —.

So if $a+b$ were given for a Root, its Square ould be $aa+2ab+bb$, which is compoſed of (aa nd bb) the Squares of the Parts of which the oot is compoſed, and of ($2ab$) the double

Product or Rectangle made by the Multiplication of the ſaid Parts (a and b) one by the other. See the Work.

$$\begin{array}{l} a+b \text{ the Root}\\ a+b\\ \hline aa+ab\\ \quad ab+bb\\ \hline aa+2ab+bb \text{ the Square.}\end{array}$$

So if it were required to find the Square of the Binomial $a-b$, or $b-a$, it (being multiplied by itſelf) would be $aa-2ab+bb$, which is compoſed of aa and bb, the Sum of the Squares of the Parts, and their double Rectangle, as before, but $2ab$, the double Rectangle of the Parts, is ſigned with —; ſo that the Squares of the Difference of any two Numbers or Quantities is equal to the Sum of the Squares of the ſaid Quantities or Numbers made leſs by their double Rectangle: as by the Work.

$$\begin{array}{ll} a-b \text{ Root} & b-a \text{ the Root}\\ a-b & b-a\\ \hline aa-ba & \quad bb-ba\\ -ba+bb & -ba+aa\\ \hline aa-2ba+bb \text{ Square.} & bb-2ba+aa \text{ the Square.}\end{array}$$

So if the Number 10 were divided into 8 and 2, *viz.* $8+2$, its Square would be $64+32+4 = 10 \times 10 = 100$. And the Square of $8-2$ is $64-32+4=6\times6=36$; for $8-2=6$, and $6\times6=36$.

No

Note, That a Binomial Root having one of its Parts ſigned with —, is by ſome Authors called a Reſidual Root, as $a-b$, and $c-d$, *&c.* are Reſiduals.

II. From what has been ſaid concerning the Square of a Binomial, may be inferred this

Theorem.

If a compound Quantity conſiſt of 3 Members, whereof two are Squares of different Names, with the Sign + prefixed to them, and the third is the double Rectangle of the Roots of thoſe Squares, having alſo the Sign + prefix'd to it, then ſhall the Square Root of ſuch a compound Quantity be the Sum of the ſquare Roots of the ſaid two ſimple ſquares; but if the ſaid double Rectangle hath the Sign — prefixed to it, the ſquare Root of the ſaid compound Quantity ſhall be the Difference of the ſaid Roots.

So, the ſquare Root of $aa+2ab+bb$ will be found to be $a+b$; for the ſquare Root of aa is a, and the ſquare Root of bb is b; which two Roots added together, give $a+b$.

Alſo, the ſquare Root of $aa+8a+16$ will be found to be $a+4$; the two Squares in the given Quantity are aa and 16, and $8a$ is the double Product of a and 4, the ſaid Roots being multiplied the one by the other.

Likewiſe, the ſquare Root of $aa-2ab+bb$ is $a-b$, or $b-a$, not $a+b$, becauſe the double Rectangle ($2ab$) is ſigned with —.

Furthermore, the ſquare Root of $9aa+12ba+4bb$ is $3a+2b$: the two ſquare Quantities in he ſaid compound Square are $9aa$, and $4b$

wh

whoſe Roots are $3a$ and $2b$, and $12ab$ is the double Product of $3a$ and $2b$ being multiplied together.

And the ſquare Root of $aa-20a+100$ is $a-10$, for the two Squares in this compound ſquare Quantity are aa and 100, whoſe ſquare Roots are a and 10, and $20a$ is the double Rectangle of 10 and a, they being multiplied together.

The foregoing Theorem being well underſtood will be of excellent uſe in the Reſolution of Queſtions, producing Quadratick Equations, as you will find by the Queſtions contained in the next Chapter.

III. When it is required to extract the ſquare Root of a Quantity whoſe Root cannot be exactly extracted, then prefix the radical Sign to it, which ſhall repreſent its ſquare Root. So the ſquare Root of bc is $\sqrt{bc}$, or $\sqrt{(2)}\ bc$, and the ſquare Root of $\overline{aa+bb}$, is thus repreſented, *viz.* $\sqrt{aa+bb}$, or $\sqrt{(2)}\ \overline{aa+bb}$, *&c.*

IV. From a due Conſideration of the foregoing Theorem, a way is diſcovered, how by having any two of the Members of a Square formed from a binomial Root, the third Member may be found out. For,

When two affirmative ſquare Quantities are given for two of the Members in a Square formed from a binomial Root, then take the Roots of thoſe two Squares and multiply them the one by the other, and double the Product, ſo ſhall that Product being doubled be the third Member, which being annexed to the two given Squares, either by +, or —, it will make an exact compound Square, whoſe Root ſhall be a Binomial

So if $aa+bb$ were given for two of the Memers of a Square, firſt I find their Roots to be and b, which being multiplied the one by the ther, produce ab, and that Product being douled, gives $2ab$ for the middle term of the comound ſquare Quantity to make it a compleat quare, the Root whereof is a Binomial, *viz.* $a+2ab+bb$: if the ſaid double Product be joined o the ſaid Sum of the Squares by the Sign —, : will give the compound ſquare Quantity $aa-ab+bb$ whoſe Root is $a-b$.

Alſo if $25aa+16bb$ were given for two of the Members of a Square whoſe Root is a Binomial, he ſaid Square being compleated, will be $25aa+40ab+16bb$, or $25aa-40ab+16bb$, whoſe Root is either $5a+4b$, or $5a-4b$.

V. When the two given Members of a comound ſquare Quantity whoſe Root is binomial, are the double Product or Rectangle, and ne of the two affirmative Squares, divide half he ſaid double Product by the Root of the given quare, and ſquare the Quotient, ſo ſhall that quare be the third Member ſought, which being joined to the two given Quantities with the ign +, it will give you a compleat Square, having for its Root a Binomial.

As for Example: Let $aa+2ba$ be propoſed or two of the Members of a Square whoſe Root is a Binomial. Firſt, I take half of $2ba$, (the ſaid double Product) and it is ba, which being divided by a, (the Root of aa, the given Square) the Quotient is b, whoſe Square is bb for the third Member ſought.

Again, let $25aa+40a$ be the two propoſed *Terms of ſuch* a Square whoſe Root is a Binomial, and let it be required to find the other Square, which ſhall make it a compleat Square raiſed from a Binomial Root; in order to which, firſt I take half ($40a$) the double Product, *viz.* $20a$, and divide it by the Root of ($25aa$) the given Square, which is $5a$, and the Quotient is 4, which being ſquared, gives 16 for the third Member required; which being joined to the reſt, gives $25aa+40a+16$ for the Square compleated.

VI. When the two given Members of a Square raiſed from a binomial Root, are ſuch that one of them is a ſquare Affirmative without any Number or Quantity prefixed to it, and the other is the Root of the ſaid Square multiplied by ſome other Quantity, then is that other Quantity by Artiſts called the Coefficient; and if you ſquare half the ſaid Coefficient, or (which is all one) take $\frac{1}{4}$ of the Square of the Coefficient, that ſhall be the third Member required; which being joined to the two given Quantities by the Sign $+$, it will give you a compleat Square raiſed from a Binomial Root.

Example. Let the two given Members of a Square be $aa+2ba$, and let it be required to find out the third Member: Here the Coefficient is $2b$, half of which is b, which being ſquared, gives bb for the third Member which was ſought; ſo is the Square compleated, $aa+2ab+bb$.

In like manner, if the two given Members of a Square were $aa+ba$, and it were required to find out the third Member;

Here the Coefficient is b; half of which is $\frac{1}{2}b$, or $\frac{b}{2}$, whoſe Square is $\frac{1}{4}bb$, or $\frac{bb}{4}$ for the Member ſought. Alſo, let the two given Members of a Square be $aa+8a$, and let it be required to find out the third Member. Here the Coefficient is 8, half of which is 4, whoſe Square is 16 for the third Member required; ſo is $aa+8a+16$, a compleat Square, whoſe Root is $a+4$.

Again, if the two given Members of a Square be $aa-ca$, and the third is required; Firſt, I take half the Coefficient c, *viz.* $\frac{1}{2}c$, and then ſquare it, and it gives $\frac{1}{4}cc$, or $\frac{cc}{4}$ for the Member ſought, and ſo is the Square compleated $aa-ca+\frac{1}{4}cc$, whoſe Root is $a-\frac{1}{2}c$.

In like manner, if it were required to make $aa+3ba$ a compleat Square, take half the Coefficient ($3b$) which is $\frac{3}{2}b$ or $\frac{3b}{2}$, whoſe Square is $\frac{9}{4}bb$, or $\frac{9bb}{4}$, which being joined to the two given Terms with the Sign +, it gives $aa+3ba+\frac{9}{4}bb$, whoſe Root is $a+\frac{3}{2}b$.

The ſame Rule is to be obſerved for the ſquaring of half the Coefficient when it is a Fraction.

As for Example: Let the two Members of a Square raiſed from a Binomial given be $aa+\frac{bd+3c}{f}$; and let it be required to find the third Member. Here half the Coefficient is $\frac{bd+3c}{2f}$, which being ſquared, gives $\frac{bbdd+6bdc+9cc}{4ff}$ for the Member ſought, and ſo the Square being compleated, is $aa+\frac{bd+3c}{f}+\frac{bbdd+6bdc+9cc}{4ff}$, whoſe Root is $a+\frac{bd+3c}{f}$

VII. When the Root of the given Square hath no Coefficient, then the Number 1 is ſuppoſed to be the Coefficient, half whereof (*viz.* $\frac{1}{2}$) being ſquared, gives ($\frac{1}{4}$) the third Member ſought to make it a compleat Square.

So $aa+a$ being given for 2 of the Members of a Square raiſed from a Binomial, its 3d Member, to make the Square compleat, will be $\frac{1}{4}$, for $aa+a = aa+1a$, where the Coefficient is 1, whoſe half is $\frac{1}{2}$, which being ſquared gives $\frac{1}{4}$ for the third Member ſought; ſo the Square being compleated, is $aa+a+\frac{1}{4}$, whoſe Root is $a+\frac{1}{2}$.

This Chapter ought to be well underſtood before any farther Progreſs be made; for the manner how to reſolve Queſtions which produce quadratick (or ſquare) Equations, doth principally depend thereupon.

CHAP. XV.

Concerning the Reſolution of Queſtions producing Quadratick Equations.

I. QUadratick (or Square) Equations, are ſuch adfected or compound Equations as conſiſt of three Terms, the higheſt of which is a Square, and is called the higheſt Term in the Equation, of which three Terms two are always unknown, and the third is always known: The firſt of the three is the Square of the Quantity or Number ſought, and the ſecond Term is the Product of the Quantity ſought, being multiplied by ſome known Number or Quantity, and is called the middle Term of an Equation; and the third Term is a Number or Quantity purely known.

So in this Equation, *viz.* $aa+ba=d$, the firſt and higheſt Term or Member is aa, which is the Square of the Quantity or Number ſought, and ba is the middle Term of the Equation, which is the Product of the Quantity ſought, it being drawn into b (which is known,) and the third Term or Member of this Equation is d, which is really known, and is uſually called the abſolute Number or Quantity given.

II. The Equations of this kind are of three Forms, which are laid down by Mr. *Kerſey*, in the 15th Chapter of the firſt Book of his Elements of Algebra, as follows, *viz.*

Equations of the firſt Form.

$$\begin{array}{rcl||rcl} aa+6a & = & 55 & aa+ca & = & b \\ aaaa+8aa & = & 48 & aaaa+daa & = & f \\ aaaaaa+4aaa & = & 837 & aaaaaa+gaaa & = & h \end{array}$$

Equations of the ſecond Form.

$$\begin{array}{rcl||rcl} aa-13a & = & 24 & aa-ba & = & k \\ aaaa-6aa & = & 27 & aaaa-paa & = & d \\ aaaaaa-2aaa & = & 48 & aaaaaa-maaa & = & z. \end{array}$$

Equations of the third Form.

$10a - aa = 24$ | $ca - aa = n$
$5aa - aaaa = 4$ | $raa - aaaa = s$
$9aaa - aaaaaa = 8$ | $raaa - aaaaaa = s$

III. The Reſolution of Equations which fall under the firſt Form.

When an Equation is compoſed after any of the three foregoing Forms, and any known Quantities are mixed with unknown, let it be ſo reduced by Tranſpoſition (according to the Rules of the 11th Chapter) as that the known Quantities may poſſeſs one ſide, and the unknown Quantities the other ſide of the Equation.

Example. Let this Equation be given, *viz.* $aa+b=-ba+bdc$.

By the Tranſpoſition of b on the firſt part of the Equation, and $-ba$ on the ſecond part, it will be reduced to this Equation, *viz.* $aa+ba=bdc-b$, which is an Equation of the firſt Form.: And when your Equation is ſo reduced, add to each part of the Equation the Square of half the Coefficient, and ſo will the firſt part of the Equation be an exact and compleat Square; then according to the 2d and 3d Rules of the 14th Chapter extract the ſquare Root of both parts of the Equation, and from the ſquare Roots of both parts of the Equation ſubtract half the Coefficient, and then you will diſcover the Value of a; as in the following Examples.

Queſt.

Queſt. 1.

What Number is that; which being ſquared, and multiplied by 8 (or *b*), the Sum of the ſaid Square and Product is equal to 384 (or *c*)?

Reſolution.

1. For the Number ſought, put a
2. Whoſe Square is aa
3. Its Product by 8 (or *b*) is ba
4. The Sum of the 2d and 3d Steps muſt be equal to 384 (or *c*) whence this Equation $aa+ba=c$
5. To each part of the Equation add the Square of (½*b*) half of the Coefficient, then will it be $aa+ba+\frac{1}{4}bb=c+\frac{1}{4}bb$
6. Then by extracting the ſquare Root of both parts of the Equation by the 2d and 3d Rules of the 14th Chap. it will be reduc'd to $a+\frac{1}{2}b=\sqrt{:\overline{c+\frac{1}{4}bb}}$
7. By the Tranſpoſition of ½*b* to the ſecond part of the Equation, the Value of *a* is diſcover'd to be $a=\sqrt{:\overline{c+\frac{1}{4}bb}}-\frac{1}{2}b$.

Which Equation is thus expreſſed in words, *viz.* the Number ſought is equal to the Remainder, when (½*b*) 4 is ſubtracted from the ſquare Root of the Sum of (*c*) 384, and ¼ of the Square of (*b*) 8 (added together,) which is 16; ſo that the Value of *a* is 16; for $c+\frac{1}{4}bb=400$, and $\sqrt{(2)}\ 400=20$, and $20-4=16$.

Queſt. 2.

What Number is that, whoſe Square being multiplied by 4 (or b) and its Biquadrate (or fourth Power) multiplied by 6 (or c) and the Products added together, the Sum is 3850 (or d)?

Reſolution.

1. For the Number ſought, put a
2. Its Square multiplied by b is baa
3. Its Biquadrate multiplied by c is $caaaa$
4. The Sum of the ſecond and third Steps muſt be equal to 3850 (or d), whence this Equation, *viz.* $caaaa+baa=d$
5. And becauſe the higheſt Power of the Equation is multiplied by c, therefore each part being divided by c, the Equation is $aaaa+\frac{baa}{c}=\frac{d}{c}$
6. To each part of the Equation add half the Square of the Coefficient $\left(\frac{b}{c}\right)$ and the Equation will be $aaaa+\frac{baa}{c}+\frac{bb}{4cc}=\frac{d}{c}+\frac{bb}{4cc}$
7. Then the ſquare Root of each part of the Equation in the 6th Step being extracted by the ſecond and third Rules of the 4th Chapter, the Equation will be $aa+\frac{b}{2c}=\sqrt{:\frac{d}{c}+\frac{bb}{4cc}}$

8. And by the Tranſpoſition of $\frac{b}{2c}$ to the ſecond part of the Equation, the Value of aa is found to be $aa=\sqrt{(2)}\overline{\frac{d}{c}+\frac{bb}{4cc}}\,\overline{\frac{2c}{2c}}$

9. And becauſe the Equation in the 8th Step is the value of aa, therefore if the ſquare Root of each part of that Equation be extracted, the value of a itſelf will be diſcover'd to be $a=\sqrt{(2)}\sqrt{:}\overline{\overline{\frac{d}{c}+\frac{bb}{4cc}}-\frac{b}{2c}}$.

which in words is as much as to ſay, the Number ſought (or a) is equal to the ſquare Root of the Remainder when $\left(\frac{b}{2c}\right)\frac{1}{3}$ is ſubtracted from the ſquare Root of the Sum of $\frac{3850}{6}$ $\left(\text{or}\frac{d}{c}\right)$ and $\frac{1}{144}$ $\left(\text{or }\frac{bb}{4cc}\right)$ being added together; ſo that the Value of a (or the Number ſought) is 5. For, $\sqrt{:}\frac{d}{c}+\frac{bb}{2cc}=\frac{76}{3}$ and $\frac{76}{3}-\frac{1}{3}\left(\frac{b}{2c}\right)=\frac{75}{3}=25$, and $\sqrt{:}25=5$, which is the Number ſought.

The Proof.

$$4\times5\times5+6\times5\times5\times5\times5=3850$$

You muſt remember always to reduce a Fraction to its loweſt Terms before you extract its Root.

IV. The Reſolution of Equations which fall under the ſecond of the three Forms before mentioned.

Queſt. 1.

What Number is that, which having 8 (or b) times itſelf ſubtracted from its Square, the Remainder is 48 (or c)?

Reſolution.

1. For the Number ſought, put a
2. Then will its Square be aa
3. The firſt Step multiplied by b is ba
4. If the 3d Step be ſubtracted from the 2d, the Remainder will be 48 (or c) whence this Equation $aa-ba=c$
5. To each part of that Equation add the Square of ($\frac{1}{2}b$) half the Coefficient, and then it will be $aa-ba+\frac{1}{4}bb=c+\frac{1}{4}bb$
6. Extract the ſquare Root of each part of the laſt Equation by the ſecond and 3d Rules of the 14th Chapter, and it is $a-\frac{1}{2}b=\sqrt{\overline{c+\frac{1}{4}bb}}$
7. And by the Tranſpoſition of $\frac{1}{2}b$ to the 2d part of the Equation, the Value of a is diſcover'd to be 12 $a=\sqrt{:\overline{c+\frac{1}{4}bb}}+\frac{b}{2}$

which in words is as much as to ſay, that the Number ſought (or a) is equal to the Sum of the univerſal ſquare Root of the Sum of 48 (or c) and a 4th part of the Square of b (or $\frac{1}{4}bb$) being added to 4 (or $\frac{b}{2}$) which is 12; for $c=48$, and $\frac{1}{4}bb=16$, and $48+16=64$, and $\sqrt{64}=8$, and $8+4\ (\frac{1}{2}b)=12$.

The Proof.

$$12\times12-8\times12=48.$$

Queſt. 2.

What Number is that, which having 12 (or b) times its Square ſubtracted from its Biquadrate, or 4th Power, the Remainder is 3328 (or c)?

Reſolution.

1. For the Number ſought, put a
2. Then its Biquadrate is $aaaa$
3. And its Square multiplied by 12 (or b) is baa
4. The Difference of the 2d and 3d Steps muſt be equal to 3328 (or c) whence this Equation, *viz.* $aaaa-baa=c$
5. Square half the Coefficient, and add to it each part of the Equation, and then it will be $aaaa-baa+\frac{1}{4}bb=c+\frac{1}{4}bb$
5. Extract the ſquare Root of both parts of the Equation, by the 2d and 3d Rules of the 14th Chapter, and then the Equation will be $aa-\frac{1}{2}b=\sqrt{:c+\frac{1}{4}bb}$
7. By the Tranſpoſition of $-\frac{1}{2}b$ to the contrary Coaſt, it will be. $aa=\sqrt{:c+\frac{1}{4}bb}+\frac{1}{2}b$

8. By extracting the ſquare Root of both parts of the Equation in the 7th Step, the Value of *a* is found to be 8.	$a = \sqrt{(2)}\sqrt{:c+\frac{1}{4}bb}+\frac{1}{2}b$.

which is as much as to ſay, that the Number ſought (or *a*) is equal to the univerſal ſquare Root of the Sum of 6 (or $\frac{1}{2}b$) being added to the univerſal ſquare Root of the Sum of 3328 (or *c*) and 36 (or $\frac{1}{4}bb$) which upon tryal you will find to be 8.

For, $c=3328$, and $b=12$, and $\frac{1}{4}bb=36$, wherefore $3328+36=3364$, and $\sqrt{:3364}=58$, and $58+6$ $(\frac{1}{2}b)=64$, and $\sqrt{:64}=8$, which is the Number ſought.

V. The manner of reſolving Equations which fall under the laſt of the three Forms beforementioned.

Let the Equation propoſed (if it falls under the third and laſt Form) be reduced to an Equation of the ſecond Form, by the Tranſpoſition of its Terms, as in the following Queſtions, *viz.*

What Number is that, whoſe Square being ſubtracted from 12 (or *b*) times itſelf, the Remainder is 32 (or *c*)?

Reſolution.

1. For the Number ſought, put	a
2. Its Product by 12 (or *b*) is	ba
3. If from the 2d Step you ſubtract (*aa*) the Square of the firſt Step, the Remainder is	$ba-aa$

4. The Remainder in the 3d Step is equal to 32 (or c) whence this Equation. } $ba-aa=c$

Now by Tranſpoſition I reduce it to an Equation of the ſecond of the foreſaid Forms. And firſt,

5. By Tranſpoſition of aa to the contrary part, the Equation is } $ba=c+aa$

6. Then by Tranſpoſition of c in the 5th Step, the Equation is } $ba-c=aa$

7. And by Tranſpoſition of ba in the 6th Step, the Equation will be } $-c=aa-ba$ or $aa-ba=-c$.

So that from a due Conſideration of the Method uſed in reducing Equations of the 3d Form to Equations of the ſecond Form, you may eaſily perceive that the Work of Tranſpoſition in the 5th, 6th, and 7th Steps is performed only by changing the Signs of all the Terms of the Equation in the 4th Step, *viz.* by changing $+$ into $-$, and $-$ into $+$.

So the Equation in the fourth Step is $ba-aa=c$.

And by changing the Signs of $ba-aa$ on the firſt part of the Equation, and of c in the ſecond part, into $-ba+aa$, and $-c$, the Equation will then be $-ba+aa=-c$, or $aa-ba=-c$, which is the ſame with that in the 7th Step; and it is now an Equation of the ſecond of the three foregoing Forms; *ſo that now* I proceed to the Solution of *the Equation.*

8. The Square of half the Coefficient ($\frac{1}{2}b$) in the 7th Step being added to each part of the Equation, it will then be $aa-ba+\frac{1}{4}bb=-c+\frac{1}{4}bb$

9. The ſquare Root of each part of the laſt Equation being extracted by the 2d and 3d Rules of the 14th Chapter, the Equation will then be $a-\frac{1}{2}b=\sqrt{:-c+\frac{1}{4}bb}$

10. And by the Tranſpoſition of $-\frac{1}{2}b$ in the ninth Step to the contrary part, the Value of *a* will be found to be (8) $a=\sqrt{-c+\frac{1}{4}bb}+\frac{1}{2}b$

which is as much as to ſay, that the Number ſought (or *a*) is equal to the Sum of 6 (or $\frac{1}{2}b$) being added to the ſquare Root of the Remainder, when 32 (or *c*) is ſubtracted from 36 (or $\frac{1}{4}bb$) which is 8. For, $\frac{1}{4}bb-c=4$ whoſe ſquare Root is 2, and $2+6$ (or $\frac{1}{2}b$)$=8$, which is the Number ſought.

The Proof.

$12\times8=96$

And $96-64$ (*aa*) $=32$ (or *c*) which was propounded.

VI. The Reſolution of various Queſtions producing Quadratick Equations.

Queſt. 1.

There are two Numbers whoſe Sum is 12 (*or b*) *and the* Sum of their Squares is 80 (or c) I demand what are thoſe Numbers?

Reſo

Reſolution.

1. For one of the Numbers ſought, put a
2. Then the other will be $b-a$
3. Then the Sum of their Squares will be $bb-2ba+2aa$.
4. Which Quantity in the 3d Step is equal to 80 (or c) whence this Equation $bb-2ba+2aa=c$
5. Which Equation being duly reduced by the Rules of the 11th Chapter, giveth this Equation $aa-ba=\frac{c}{2}-\frac{bb}{2}$
6. Which Equation being ſolved according to the 3d Rule of this Chapter, the Value of a is diſcovered to be $a=\sqrt{\frac{c-bb}{2}+\frac{bb}{4}}+\frac{b}{2}$
7. Wherefore I conclude the Numbers ſought are 8 and 4, for their Sum is 12, and the Sum of their Squares is 80.
8. Moreover, the Equation in the ſixth Step will give this

Canon.

If from half the given Sum of the Squares you ſubtract half the Square of the given Sum, and to the Remainder you add half the given Sum, the ſquare Root thereof being added to the ſaid half Sum of the Numbers, the Sum of this Addition will give you the greater Number ſought; and the greater Number being ſubtracted from the *given Sum of the* Numbers, will give the leſſer *Number ſought.*

Queſt. 2.

There are two Numbers, the Product of wl Multiplication is *96* (or b) and the Sum of tl Squares is 208 (or c) I demand what are tl Numbers.

Reſolution.

1. For one of the Numbers ſought, put a
2. Then by dividing *96* (or b) by a, the Quotient will give the other, which is $\frac{b}{a}$
3. The Square of the Number in the firſt Step is aa
4. The Square of the other Number in the 2d Step is $\frac{bb}{aa}$
5. And the Sum of their Squares in the 3d and 4th Step is $aa+\frac{bb}{aa}$
6. Which Sum in the 5th Step muſt be equal to the given Sum of the Squares 208 (or c) whence followeth this Equation $aa+\frac{bb}{aa}=c$
7. Which Equation in the laſt Step being reduced by the Rules of the 11th Chapter, Value of a will be diſcover'd to be

$$a=\sqrt{(2)}\,\overline{\sqrt{\tfrac{1}{4}cc-bb}+\tfrac{1}{2}c}.$$

8. So that I conclude the Numbers ſought t *12* and 8; for their Product is 96, an Sum of their Squares is 208.

9. Moreover the Equation in the 7th Step giveth this

Canon.

From ¼ of the Square of the given Sum of the Squares, ſubtract the Square of the given Product of the Multiplication of the Numbers ſought, and extract the ſquare Root of the Remainder, and to the ſaid ſquare Root add half the given Sum of the ſaid Squares, and then extract the ſquare Root of the Sum of that Addition, ſo ſhall that ſquare Root be one of the Numbers ſought, by which if you divide the given Product, the Quotient will be the other Number ſought.

Queſt. 3.

There are two Numbers whoſe Sum is 12 (or b) and the Product of their Multiplication is 20 (or c) what are the Numbers?

Reſolution.

1. For one of the Numbers ſought, put } a
2. Which if you ſubtract from (12) b, the given Sum, the Remainder will be the other Number, *viz.* } $b-a$
3. And if the 1ſt and 2d Steps be multiplied the one by the other, the Product will be } $ba-aa$

4. Which Product the Queſtion requires to be equal to 20 (or c) from whence this Equation $\Big\}$ $ba - aa = c$

5. Which Equation is of the third and laſt Form, mentioned in the beginning of this Chapter, which being duly reduced by the Rules of the 11th Chapter, it will be

$$aa - ba = -c$$

6. Which Equation being ſolved according to the Method uſed in the 4th Rule of this Chapter, the Value of a will be diſcovered to be

$$a = \sqrt{:\tfrac{1}{4}bb - c} + \tfrac{1}{2}b$$

7. So that I conclude the Numbers ſought to be 10 and 2, whoſe Sum is 12, and their Product 20, according to the Conditions of the Queſtion. Moreover, the Equation in the 6th Step will preſent you with this

Canon.

From the Square of the half given Sum of the Number ſought, ſubtract their given Product, and extract the ſquare Root of the Remainder, and to its ſquare Root add half the given Sum of the Numbers ſought, ſo ſhall the Sum of that Addition be the greater Number ſought, which being ſubtracted from the ſaid given Sum, will leave the leſſer.

Queſt. 4.

There are three Numbers which are Geometrical Proportionals continued, the Mean whereof is 12 (or b) and the two Extreams are ſuch, that their Difference is 18 (or c) I demand what are thoſe Extreams.

Reſolution.

1. For the leſſer Extream, put a
2. Then the greater will be $a+c$
3. Then will the Product made by the Multiplication of the Extreams of the 1ſt and 2d Steps, be $aa+ca$
4. Which Product (or Rectangle) in the 3d Step muſt be equal to the Square of 12 (or b) the Mean, whence this Equation $aa+ca=bb$
5. Which Equation being ſolved by the 2d Rule of this Chapter, the Value of a will be found to be $a=\sqrt{:bb+\frac{1}{4}cc}-\frac{1}{2}c$
6. I ſay the extream Proportionals ſought are 6 and 24, whoſe Difference is 18: For

$$6 : 12 :: 12 : 24, \text{ or } a : b :: b : \frac{bb}{a}.$$

7. The Equation in the 5th Step being well conſidered, will preſent you with this

Canon.

If to the Square of the given Mean you add the Square of half the Difference of the Extreams, or (which is all one) ¼ part of the Square of the given Difference of the Extreams, and extract the ſquare Root of the Sum of that Addition, and then from that ſquare Root ſubtract half the ſaid Difference, the Remainder will be the leſſer Extream; and if thereto you add the given Difference, that Sum will be the greater Extream.

Queſt. 5.

A Draper ſold a Piece of Cloth for 24 *l.* (or *b*) and gained as much *per Cent.* (or *c*) as the Cloth coſt him, I demand how much it coſt?

Reſolution.

1. For the Price which the Cloth coſt, put } a
2. Then will the Gain by its Sale be } $b-a$
3. Then by the Rule of Three find how much is gained *per Cent.* ſaying, $a : b-a :: c : \frac{cb-ca}{a}$ ſo that his Gain *per Cent.* was } $\frac{cb-ca}{a}$
4. Which Quantity in the 3d Step, according to the Tenour of the Queſtion, muſt be equal to what the Cloth coſt in the 1ſt Step, whence this Equation, *viz.* } $a = \frac{cb-ca}{a}$

5. Which being reduc'd by the Rules of the 11th Chapter, it will be } $aa+ca=bc$

6. Which (being an Equation of the firſt of the 3 Forms deliver'd in the beginning of the 15th Chapter) being ſolved by the 2d & 3d Rule of the 14th Chap. the Value of *a* is diſcover'd to be } $a=\sqrt{:cb+\frac{cc}{4}}-\frac{c}{2}$

I ſay, the Cloth coſt 20 *l.* which is the Value of *a*; for $\sqrt{:cb+\frac{cc}{4}}=70$, and $70-\frac{c}{2}=20$, ſo that he gained 4 *l.* in laying out 20; for,

$$\overset{l.}{20} : \overset{l.}{4} :: \overset{l.}{100} : \overset{l.}{20}$$

and ſo the Conditions of the Queſtion are ſatisfied.

Queſt. 7.

A Merchant bought a certain number of Pieces of Cloth, and paid 30 Pounds (or *b*) *per* Cloth, and ſold them again at ſuch a rate *per* Cloth, that if the Pounds he ſold a Cloth for, be multiplied by the Pounds he gained *per* Cloth, the Product will be equal to the Cube of the number of Pounds gained *per* Cloth; I demand what he gained *per* Cloth, and what he ſold each Cloth for?

Reſolution.

1. For the number of Pounds gained *per* Piece, put	a
2. To which if you add 30 (or b) the Sum will be the Number of Pounds it was ſold for *per* Piece, *viz.*	$a+b$
3. And if (according to the Tenour of the Queſtion) the 2d Step be multiplied by the firſt, the Product will be	$aa+ba$
4. Which Product in the 3d Step, muſt (according to the nature of the Queſtion) be equal to the Cube of the Pounds gained *per* Cloth in the firſt Step, whence this Equation, *viz.*	$aa+ba=aaa$
5. Which Equation being reduced by the 3d and 6th Rules of the 11th Chapter, it will then be	$aa-a=b$
6. Which Equation in the 5th Step being ſolved by the 7th Rule of the 14th Chapter, and the 6th Rule of this Chapter, the Value of a will be diſcover'd to be	$a=\sqrt{:b+\frac{1}{4}}+$

which is as much as to ſay in words, a (or Gain *per* Cloth) is equal to the Sum, when added to the ſquare Root of the Sum of 30 a added together, (*viz.* the ſquare Root of 3 which is $5\frac{1}{2}+\frac{1}{2}=6$.

I ſay, he gained 6 Pounds *per* Cloth, and he ſold it for 36 Pounds *per* Cloth; which two Numbers will ſatisfy the Conditions of the Queſtion.

The Proof.

$$6 \times 36 = 6 \times 6 \times 6 = 216$$

Queſt. 8.

A Bricklayer and a Labourer wrought together at the building of a certain Houſe 42 Days, (or b) and the Labourer wrought 4 (or c) Days more than the Bricklayer did to gain one Pound, and at the end of the 42 Days, the Bricklayer received for his Work $1\frac{3}{4}$ Pounds (or d) more than the Labourer; I demand how many Days each of them wrought for 1 *l*?

Reſolution.

1. For the number of Days which the Bricklayer wrought for 1 *l.* put } a
2. Then, according to the Conditions of the Queſtion, the number of Days that the Labourer wrought for 1 *l.* will be } $a+c$
3. By the Rule of Proportion find how many Pounds the Bricklayer received for the Work of 42 Days, as followeth, } $\frac{b}{a}$

$a : 1 :: b : \frac{b}{a}$

which is

4. Then find out by the Rule of 3 how many Pounds the Labourer received for his Work in 42 Days, thus; $a+c : 1 :: b : \frac{b}{a+c}$ which is

$$\frac{b}{a+c}$$

5. But the Bricklayer received $1\frac{1}{2}$ (or *d*) Pounds more than the Labourer for 42 Days Work, wherefore if to the 4th Step you add *d*, it will be equal to what the Bricklayer receiv'd in the third Step, whence this Equation, *viz.*

$$\frac{b+da+dc}{a+c}=\frac{l}{a}$$

6. Which Equation being reduced by the 4th, 2d, and 7th Rules of the 11th Chapter, it will then be

$$aa+ca=\frac{bc}{d}$$

7. The Equation in the 5th Step being ſolved by the Rule of this Chapter, the Value of *a* will be diſcovered, *viz.*

$$a=\sqrt{\frac{bc}{d}+\frac{cc}{4}}\cdot$$

which is as much as to ſay *a* (or the numbe Days which the Bricklayer wrought) is equ the Difference when $(\frac{c}{2})$ 2 is ſubtracted fron ſquare Root of the Sum of $(\frac{bc}{d})\frac{672}{7}$ or 96 and 4 (which is 100)$=10-2=8$.

I ſay the Bricklayer wrought 8 Days *twenty* Shillings, and the Labourer wrc

$8+4=12$ days, which two Numbers will ſatisfy the Conditions of the Queſtion, as will appear by the

Proof.

Firſt, by the Rule of Three find what the Bricklayer received for the 42 days, ſaying,

days		*l.*		days		*l.*	*l.*	*s.*
8	:	1	::	42	:	$\frac{42}{8}=$	5	5

Then, find how much the Labourer received for his 42 days Work by the Rule of Three, ſaying,

days		*l.*		days		*l.*	*l.*	*s.*
12	:	1	::	42	:	$\frac{42}{12}=$	3	10

So that I find the Bricklayer for his 42 days Work received 5 *l.* 5 *s.* and the Labourer 3 *l.* 10 *s.* which is 1 *l.* 15 *s.* or $1\frac{3}{4}$ *l.* leſs than the Bricklayer received.

Queſt. 9.

A Gentleman bought a Houſe, and ſold it again for 280 Pounds (or *b*) and by its Sale he gained ſo many Pounds, that their Square being added to the Square of the number of Pounds it coſt him, the Sum will amount to 52000 (or *c*) Pounds; now I demand how much the Houſe coſt him?

Reſolution.

1. For the number of Pounds which the Houſe coſt, put } a.
2. Then will the Gain by its Sale be } $b-a$
3. The Square of (a) its firſt Coſt, is } aa
4. The Square of the Gain by Sale is } $aa-2ba+bb$
5. The Sum of the two Quantities in the third and 4th Steps is } $2a-2ba+bb$
6. Which Quantity in the 5th Step is equal to 52000*l.* (or c) whence this Equation } $2aa-2ba+bb=c$
7. Which Equation being reduced by the 3d and 7th Rules of the 11th Chapter, it will be } $aa-ba=\frac{c-bb}{2}$
8. Which Equation being ſolved by the 3d Rule of this Chapter, the Value of a will be diſcover'd, *viz.* } $a=\sqrt{\frac{c-bb}{2}+\frac{bb}{4}}+\frac{1}{2}b$

which is as much as to ſay in words, (a) the Price which the Houſe coſt, is equal to the Sum, when half what he ſold it for is added to the ſquare Root of the Sum of half the given Sum of the Squares added to a 4th part of the Square of what it was ſold for, that Sum being made leſs by half the Square of what it was ſold for, which was 220 *l.* and he gained by the Sale 60 *l.* For, $\frac{c}{2}=26000$, and $\frac{bb}{4}=19600$, and $\frac{bb}{2}=39200$, now

26000

$26000 + 19600 = 45600$, and $45600 - 39200 = 6400$, and $\sqrt{(2)}\ 6400 = 80$, and $80 + (\frac{1}{2}b)\ 140 = 220$, which is the number of Pounds the Houſe coſt, and $280 - 220 = 60$, which is the number of Pounds he gained by the Sale of the Houſe, as you will find by

The Proof.

$$220 \times 220 = 48400, \text{ and}$$
$$60 \times 60 = 3600, \text{ and}$$
$$48400 + 3600 = 52000$$

whereby the Conditions of the Queſtions are anſwered.

Queſt. 10.

A Draper ſold 2 Pieces of Cloth (whereof one contained 6 (or b) Yards more than the other) for two equal numbers of Shillings, the leſſer Piece he ſelleth for 2 (or c) Shillings *per* Yard more than the other, and the number of Shillings which one Piece was ſold for, did exceed the number of Yards in both Pieces by 186 or (d): the Queſtion is, what was the number of Yards in each Piece, and what each Piece was ſold for *per* Yard?

Reſolution.

1. For the number of Yards in the leaſt Piece, put } a
2. Then will the Yards in the great Piece be } $a+b$
3. Then will the Sum of the *Yards in both* Pieces be } $2a+b$

4. The

4. Then if (according to the Nature of the Queſtion) to the Sum of the Yards in the third Step, you add 186 (or *d*) the Sum will be the number of Shillings which each Piece was ſold for, *viz.* $2a+b+d$

5. And if the Quantity in the fourth Step be divided by (*a*) the Quantity in the firſt Step, the Quotient will give the number of Shillings that 1 Yard of the leaſt Piece was ſold for $\frac{2a+b+d}{a}$

6. And if the ſaid Quantity in the 4th Step be divided by the number of Yards in the biggeſt Piece, (which is the Quantity in the 2d Step) the Quotient will give the number of Shillings that a Yard of the biggeſt Piece was ſold for, which is $\frac{2a+b+d}{a+b}$

7. If to the Quantity in the 6th Step you add 2 (or *c*) Shillings, it will then be $\frac{2a+b+d+ca+cb}{a+b}$

8. Which Quantity in the 7th Step (as the Queſtion requires) is equal to the Quantity in the 5th Step, whence this Equation, *viz.*

$$\frac{2a+b+d+ca+cb}{a+b}=\frac{2a+b+d}{a}$$

9. Which

Which Equation in the 8th Step being reduced by the Rules in the 11th Chapter, it will then be

$$aa+\frac{cb-2b}{c}+a=\frac{bb+bd}{c}$$

. The Equation in the laſt Step being ſolved by the 3 Rule of this Chapter, the Value of *a* will be diſcover'd to be

$$a=\sqrt{\frac{bb+bd}{c}+\frac{ccbb-4cbb}{4cc}}+\frac{4bb-cb-2b}{2c}$$

11. But if you conſider well the Equation in e 9th Step, you will find the Coefficient to 0, for $\frac{cb-2b}{c}=0$; and therefore $\frac{cb-2b}{2c}a=0$, hence the middle Term in that Equation 0; and therefore the middle Term being removv'd, the Equation will be $aa=\frac{bb+bd}{c}$ ich is a Simple Equation, and if the Square oot of both parts of that Equation be extracted, Value of *a* will be diſcover'd to be $a=\sqrt{\frac{bb+bd}{c}}$ 24, which is the ſame with the Value of *a* in 10th Step, as you may eaſily find upon tryal, erefore I ſay,

The number of Yards in the leaſt Piece is 24.
And the number of Yards in the biggeſt Piece 24+6=30; which two Numbers will ſatisfy Conditions of the Queſtions, as will appear

The

The Proof.

The number of Yards in both Pieces is 24+30 =54, which if added to 186 (as the Queſtion requires) it will give the number of Shillings which one Piece was ſold for, which is 54+186 =240, and the leaſt Piece was ſold at 10 Shillings *per* Yard.

yards. *s.* yard *l.* *s.*
$24 : 240 :: 1 : \frac{240}{24} = 10$

And the Price of a Yard of the biggeſt Piece was 8 *s.* For,

yards *s.* yard *l.* *s.*
$30 : 240 :: 1 : \frac{240}{30} = 8$

which is two Shillings *per* Yard leſs than the leſſer Piece was ſold for *per* Yard, and therefore the Anſwer is true, and the Conditions of the Queſtion are ſatisfied.

* * *

CHAP.

CHAP. XVI.

The Doctrine of Surd Quantities.

ALL Quantities or Numbers whatſoever, whether Integral or Fractional, are alled Rational; but when the Root of any Power annot be exactly extracted, ſuch Root is called rational or Surd, and is expreſſed by putting e Radical Sign before the Number out of which e Root propoſed ought to be extracted; as $\surd$ r $\surd(2)$ placed before any Number or Quantity gnifieth the ſquare Root of the Quantity or umber, and $\surd(3)$ the Cube Root, and $\surd(4)$ the iquadrate Root, *&c.* So $\surd 12$, or $\surd(2)\,12$ ſignifieth the Square Root of 12, and $\surd(3)\,12$ its Cube oot, *&c.*

II. Surd Numbers are two-fold, *viz.* Simple nd Compound: a Simple Surd Quantity is when e Radical Sign is prefixed to a Simple Quantity, s $\surd(3)\,5$ or $\surd(4)\,ab$.

A Compound Surd Quantity conſiſts of ſeveral imple Surds, which are connected together by $+$ or $-$, as $\surd 4+\surd 6$, and $\surd ab+\surd ac+\surd d$, and $(2)\overline{ab+dc}$, which laſt Compound Surd is uſually lled *an univerſal* Root.

III. To reduce Simple Surd Quantities that have different Radical Signs to a common Radical Sign.

Let the Indices of the given Powers be reduced to the loweſt Terms by their common Meaſurer, and ſet the Quotients under their reſpective Dividends, and multiply croſs-wiſe, ſo ſhall the Product be the Index required, before which placing $\surd$, it will then be the common Radical Sign requir'd; then raiſe the Powers of the given Roots to the Powers ſignified by the ſaid altern. Quotients, before which ſaid Powers place the common Radical Sign found as before, ſo will you have new Surd Quantities equal to the given Quantities, and having equal Radical Signs.

Example. Let it be requir'd to reduce $\surd(6)\,8$, and $\surd(8)\,12$ to two other Roots equivalent to the former, having a common Radical Sign.

$$\begin{array}{ccc} \surd(6)8 & & \surd(8)12 \\ 3 & \times & 4 \\ \surd(24)4096 & & \surd(24)1728 \end{array}$$

Firſt, the Exponents 6 and 8 are reduced to 3 and 4, which being placed under the given Exponents 6 and 8 as you ſee, and having multiplied croſs-wiſe, *viz.* 3×8, or 4×6, you have 24 for a new Index, to which prefix $\surd$, and it is $\surd(24)$ for the common Radical Sign, and then raiſing 12 to the third Power thereof, and 8 to the fourth, you have $\surd(24)4096$ and $\surd(24)1728$ equal to $\surd(6)8$, and $\surd(8)12$.

So if it were required to reduce $\surd(4)a$, and $\surd(6)b$ to Surd Roots equivalent thereto, having a common Radical Sign, it will be as followeth:

$\surd(4)$

$$\begin{array}{ccc} \surd(4)a & \times & \surd(6)b \\ 2 & & 3 \\ \surd(12)aaa & \text{and} & \surd(12)bb \end{array}$$

IV. Multiplication in Simple Surd Quantities.

1. If the Quantities given to be multiplied have a common Radical Sign, then multiply them together without any regard to the Sign, and to the Product prefix the given Radical Sign, which new Quantity ſhall be the Product ſought.

So if $\surd 6$ be to be multiplied by $\surd 8$, the Product will be $\surd 48$, and $\surd(3)4$ by $\surd(3)8$ produceth $\surd(3)32$, and $\surd a$ by $\surd b$ produceth $\surd ab$, and $\surd(3)c$ by $\surd(3)bd$ produceth $\surd(3)cbd$, *&c.*

2. But if the Quantities given to be multiplied have not a common Radical Sign, let them be reduced to ſuch by the third Rule foregoing, and then proceed as before.

Example. What is the Product of $\surd(4)a$ by $\surd(6)b$? The ſaid Quantities being reduced to a common Radical Sign, will be $\surd(12)aaa$ and $\surd(12)bb$; which being multiplied together, produce $\surd(12)aaabb$, which is the Product ſought.

So the $\surd(2)b$ being multiplied by $\surd(3)c$, they being reduced to a common radical Sign, are $\surd(6)bbb$, and $\surd(6)cc$, which being multiplied produce $\surd(6)bbbcc$.

3. When a ſurd Quantity is to be multiplied by a rational Quantity, then firſt raiſe the given rational Quantity to the Power of the given Quantity, whoſe Root is irrational or ſurd; and then proceed as before.

So if it were requir'd to multiply $\surd 5$ by 5, the rational Number 5 being raiſed to the ſecond Power is 25, and then you will have to multiply $\surd 5$ *by* $\surd 25$, *whoſe* Product is $\surd 125$.

Likewiſe $\surd(3)b$ being to be multiplied by a, the Product will be $\surd(3)baaa$; for a being raiſed to the third Power is aaa, and $\surd(3)b$ by $\surd(3)aaa$ produceth $\surd(3)baaa$, as before.

V. Diviſion in ſimple ſurd Quantities.

1. Reduce the ſurd Quantities given to be divided, to a common radical Sign by the third Rule of this Chapter, and then divide the Quantity following the radical Sign of the Dividend by the Quantity following the radical Sign of the Diviſor, and to the Quotient prefix the ſaid common radical Sign, ſo ſhall that ſurd Quantity be the Quotient ſought.

Example. There being given $\surd 15$ to be divided by $\surd 3$, the Quotient will be $\surd 5$. And $\surd b$ being to be divided by $\surd a$, the Quotient will be $\surd\frac{b}{a}$ and $\surd(2)a$ being given to be divided by $\surd(3)bc$, the Quotient will be $\surd(6)\frac{aaa}{bbcc}$, for the given Quantities being reduced to a common radical Sign, are $\surd(6)aaa$, and $\surd(6)bbcc$.

VI. Addition and Subtraction of ſimple ſurd Quantities.

1. When the ſurd Roots to be added together are equal, multiply any one of them by the given number of ſurd Quantities, ſo ſhall that Product be the Sum required, before which prefix the radical Sign given; ſo the Sum of $\surd 6$, and $\surd 6$ is $\surd 24$, for the given Number of Roots is 2, whoſe Square is 4, and $\surd 4 \times \surd 6 = \surd 24$; ſo $\surd(3)b$ being to be added to $\surd(3)b$, their Sum is $\surd(3)8b$, and $\surd(3)a$ being to be added to $\surd(3)a$, and $\surd(3)a$, their Sum will be $\surd(3)27$

$\surd(3)27a$, for the given Number of Surds is 3, and $\surd(3)a$ being multiplied by 3, *viz.* $\surd(3)27$ (by the third part of the fourth Rule) the Product is $\surd(3)27a$, which is the Sum of $\surd(3)a$, $\surd(3)a$, and $\surd(3)a$, which was required.

2. When two unequal ſurd Roots which have the ſame radical Sign prefixed to each of them, be to be added together, or when the leſſer of them is to be ſubtracted from the greater, then you muſt firſt try whether they be commenſurable, or not; that is, if after they have been divided by their greateſt common Meaſurer, the Quotients be rational Quantities, then multiply the Sum of thoſe rational Quantities by the ſaid common Diviſor, and the Product ſhall be the Sum of the ſurd Quantities propounded; and if the Difference of thoſe rational Quotients be multiplied by the ſaid common Meaſurer, then will the Product be the Difference of the ſurd Quantities propounded.

Example. Let it be required to find the Sum and Difference of $\surd 50$, and $\surd 8$, their greateſt common Meaſurer is $\surd 2$, by which they being divided, the Quotients are $\surd 25$ and $\surd 4$, *viz.* 5 and 2, whoſe Sum is 7; which being multiplied by $\surd 2$, the Product is $7\surd 2$, or $\surd 98$, which is the deſired Sum of the ſurd Quantities propounded. And if the Difference of the ſaid rational Quotients, *viz.* $5-2$ (or 3) be multiplied by the ſaid common Diviſor ($\surd 2$), the Product will be $3\surd 2=\surd 18$, which is the Difference of the ſurd Quantities given, the leſſer being ſubtracted from the greater.

But if the ſimple ſurd Quantities given to be added or ſubtracted, be incommenſurable, neither *heir Sum nor Difference* can be expreſs'd by any ſimple Term or Root, but their Sum and Difference muſt be expreſs'd by + and —; as ſuppoſe you were to add $\sqrt{10}$ and $\sqrt{13}$ together, their Sum would be $\sqrt{13}+\sqrt{10}$, and their Difference $\sqrt{13}-\sqrt{10}$. The like of other Quantities expreſs'd by Letters.

CHAP. XVII.

The Parts of Numeration in Compound Surd Quantities.

I. *Addition and Subtraction in Compound Surd Quantities.*

THE Addition and Subtraction of compound ſurd Quantities is the ſame with the ſimple Surds, having reſpect to the Signs of Affirmation and Negation, *viz.* + and —.

So if to $6+\sqrt{18}$ $(3\sqrt{2})$ you add $4+\sqrt{8}$ $(2\sqrt{2})$ the Sum will be $10+\sqrt{50}$ $(5\sqrt{2})$ and if from $6+\sqrt{18}$ $(3\sqrt{2})$ you ſubtract $4+\sqrt{8}$ $(2\sqrt{2})$ the Difference will be $2-\sqrt{2}$.

Likewiſe, if to $\sqrt{320}+\sqrt{108}$ $(85\sqrt{}+6\sqrt{3})$ you are to add $\sqrt{80}-\sqrt{27}$ $(4\sqrt{5}-3\sqrt{3})$ the Sum is $\sqrt{720}+\sqrt{27}$ $(12\sqrt{5}+3\sqrt{5})$ and if you ſubtract

:ter from the former, the Remainder will
o $+-\sqrt{243}\,(4\sqrt{5}+9\sqrt{3})$.
ſe two Examples are of compound ſurd
ities which are commenſurable, and the
of compound ſurd Quantities, partly com-
able, and partly incommenſurable. As,
it be required to add $\sqrt{12}(2\sqrt{3})+\sqrt{5}$ to
$\sqrt{3})+\sqrt{8}$; the Sum will be $\sqrt{7}(5\,5\sqrt{3})+\sqrt{8}$
; and if the former be ſubtracted from the
the Remainder will be $\sqrt{3}+-\sqrt{8}-\sqrt{5}$.
ſame is to be obſerved in Addition and
ction of compound ſurd Quantities altoge-
commenſurable: as in the following Ex-

ind from $\sqrt{10}+\sqrt{7}$
and ſubtract $\sqrt{3}+\sqrt{2}$

$\sqrt{10}+\sqrt{7}+-\sqrt{3}+\sqrt{2}$

$\sqrt{:}17+\sqrt{280}+\sqrt{:}5+\sqrt{24}$

erence is $\sqrt{10}+\sqrt{7}-\sqrt{3}\sqrt{2}$

$\sqrt{:}17+\sqrt{280}-\sqrt{:}5+\sqrt{24}$

ind from $\sqrt{(3)}10+\sqrt{(3)}7$
and ſubtract $\sqrt{(3)}3-\sqrt{(3)}2$

is $\sqrt{(3)}10+-\sqrt{(3)}7+\sqrt{(3)}3-\sqrt{(3)}2$

erence is $\sqrt{(3)}10+\sqrt{(3)}7-\sqrt{(3)}3+\sqrt{(3)}2$

II. *Multiplication in Compound Surd Quantities.*

Multiplicand $\sqrt{180}+\sqrt{48}\ (6\sqrt{5}+4\sqrt{3})$
Multiplier $\sqrt{125}+\sqrt{12}\ (5\sqrt{5}+2\sqrt{3})$

$150+20\sqrt{15}$
$+12\sqrt{15}+24$

Product $150+32\sqrt{15}+24$
Product contracted $174+32\sqrt{15}$

Multiplicand $\sqrt{abb}+\sqrt{cff}\ (b\sqrt{a}+f\sqrt{c})$
Multiplier $\sqrt{add}+\sqrt{caa}(d\sqrt{a}+a\sqrt{c})$

$bda+fd\sqrt{ca}$
$+ba\sqrt{ca}+fac$

Product $bda+\overline{fd+ba}\times\sqrt{ca}+fac$

Multiplicand $\sqrt{bc}+a$
Multiplier $\sqrt{bc}-a$

Product $bc-aa$

III. *Diviſion in Compound Surd Quantities.*

Dividend	$\sqrt{21}+15$	$\sqrt{(3)14}-\sqrt{(3)28}$
Diviſor	$\sqrt{3}$	$\sqrt{(3)7}$
Quotient	$\sqrt{7}+\sqrt{5}$	$\sqrt{(3)\,2}-\sqrt{(3)4}$

Diviſor. Dividend.

$$\begin{array}{ll}
a+\sqrt{bc}) & ab+b\sqrt{bc}\ (b \text{ Quotient.} \\
 & ab+b\sqrt{bc} \\
\hline
 & 0 \quad 0
\end{array}$$

$$\begin{array}{lll}
a+\sqrt{bc}) & aaa+bc\sqrt{bc}\ (aa+bc-a\sqrt{bc} & \\
 & aaa+aa\sqrt{bc} & \\
\hline
 & +bc\sqrt{bc}-aa\sqrt{bc} & \\
 & +bc\sqrt{bc}+abc & \\
\hline
 & \quad -aa\sqrt{bc}-abc & \\
 & \quad -aa\sqrt{bc}-abc & \\
\hline
 & \quad 0 \qquad 0 &
\end{array}$$

Theſe Examples will not ſeem difficult to the Ingenious, if what is before delivered concerning ſurd Quantities be duly conſidered.

CHAP. XVIII.

The Parts of Numeration in univer Surd Roots.

WHEN it is required to extract the R of any Compound Quantity, whe Square, Cube, Biquadrate, *&c.* if they canno exactly extracted without any Remainder; t if to ſuch given Compound Quantity you pr the Radical Sign, ſuch Roots are called Univ Surd Roots. And firſt, concerning

I. *Multiplication in Univerſal Surds.*

1. When any univerſal Root is to be m plied by a Rational-Quantity, or by any S multiply the Square of the Multiplicand by Square of the Multiplier, when the univ radical Sign is quadratick; or the Cube of Multiplicand by the Cube of the Multiplier, w the univerſal radical Sign is cubical, and be that Product prefix the given univerſal rac Sign, ſo ſhall that new univerſal Root be Product ſought.

Example. Let it be required to multiply by 2 this univerſal ſquare Root, *viz,* $\overline{\surd 10{:}+\surd 40}$, I take the Square of 2, which is 4, and the Square of $\overline{\surd 10+\surd 40}$, which is $10+40$, and multiply it by 4, and the Product is $40+4\times 40$, whoſe univerſal ſquare Root is the Product ſought, *viz.* $\overline{\surd{:}40+4\surd 40}$.

Alſo, if $\surd(3){:}\surd(3)64+\surd(3)27$ were to be multiplied by 2, or doubled, take the Cube of the univerſal Root given, which is $\surd(3)64+\surd(3)27$, and multiply the ſame by the Cube of 2, which is 8, and the Product is $8\surd(3)64+8\surd(3)27$, the Cube Root of which is the Product ſought, *viz.* $\overline{\surd{:}(3)8\surd(3)64+8\surd(3)27}$, and it is double to $\surd(3){:}\surd(3)64+\surd(3)27$, the ſurd Root given.

In like manner, if it were required to multiply $\surd{:}\overline{12+\surd 6}+\surd\overline{12-\surd 6}$ into itſelf, or to find its Square; the Squares of the Parts are $12+6$ and $12-6$, the Sum of which is 24, and the Product made by the multiplication of the Parts one into the other, *viz.* $\surd{:}\overline{12+\surd 6}$, into $\surd{:}\overline{12-\surd 6}$, is $\surd 138$, for the Difference of the Squares of 12 and $\surd 6$ is 138, whoſe ſquare Root is $\surd 138$, and the double of the ſaid Product is $2\surd 138$, which added to 24 (the Sum of the Squares of the Parts) makes $24+2\surd 138$, which is the Square of $\surd{:}12+\surd 6+\surd{:}12-\surd 6$.

Likewiſe, if $6+\surd{:}20-\surd 16$ is to be multiplied by $6-\surd{:}20-\surd 16$, the Product will be found to be 20; for if $20-\surd 16$ (which is the Square of $\surd{:}\overline{20-\surd 16}$) be ſubtracted from 36 (the Square of 6) there will *remain* $16+\surd 16$, which is 20, the

Product ſought. Alſo, $\overline{6+\surd:20-\surd:10}=10$, and $\overline{6-\surd:20-\surd 16}=2$, and $2\times 10=20$, as before.

Again, if it be required to multiply $\overline{\surd:aa+bb}$ by a, the Square of the given Quantities are $aa+bb$ and aa, which being multiplied the one into the other, the Product will be $aaaa+bbaa$, the univerſal ſquare Root of which is the Product ſought, *viz.* $\overline{\surd:aaaa+bbaa}$; which may be more compendiouſly expreſs'd thus, $\overline{a\surd:aa+bb}$.

II. *Diviſion in Univerſal Surds.*

As in Multiplication you multiplied the Square of the Multiplicand by the Square of the Multiplier, the given radical Sign being quadratick, *&c.* ſo in Diviſion of univerſal ſurd Roots you are to divide the Square of the Dividend by the Square of the Diviſor, when the univerſal radical Sign is quadratick; and divide the Cube of the Dividend by the Cube of the Diviſor, when the univerſal radical Sign is cubical, *&c.* ſo ſhall the Quotient, when the univerſal radical Sign given is prefix'd thereto, be the Quotient required.

Example. What is the Quotient when $\overline{\surd:40+4\surd 40}$ is divided by 2? Here I divide $40+\surd 40$ (which is the Square of $\surd:40+4\surd 40$ the Dividend) by 4 (the Square of the given Diviſor) and there ariſeth $10+\surd 40$; the univerſal ſquare Root of which, *viz.* $\surd:10+\surd 40$ is the Quotient required.

Alſo, if it were requir'd to divide $\surd(3)\overline{8\surd(3)64+8\surd(3)27}$ by 2, the Quotient would be found to be $\surd:(3)\surd(3)64+\surd(3)27$. Here the Cube of the given Dividend is $8\surd(3)64+8\surd(3)27$ which being divided by 8 (the Cube of 2) there will ariſe $\surd(3)64+\surd(3)27$, to which if you prefix the univerſal radical Sign of its Cube Root, it will be $\overline{\surd{:}(3)\surd(3)64+\surd(3)27}{:}$ which is the Quotient ſought.

Likewiſe, if it be requir'd to divide $\surd{:}\overline{aaaa+bbaa}$ by a, the Quotient will be found to be $\surd{:}\overline{aa+\surd bb}$; for the Square of the Dividend is $aaaa+bbaa$, and the Square of the Diviſor is aa, and when the Diviſion is ended, there will ariſe $aa+bb$, the univerſal ſquare Root of which is $\surd{:}\overline{aa+bb}$; which is the Quotient ſought.

But when the Work of Diviſion in univerſal ſurd Quantities happens to be intricate, and its Operation cannot be finiſh'd without a Remainder, you may ſet the Power of the Dividend for a Numerator, and the Power of the Diviſor for a Denominator, and againſt the Line of Separation, place or prefix the univerſal radical Sign, which univerſal Root ſo ſignified ſhall be the Quotient ſought.

As, if it were required to divide $\surd{:}\overline{\surd ab+bc}$ by $\surd{:}\overline{\surd a+c}$ the Quotient will be $\surd{:}\dfrac{\surd ab+bc}{\surd a+c}$.

III. *Addition and Subtraction in Univerſal Surd Quantities.*

I. If two univerſal ſurd Quantities that are commenſurable, are propoſed to be added together, or ſubtracted, the Operation may be performed like ſimple Surds: as for Example; If the Sum and Difference of $\surd{:}8+4\surd 3$ and $\surd{:}2+\surd 3$ *were required;*

Here each of the ſaid Quantities being divided by their greateſt common Meaſurer, $\sqrt{:2+\sqrt{3}}$, the Quotients are $\sqrt{4}$ and $\sqrt{1}$, *viz.* 2 and 1, which are rational Numbers expreſſing the Proportion of the Surds propounded; therefore if their common Diviſor be multiplied 2+1 (*viz.* 3.) it giveth $3\sqrt{:2+\sqrt{3}}$ for the Sum required; and the ſaid common Diviſor being multiplied by (2−1) the Difference of the ſaid 2 and 1, it will produce $\sqrt{:2+\sqrt{3}}$ for the Difference of the Roots propoſed.

Likewiſe, if it were requir'd to find the Sum and Difference of $\sqrt{:aaaa+aabb}$ and $\sqrt{:aabb+bbbb}$.

The ſaid Quantities being reduc'd, are $a\sqrt{:aa+bb}$ and $b\sqrt{:aa+bb}$.

Therefore is their Sum $\overline{a+b}\times\sqrt{:aa+bb}$; and their Difference is $\overline{a \backsim b}\sqrt{:aa+bb}$.

2. When the Root of a Reſidual is to be added to, or ſubtracted from the Root of its correſpondent Binomial, then may thoſe Roots be connected together by the Signs + and −; and then the whole being multiplied by itſelf, the univerſal Root of the Product ſhall be the Sum or Difference of the Roots propounded.

As, ſuppoſe $\sqrt{:12+\sqrt{6}}$ were propounded to be added to $\sqrt{:12-\sqrt{6}}$; the given Roots being connected together by +, make $\sqrt{:12+\sqrt{6}}+\sqrt{12-\sqrt{6}}$; which compoſed Quantity being multiplied by itſelf, produceth $24+2\sqrt{138}$, whoſe univerſal ſquare Root $(\sqrt{24+2\sqrt{138}})$ ſhall be the Sum of the Quantities propoſed to be added.

But if $\sqrt{:12+\sqrt{6}}-\sqrt{:12-\sqrt{6}}$ be multiplied *into* itſelf, the Product will be $24-2\sqrt{138}$, whoſe univerſal ſquare Root is the Difference of the two given Roots.

3. But if the univerſal Roots to be added or ſubtracted are not commenſurable, *&c.* then they are to be added by $+$, and ſubtracted by $-$.

So, if it were required to add $\surd:5+\surd 3$ to $\surd:3-\surd 2$, their Sum would be $\surd:5+\surd 3+\surd:3-\surd 2$. And the latter being ſubtracted from the former, the Remainder would be $\surd:5+\surd 3:-\surd 3:-\surd 2$. And the Sum of $\surd:ab+c$ being added to $\surd:d+b$ will be $\surd:ab+c+\surd:d+b$; and the latter being ſubtracted from the former, the Remainder will be $\surd:ab+c:-\surd:d+b$.

IV. *The Extraction of the Square Root out of Binomials and Reſiduals.*

Subtract the Square of the leſſer part of the given Binomial, from the Square of the greater part, and add the ſquare Root of the Remainder to the greater part, and alſo ſubtract it therefrom, and then extract the ſquare Roots of the Sum and Remainder, and join them together by $+$, if the Quantity propoſed be a Binomial, but by $-$ if it be a Reſidual; which Roots ſo joined, are the ſquare Root of the given Binomial, or Reſidual.

Example 1.

Extract the ſquare Root of this Binomial, *viz.*	$38+\surd 1300$
1. From the Square of the greater part 38, *viz.* from	1444

2. Sub-

2. Subtract the Square of the leſſer part, *viz.* $\sqrt{1300}$, which is	1300
3. The Remainder is	144
4. The ſquare Root of the Remainder is	12
5. To which Root if you add the greater part 38, the Sum is	50
6. The half of which Sum is	25
7. The ſquare Root of the ſaid half Sum is the greater part of the Root ſought, which is	5
8. From the greater part of the given Binomial, *viz.* 38, ſubtract the ſquare Root in the 4th Step, *viz.* 12, the Remainder is	26
9. The half of which is	13
10. The ſquare Root of the ſaid half Remainder is the leſſer part of the Root ſought	$\sqrt{13}$
11. To which if you add the Quantity in the 7th Step, the Sum will be the ſquare Root ſought, *viz.*	$5+\sqrt{13}$

which is the ſquare Root of the given Binomial: but if the given ſurd Quantity had been a Reſidual, *viz.* if it had been required to extract the ſquare Root of $38-\sqrt{1300}$, then the Root would have been $5-\sqrt{13}$.

Example 2.

Extract the ſquare Root of this Binomial, *viz.* $7+\sqrt{20}$

1. The Square of the greater part 7, is 49

2. From which ſubtract the Square of the leſſer part, *viz.* $\sqrt{20}$, which is 20

3. The Remainder is 29

4. The ſquare Root of that Remainder is $\sqrt{29}$

5. To which ſquare Root add the greater part of the given Binomial, *viz.* 7, and the Sum is $7+\sqrt{29}$

6. The half of which Sum is $\frac{7}{2}+\sqrt{\frac{29}{4}}$

7. The ſquare Root of the ſaid Sum is the greater part of the Root ſought, which is $\overline{\sqrt{:\frac{7}{2}+\sqrt{\frac{29}{4}}:}}$

8. From the greater part of the given Binomial, (*viz.* from 7) ſubtract $\sqrt{29}$ in the 4th Step, and the Remainder is $7-\sqrt{29}$

9. The half of which Remainder is $\frac{7}{2}-\sqrt{\frac{29}{4}}$

10. The ſquare Root of the ſaid half Remainder is the leſſer part of the Root ſought, which is $\overline{\sqrt{:\frac{7}{2}-\sqrt{\frac{29}{4}}:}}$

11. Which being joined to the greater part of the Root ſought in the 7th Step by the Sign $+$, the Sum will be the ſquare Root ſought, which is

$$\sqrt{:\tfrac{7}{2}+\sqrt{\tfrac{29}{4}}};+\sqrt{:\tfrac{7}{2}-\sqrt{\tfrac{29}{4}}}.$$

But if the leſſer part of the ſaid Root found in the 10th Step be joined to the greater part found in the 7th Step, by interpoſing the Sign $-$ inſtead of $+$, it will then be the ſquare Root of the Reſidual $7-\sqrt{20}$.

Example 3.

Let it be required to extract the ſquare Root of this binomial, *viz.* $aa+d$ added to $2a\sqrt{d}$, ſuppoſing the greater part of the given Binomial to be $aa+d$. Then,

1. The Square of the greater part is } $aaaa+2aad+dd$
2. From which ſubtract the Square of the leſſer part ($2a\sqrt{d}$) *viz.* $4daa$, and the Remainder is } $aaaa-2add+dd$
3. The ſquare Root of that Remainder is } $aa-d$
4. To which ſquare Root add the greater part of the given Binomial, *viz.* $aa+d$, and the Sum is } $2aa$
5. The half of which Sum is aa
6. The ſquare Root of which half Sum is the greater part of the Root ſought, which is } a

7. From

7. From $(aa+d)$ the greater part of the given Binomial ſubtract the ſquare Root found in the 3dStep$(aa-d)$ and the Remainder is	$2d$
8. The half of which Remainder is	d
9. The ſquare Root of which half Remainder is the leſſer part of the Root ſought, *viz.*	$\sqrt{d}$
10. Which ſaid Root being joined to the greater part found in the 6th Step by the Sign +, it will be the Root ſought, *viz.*	$a+\sqrt{d}$

But if the Quantity in the 9th Step be joined to the Quantity in the 6th Step, by interpoſing the Sign —, it will then be the ſquare Root of the Reſidual, $aa+d-2a\sqrt{d}$.

Example 4.

Let it be required to extract the ſquare Root of	$\overline{e+d}\sqrt{ed}+2ed$
Suppoſing the greater part to be	$\overline{e+d}\sqrt{ed}$, then
1. The Square of the greater part is	$eeed+2eedd+eddd$
2. From which ſubtract the Square of the leſſer part, which is	$4eedd$
3. And the Remainder is	$eeed-2eedd-eddd$
4. The ſquare Root of that Remainder is	$e-d\sqrt{ed}$

5. To which if you add the greater part of the given Binomial, the Sum is $2e\sqrt{ed}$
6. The half of which Sum is $e\sqrt{ed}$
7. The ſquare Root of the ſaid half Sum is the greater part of the Root ſought, which is $\sqrt{:e\sqrt{ed}}$
8. From the greater part of the given Binomial ſubtract the ſquare Root found in the 4th Step and the Remainder is $2d\sqrt{ed}$
9. The half of which Remainder is $d\sqrt{ed}$
10. The ſquare Root of the ſaid half Remainder is the leſſer part of the Root ſought, which is $\sqrt{:d\sqrt{ed}}$
11. If to the greater part of the Root ſough the 7th Step, you join the leſſer part in 10th Step, by interpoſing the Sign +, it then be the Root ſought, which is

$$\sqrt{:e\sqrt{ed}}+\sqrt{:d\sqrt{ed}}$$

But if the two ſaid Quantities are joined gether by the Interpoſition of the Sign —, it then be the ſquare Root of the Reſidual $e+d$ $-2ed$.

V. *Some Queſtions to exerciſe the Rules of this* *foregoing Chapters.*

Queſt. 1.

Let it be required to divide 100 (or c) i two ſuch unequal parts, that 100 multiplied the leſſer part may be equal to the Square of the greater.

Reſolution.

1. For the greater Number put a
2. Then will the leſſer be $c-a$
3. By which if you multiply 100 (or c) the Product will be $cc-ca$
4. Which Quantity in the 3d Step muſt be equal to the Square of the Quantity in the firſt Step, whence this Equation $aa=cc-ca$
5. Which Equation being reduced by the Rules of the 11th Chapter, and ſolved, the Value of a will be diſcover'd to be $a=\sqrt{:cc+\frac{cc}{4}}-\frac{1}{2}c$

which Equation in the laſt Step being duly conſider'd, will preſent you with this

Theorem.

To the Square of the given Line or Number add a fourth part of its Square, and extract the ſquare Root of that Sum; then from the ſaid ſquare Root ſubtract half the given Line, ſo ſhall the Remainder be the greater Segment or Number ſought.

Queſt. 2.

What Number is that, whoſe Square being made leſs by the Rectangle of itſelf drawn into 12 (or b) the Remainder is equal to f?

1. For the Number ſought, put a
2. The Square of which is aa
3. The Rectangle of a into b is ba
4. If the Quantity in the 3d Step be ſubtracted from the Quantity in the 2d Step, the Remainder is equal to f, whence this Equation $aa-ba=f$
5. Which Equation being ſolved by the Rules of the 11th Chapter, the Value of a will be found to be $a=\frac{1}{2}b+\sqrt{:f+\frac{bb}{4}}$

The Proof.

6. If $a=\frac{1}{2}b+\sqrt{:\overline{f+\frac{1}{4}bb}}$
7. Then by ſubtracting $\frac{1}{2}b$ from each part of the Equation, there remaineth $a-\frac{1}{2}b=\sqrt{:f+\frac{1}{4}bb}$.
8. Then by ſquaring each part of the Equation, you have $aa-ba+\frac{1}{4}bb=f+\frac{1}{4}bb$
9. And by ſubtracting $\frac{1}{4}bb$ from both ſides of the Equation, there remaineth $aa-ba=f$

which was to be proved.

Queſt. 10.

1. Let c and d be put for two ſuch known Quantities that d not $\sqsubset$ $\frac{1}{4}cc$, and let a be put for a Quantity unknown, and let it be granted that $ca-aa=d$, what is the Value of a?

2. The given Equation in the firſt Step is one of the 3d Form, mentioned in the beginning of the 15th Chapter, and it will be found that the two Values of a are

$a=\frac{1}{2}c+$

$$a=\tfrac{1}{2}c+\sqrt{:\tfrac{1}{4}cc-d}$$

And

$$a=\tfrac{1}{2}c-\sqrt{:\tfrac{1}{4}cc-d}$$

By either of which Values of *a* the Equation pounded in the first Step may be expounded, will appear by the

Demonstration.

If $a=\frac{1}{2}c+\sqrt{:\frac{1}{4}cc-d}$

Then by the Transposition of $\frac{1}{2}c$ to the contrary Coast, it is $a-\frac{1}{2}c=\sqrt{:\frac{1}{4}cc-d}$

And by squaring each part of the last Equation, it is $aa-ca+\frac{1}{4}cc=\frac{1}{4}cc-d$

And by subtracting $\frac{1}{4}cc$ from each part of the Equation, it is $aa-ca=-d$

And by changing the Signs of the Quantities on each ide of the Equation, it is $ca-aa=d$

hich was to be demonstrated.

Again, if $a=\frac{1}{2}c-\sqrt{:\frac{1}{4}cc-d}$

Then by Transposition of $-\sqrt{:\frac{1}{4}cc-d}$ to the other ide, it is $a+\sqrt{:\frac{1}{4}cc-d}=\frac{1}{4}cc$

And by Transposition of a, it is $\sqrt{:\frac{1}{4}cc-d}=\frac{1}{4}cc-a$

And by squaring each part of the Equation, it will then be $\frac{1}{4}cc-d=\frac{1}{4}cc-ca+aa$

10. And by ſubtracting $\frac{1}{4}cc$ from both parts of the Equation, it is $-d=-ca+aa$

11. And the Quantities on both ſides of the Equation being tranſpoſed to the contrary Coaſt, and the Signs of each thereby changed, the Equation will then be $ca-aa=d$

Which was likewiſe to be proved.

Queſt. 2.

Let it be required to divide 100 into two ſuch parts, that if each part be divided by the other, the Sum of the Quotients may be 3. This is *Qu.* 1. of the 9th Chapter of the 2d Book of *Kerſey*'s Elements of Algebra, and it is thus wrought, *viz.*

1. For one of the parts ſought, put a

2. Then will the other be $100-a$

3. Each of which Quantities in the firſt and ſecond Steps being mutually divided by each other (according to the Import of the Queſtion) this Equation ariſeth $\frac{a}{100-a}+\frac{100-a}{a}=3$

4. Which Equation being duly reduced, gives $100a-aa=2000$

5. Which is an Equation of the third Form mentioned in Chapter 15, and being ſolved according to the Method there given, the two Values of it will be found to be $a=\begin{cases}50+10\sqrt{5}\\50-10\sqrt{5}\end{cases}$

Which you may eaſily prove at your leiſure.

CHAP.

CHAP. XIX.

lgebraical Queſtions reſolved by various Poſitions.

MR. *Kerſey*, in the 12th Chapter of the 2d Book of his Elements of Algebra, hath id down Rules for the Solution of Queſtions al- braically; by various Poſitions; aſſuming a culiar Letter to repreſent every one of the uantities ſought, *viz.* *a* for one unknown Quan- ty, *e* for another, and *y* for a third, *&c.* and for e performance of the Work he hath laid down ree Rules, which are as followeth, *viz.*

Rule 1.

When many Quantities are ſought in a Queſ- on, let them be repreſented by various Let- rs, and let the Tenour of the Queſtion be repre- nted by Equations; which done, by Tranſpoſi- on find what any ſingle Letter in the firſt Equa- on is equal to; then whereſoever that Letter found in the other Equations, inſtead thereof ke what it is found equal to, ſo will that Letter ite vaniſh out of the following Equations; en by Tranſpoſition ſet a ſecond Letter alone one of thoſe Equations out of which the firſt *etter was* cancelled, and proceed as before, ſo at leng length one of the Letters will be made known, by help of which the reſt will be eaſily diſcovered.

Rule 2.

When the ſame Quantity (ſuppoſe *a*) is found in two ſeveral Equations, and equal Numbers are prefixed to thoſe Quantities, then if their Signs be both +, or both −, ſubtract the leſſer Equation from the greater; but if the Signs be one +, and the other −, then add thoſe two Equations together, ſo will the ſaid Quantity *a* quite vaniſh.

Rule 3.

When the ſame Quantity (ſuppoſe *a*) is found in two ſeveral Equations, but the Numbers prefixed to thoſe equal Quantities are unequal, thoſe two Equations may be reduced to two others which ſhall have equal Numbers prefixed to the ſaid Quantity *a*, thus, *viz.* multiply all the Quantities in the firſt Equation by the Number prefixed to *a* in the ſecond Equation; and alſo multiply all the Quantities in the ſecond Equation by the Number prefixed to the ſame Quantity *a* in the firſt Equation; ſo by ſuch alternate multiplication two new Equations will be produced, wherein the Numbers prefixed to the ſaid Quantity *a* will be equal to one another; and then proceed according to the ſecond Rule, and expel the ſame Quantity out of the reſt of the Equations: proceed in like manner with a ſecond Quantity, till at length ſome one Quantity be made known; by which all the reſt may be found out. The three foregoing Rules will be exerciſed in the Reſolution of the following Queſtions.

Queſt. 1.

Divide 100 (or c) into two ſuch Numbers, (*viz.* a and e) that $\frac{a}{3}+\frac{e}{5}$ may be equal to 30 (or d) I demand the Numbers a and e.

Reſolution.

1. If $a+e=c$
2. And $\frac{a}{3}+\frac{e}{5}=d$
3. Then by Tranſpoſition of e in the firſt Step, you will have $a=c-e$
4. By reducing the Equation in the 2d Step, ſo as a may ſolely poſſeſs one ſide thereof, you will have $a=\frac{15d-3e}{5}$
5. If inſtead of a in the 4th Step, you take what a is equal to in the third Step, you will have this Equation, *viz.* $c-e=\frac{15d-3e}{5}$
6. The firſt part of the Equation in the 5th Step being multiplied by 5, will give $5c-5e=15d-3e$
7. By the tranſpoſition of $-5e$ it is $5c=15d+2e$
8. And by the Tranſpoſition of $15d$ in the laſt Step, you have $5c-15d=2e$

9. Each part of the Equation in the laſt Step being divided by 2, will give the Value of *e*, *viz.* $\frac{5c-15d}{2}=e$.

I ſay, the Value of *e* is 25, and the Value of *a* is $100-25=75$, which will anſwer the Conditions of the Queſtion; as appears by

The Proof.

$$25+75=100, \text{ and } \tfrac{25}{5}+\tfrac{75}{3}=30$$

Queſt. 2.

There are two Numbers (*a* the greater, and *e* the leſſer) whoſe Difference is 4 (or *b*) and the Difference of their Squares is 64 (or *c*) what are the Numbers?

Reſolution.

1. If $a-e=b$
2. Then by the Tranſpoſition of *e* you have $a=b+e$
3. And if $aa-ee=c$
4. Then by Tranſpoſition of *ee*, it is $aa=c+ee$
5. If both parts of the Equation in the ſecond Step be ſquared, it will be $aa=bb+2be+ee$.
6. And if inſtead of *aa* in the 5th Equation, you place what it is equal to in the 4th Step, the Equation will then be

$$c+ee=bb+2be+ee$$

7. By ſubtracting ee from both parts of the laſt Equation you will have $c=bb+2be$

8. By dividing both parts of the laſt Equation by b, you will then have $\frac{c}{b}=b+2e$

9. By Tranſpoſition of b in the laſt Step, the Equation will be $\frac{c}{b}-b=2e$

10. And if both parts of the Equation in the laſt Step be divided by 2, the Value of e will then be diſcover'd to be $\frac{c}{2b}-\frac{b}{2}=e$

I ſay, the Value of e (the leſſer Number) is 6, and by the 2d Step (*a*) the greater Number is $e+b=6+4=10$; which two Numbers (*viz.* 10 and 6) will ſatisfy the Conditions of the Queſtion, as will appear by

The Proof.

$$10-6=4,\ \text{and}\ \overline{10\times10}-\overline{6\times6}=64$$

Queſt. 3.

A Maid being at Market, ſold 10 dozen of Eggs and 12 Pounds of Butter for 13 Shillings; and at another time, and at the ſame rate, ſhe ſelleth 8 dozen of Eggs and 18 Pounds of Butter for 16 Shillings; I demand how ſhe ſold her Eggs *per* dozen, and her Butter *per* Pound.

Let a repreſent the deſired Value of a dozen of Eggs, and for the Price of a Pound of Butter put *e*, and then may the Queſtion, being abſtracted from Words, be ſtated thus, *viz.*

1. If $10a+12e=13$
2. And $8a+18e=16$

What are the Values of *a* and *e*?

Reſolution.

3. By Tranſpoſition of 12*e* in the firſt Step, that Equation will be $10a=13-12e$
4. And both parts of the laſt Equation being divided by 10, it is $a=\dfrac{13-12e}{10}$
5. By Tranſpoſition of 18*e* in the 2d Step, that Equation will be $8a=16-18e$
6. Each part of the laſt Equation being divided by 8, will give $a=\dfrac{16-18e}{8}$
7. If inſtead of *a* in the 6th Step, you place what it is equal to in the 4th Step, the Equation will then be $\dfrac{13-12e}{10}=\dfrac{16-18e}{8}$
8. Both parts of the laſt Equation being reduced to Integers, will give $104-96e=160-180e$
9. By Tranſpoſition of 180*e* and 104 in the laſt Equation each to the contrary Coaſt, the Equation will be $84e=56$

10. If each part of the laſt Equation be divided by 84, the Value of e will be diſcovered to be $\left.\right\}$ $e=\frac{56}{84}=\frac{2}{3}$.

which is 8*d.* for the Price of one Pound of Butter.

11. By the 10th Step the Value of e is diſcover'd to be $\frac{2}{3}s.$ by which means the Value of a (by the Quantities in the 4th and 6th Steps) is found to be 6 *d.* For the 4th Step is $\left.\right\}$ $a=\frac{13-12e}{10}$

And it hath been found before, that $e=\frac{2}{3}s.$ ſo that $12e=12\times\frac{2}{3}s.=8s.$ and $\frac{13s.-8s.}{10}=\frac{5}{10s.}=6d.$ So that the Maid ſold her Eggs at 6 *d. per* doz. and her Butter at 8 *d. per* Pound, which will anſwer the Conditions of the Queſtion.

Queſt. 4.

Three Men, *viz.* *A*, *B*, and *C*, diſcourſe thus together concerning their Age: Quoth *B* to *A*, your Age added to mine is 54 (or *b*) Years, Quoth *C* to *B*, and my Age added to yours makes 78 (or *c*) Years. And quoth *A* to *C*, my Age added to yours is 72 (or *d*) Years. I demand the Age of each Perſon?

Let the Age of each Perſon be repreſented by the three Letters *a*, *e*, *y*, *viz.* for the Age of *A* put *a*, for the Age of *B* put *e*, and for the Age of *C* put *y*; and the Queſtion being abſtracted from Words, will be as followeth, *viz.*

1. If $a+e=b(=54)$
2. And $e+y=c(=78)$
3. And $y+a=d(=72)$

What are the Values of *a*, *e* and *y*?

Reſolution.

4. By Tranſpoſition of *e* in the firſt Step, there will ariſe $a=b-e$

5. If inſtead of *a* in the third Step you put what *a* is equal to in the 4th Step, there will ariſe $y+b-e=d$

6. By the Tranſpoſition of *d* and *e* in the laſt Step, there ariſeth $e=y+b-d$

7. And if inſtead of *e* in the 2d Step you take the latter part of the 6th Step, there will then ariſe $2y+b-d=c$

8. In which laſt Equation there is no unknown Quantity but *y*; and therefore the Equation being duly reduc'd, will diſcover the Value of *y* to be $y=\frac{c+d-b}{2}$

9. If in the 6th Step inſtead of *y* you take the latter part of the Equation in the 8th Step, the Value of *e* will be found to be $e=\frac{c+b-d}{2}$

10. And if inſtead of *e* in the 4th Step you take the latter part of the Equation in the 9th Step, the Value of *a* will be diſcover'd, *viz.* $a=\frac{b+d-c}{2}$

And thus the Work is finiſhed, and the Equations in the 8th, 9th, and 10th Steps preſent you with this

Canon.

From the Sum of every two of the three given Numbers ſubtract the 3d Number remaining, ſo ſhall the three remaining Numbers, being divided by 2, be the Numbers ſought. So the Numbers ſought in the Queſtion, *viz.* a, e and y, are found to be 24, 30, and 48, *viz.* the Age of *A* is 24, the Age of *B* is 30, and the Age of *C* is 48; which three Numbers will ſatisfy the Conditions of the Queſtion; for $24+30=54$, and $30+48=78$, and $48+24=72$.

Queſt. 5.

What two Numbers are thoſe whoſe Sum is 20 (or b) and their Difference 4 (or c)?

Let a be put for the greater Number ſought, and e for the leſſer; and then the Queſtion being abſtracted from Words, may be ſtated thus, *viz.*

1. If $a+e=b(20)$

2. And $a-e=c(4)$

What are a and e?

Reſolution.

3. Foraſmuch as $+1\,a$ is found in each of the Equations in the 1ſt and 2d Steps, therefore (by the 2d Rule) they being ſubtracted, do give *this Equation*, *viz.* $2e=b-c$

4. And by dividing both parts of the Equation in the 3d Step by 2, the Value of *e* will be diſcover'd, *viz.* $e=\frac{1}{2}b-\frac{1}{2}c$

5. And if inſtead of *e* in the 2d Step you put what *e* is equal to in the 4th Step, you will have this Equation, *viz.* $a-\frac{1}{2}b+\frac{1}{2}c=c$

6. By the tranſpoſition of $-\frac{1}{2}b$ and $+\frac{1}{2}c$ to the contrary Coaſt, the Value of *a* will be diſcover'd, *viz.* $a=\frac{1}{2}b+\frac{1}{2}c$

From the 4th and 6th Steps is raiſed this

Canon.

If from half the Sum of two Numbers you ſubtract half their Difference, the Remainder will be the leſſer Number; and if to half their Sum you add half their Difference, that Sum will be the greater Number; whereby the two Numbers ſought in this Queſtion are found to be 12 and 8, for 12+8=20, and 12—8=4.

Queſt. 6.

What 3 Numbers are thoſe, that if to the firſt there be added 121 (or *b*) the Sum will be equal to the Sum of the ſecond and third; and if to the ſecond there be added 121, the Sum will be equal to double the Sum of the firſt and third; and if to the third there be added 121, their Sum will be triple the Sum of the firſt and ſecond?

If for the Numbers ſought you put *a*, *e*, and *y*, *viz.* for the firſt Number *a*, for the ſecond *e*,

an

r the third y, then the Queſtion being ab-
l from Words, may be ſtated thus, *viz.*

$a+b=e+y$

l $e+b=2a+2y$

d $y+b=3a+3e$

What are the Numbers a, e, y?

Reſolution.

the Tranſpoſition of y
ıe firſt Equation, there
:th
$a+b-y=e$

l if inſtead of e in the 2d Equation you
: what is equal thereto in the 4th Equation,
'e ariſeth

$$a+b-y+b=2a+2y.$$

: laſt Equation, after due
luction, will be
$2b=a+3y$

d if inſtead of $3e$ in the 3d Equation you
: the triple of what e is found equal to in
4th Step, you will have the following
ıation to ariſe, *viz.*

$$y+b=3a+3a+3b-3y$$

hich Equation being
, reduced by Tranſpoſi-
ı, the Quantities will be
ıd to be
$4y=6a+2b$

d both parts of the laſt
ıation being divided by
there ariſeth
$y=\frac{1}{2}a+\frac{1}{2}b$

10. Then if inſtead of $3y$ in the 6th Equation there be taken the triple of the latter part of the 9th Equation, there ariſeth $2b = a + \frac{9}{2}a + \frac{3}{2}b$

11. After due Reduction of the Equation in the 10th Step, the Value of a will be diſcover'd, *viz.* $a = \frac{b}{11}$

12. Again, if inſtead of a in the 6th Equation, you put the latter part of the 11th Equation, there ariſeth $2b = \frac{b}{11} + 3y$

13. After due Reduction of the Equation in the 12th Step, the Value of y will be diſcover'd to be $y = \frac{7be}{11}$

14. And if for a and y in the 4th Step there be put their Equals in the 11th and 13th Steps, there will ariſe $\frac{b}{11} + \frac{7b}{11} =$

15. The Equation in the laſt Step being duly reduc'd, will diſcover the Value of e, *viz.* $e = \frac{5b}{11}$

From the 11th, 13th, and 15th Steps, is gathered this

Canon.

If the Number given to be added to the three Numbers required be divided by 11, the Quotient will give the firſt Number, and its Quintuple (or Product by 5) being divided by 11, will give the 2d Number, and its Septuple (or Product by 7) being divided by 11, will give the third Number.

By which Canon the Numbers requir'd in the Queſtion are 11, 55, and 77, the ſecond being 5 times as much as the firſt, and the third is 7 times as much as the firſt; which ſaid Numbers will ſatisfy the Conditions of the Queſtion, as appears by

The Proof.

$11+121=55+77=132$

And $55+121=2\times\overline{11+77}=176$

And $77+121=3\times\overline{11+55}=198$.

Which was to be done.

Queſt. 7.

What 2 Numbers are thoſe, that if to ten times the greater there be added ſix times the leſſer, the Sum will be 228 (or b;) and if from 4 times the greater you ſubtract two times the leſſer, the Remainder will be 56 (or c)? For the two Numbers put a and e, and then the foregoing Queſtion being abſtracted from Words, may be ſtated thus, *viz.*

1. If $10a+6e=b$
2. And $4a-2e=c$

What are the Numbers a and e?

Reſolution.

3. The firſt Equation (according to the 3d Rule) being multiplied by 4, which is prefixed to a in the 2d Equation, produceth $\Big\}\ 40a+24e=4b$

4. And the 2d Equation being multiplied by 10, which is prefixed to *a* in the firſt, it produceth

$$40a-20e=10$$

5. And if from the Equation in the 3d Step you ſubtract the Equation in the 4th Step, becauſe 40*a* is found in both, (according to the 2d Rule) there ariſeth this Equation, *viz.*

$$44e=4b-10c$$

6. Both parts of the Equation in the 5th Step being divided by 44, the Value of *e* will be diſcover'd to be

$$e=\frac{4b-10c}{44}$$

7. If inſtead of $-2e$ in the 2d Step you put double the latter part of the Equation in the 6th Step, you will have this Equation

$$4a-\frac{2b-5c}{11}=$$

8. The 7th Equation being duly reduc'd, the Value of *a* will be diſcover'd to be

$$a=\frac{6c+2b}{44}$$

By the 6th and 8th Steps the Numbers ſoug are 18 and 8, which will anſwer the Conditi of the Queſtion: as you may perceive by

The Proof.

$$10\times18+6\times8=228$$

And

$$4\times18-2\times8=56$$

Soli Deo Gloria.

FINIS.